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Section 1 – Expressions

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.A-APR.1.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MAFS.912.A-SSE.1.1	Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
MAFS.912.A-SSE.1.2	Use the structure of an expression to identify ways to rewrite it.
MAFS.912.N-RN.1.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
MAFS.912.N-RN.1.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MAFS.912.N-RN.2.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Topics in this Section

Section 1 – Topic 1
Using Expressions to Represent Real-World Situations

Jenny tweets 33 times a day. Antonio posts five tweets every day. Let d represent the given number of days.

Use an algebraic expression to describe Jenny's total posts after any given number of days.

Create an algebraic expression to describe Antonio's total posts after any given number of days.

Write an algebraic expression to describe the combined total posts for Jenny and Antonio after any given number of days.

After five days, how many tweets have Antonio and Jenny posted altogether?

Let's Practice!

1. Mario and Luigi plan to buy a Wii U™ for $299.00. Wii U™ games cost $59.99 each. They plan to purchase one console.

   a. Use an algebraic expression to describe how much they will spend before sales tax based on purchasing the console and the number of games.

   b. If they purchase one console and three games, how much do they spend before sales tax?

   c. Mario and Luigi want to purchase some extra controllers for their friends. Each controller costs $29.99. Use an algebraic expression to describe how much they spend in total, before sales tax, based on purchasing the console, the number of games, and the number of extra controllers.

d. What is the total cost, before sales tax, if Mario and Luigi purchase one console, three games, and two extra controllers?

STUDY EDGE TIP

When defining variables, choose variables that make sense to you, such as h for hours and d for days.

Try It!

2. Micah and Crystal purchase two movie tickets. Tickets cost $\$8.50$ each, drinks cost $\$3.50$ each, and boxes of candy cost $\$3.00$ each. Use an algebraic expression to describe how much they spend based on the number of ticekts, drinks and boxes of candy they buy. Identify the parts of the expression by underlining the coefficient(s), circling the constant(s), and drawing a box around the variable(s).

BEAT THE TEST!

1. José is going to have the exterior of his home painted. He will choose between Krystal Klean Painting and Elegance Home Painting. Krystal Klean Painting charges $\$175.00$ to come out and evaluate the house plus $\$14.00$ per hour. Elegance Home Painting charges $\$23.00$ per hour. Let h represent the number of hours for which José hires a painter. Which of the following statements are true? Select all that apply.

   - ☐ The expression $14h$ represents the total charge for Krystal Klean Painting.
   - ☐ The expression $23h$ represents the total charge for Elegance Home Painting.
   - ☐ The expression $175 + 14h + 23h$ represents the total amount José will spend for the painters to paint the exterior of his home.
   - ☐ If José hires the painters for 10 hours, Elegance Home Painting would be cheaper.
   - ☐ If José hires the painters for 20 hours, Krystal Klean Painting would be cheaper.

2. The Griffin family wants to buy an Xbox One + Kinect Sensor for $399.00. They also want to buy accessories and games. The wireless controllers cost $49.99 each. The headsets cost $79.00 each. The games cost $59.99 each. Peter and Lois are trying to decide how many accessories and games to buy for their family. Let x represent the number of wireless controllers, y represent the number of headsets, and z represent the number of games the Griffins will purchase. Which of the following algebraic expressions can be used to describe how much the Griffins will spend, before sales tax, based on the number of accessories and games they purchase?

Ⓐ $399 + 188.98(x + y + z)$
Ⓑ $399(49.99x + 79.00y + 59.99z)$
Ⓒ $399 + 49.99x + 79.00y + 59.99z$
Ⓓ $399x + 49.99y + 79.00z + 59.99$

Section 1 – Topic 2
Understanding Polynomial Expressions

A ***term*** is a constant, variable, or multiplicative combination of the two.

Consider $3x^2 + 2y - 4z + 5$.

How many terms do you see?

List each term.

This is an example of a ***polynomial expression***. A polynomial can be one term or the sum of several terms. There are many different types of ***polynomials***.

A monarchy has one leader. How many terms do you think a monomial has?

A bicycle has two wheels. How many terms do you think a binomial has?

A triceratops has three horns. How many terms do you think a trinomial has?

Let's recap:

Type of Polynomial	Number of Terms	Example
Monomial		
Binomial		
Trinomial		
Polynomial		

Some important facts:

- The ***degree of a monomial*** is the sum of the ____________ of the variables.
- The ***degree of a polynomial*** is the degree of the monomial term with the ____________ degree.

Sometimes, you will be asked to write polynomials in standard form.

- Write the monomial terms in ________________ _________ order.
- The ***leading term*** of a polynomial is the term with the ________________ _____________.
- The ***leading coefficient*** is the coefficient of the _____________ _________.

Let's Practice!

1. Are the following expressions polynomials? If so, name the type of polynomial and state the degree. If not, justify your reasoning.

   a. $8x^2y^3$

   b. $\frac{2a^2}{3b}$

   c. $\frac{3}{2}x^4 - 5x^3 + 9x^7$

   d. $10a^6b^2 + 17ab^3c - 5a^7$

   e. $2m + 3n^{-1} + 8m^2n$

Try It!

2. Are the following expressions polynomials?

a. $\frac{1}{2}a + 2b^2$

- ○ polynomial
- ○ not a polynomial

b. 34

- ○ polynomial
- ○ not a polynomial

c. $\frac{xy}{y^2}$

- ○ polynomial
- ○ not a polynomial

d. $2rs + s^4$

- ○ polynomial
- ○ not a polynomial

e. $xy^2 + 3x - 4y^{-1}$

- ○ polynomial
- ○ not a polynomial

3. Consider the polynomial $3x^4 - 5x^3 + 9x^7$.

a. Write the polynomial in standard form.

b. What is the degree of the polynomial?

c. How many terms are in the polynomial?

d. What is the leading term?

e. What is the leading coefficient?

BEAT THE TEST!

1. Match the polynomial in the left column with its descriptive feature in the right column.

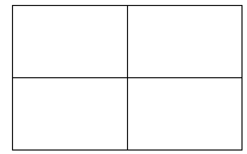
A. $x^3 + 4x^2 - 5x + 9$	I. Fifth degree polynomial
B. $5a^2b^3$	II. Constant term of -2
C. $3x^4 - 9x^3 + 4x^9$	III. Seventh degree polynomial
D. $7a^6b^2 + 18ab^3c - 9a^7$	IV. Leading coefficient of 3
E. $x^5 - 9x^3 + 2x^7$	V. Four terms
F. $3x^3 + 7x^2 - 11$	VI. Eighth degree polynomial
G. $x^2 - 2$	VII. Equivalent to $4x^9 + 3x^4 - 9x^3$

Section 1 – Topic 3
Algebraic Expressions Using the Distributive Property

Recall the ***distributive property***.

➢ If $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are real numbers, then
$\boldsymbol{a}(\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \cdot$ ____ $+ \boldsymbol{a} \cdot$ ____ .

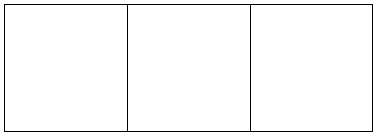
One way to visualize the distributive property is to use models. Consider $(a+3)(a+2)$.



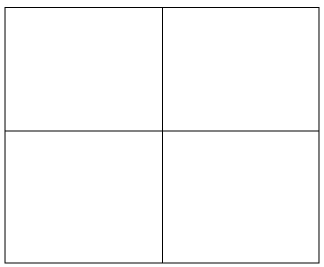
Now, use the distributive property to write an equivalent expression for $(a+3)(a+2)$.

Let's Practice!

1. Write an equivalent expression for $3(x + 2y - 7z)$ by modeling and then by using the distributive property.

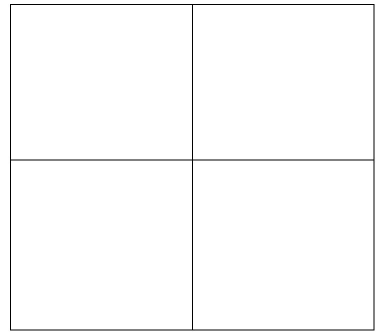


2. Write an equivalent expression for $(x - 3)(x - 2)$ by modeling and then by using the distributive property.



Try It!

3. Use the distributive property or modeling to write an equivalent expression for $(m + 5)(m - 3)$.



BEAT THE TEST!

1. Students were asked to use the distributive property to write an equivalent expression for the expression $(x-5)(x-2)$. Their work is shown below. Identify the student with the correct work. For the problems that are incorrect, explain where the students made mistakes.

Student 1

$(x-5)(x-2)$
$= x \cdot x - 5(-2)$
$= x^2 + 10$

Student 2

$(x-5)(x-2)$
$= xx - 2x - 5x - 5(-2)$
$= x^2 - 2x - 5x + 10$
$= x^2 - 7x + 10$

Student 3

$(x-5)(x-2)$
$= x \cdot x + x(-2) - 5 \cdot x - 5(-2)$
$= x^2 - 2x - 5x - 10$
$= x^2 - 7x - 10$

Section 1 – Topic 4
Algebraic Expressions Using the Commutative and Associative Properties

What is $5 + 2$? What is $2 + 5$?

Does it matter which number comes first?

What is $9 \cdot 2$? What is $2 \cdot 9$?

Does it matter which number comes first?

This is the ***commutative property***.

- The order of the numbers can be ____________ without affecting the _________ or ______________.
- If a and b are real numbers, then $a + b =$ ________ and/or $a \cdot b =$ _________.

Does the commutative property hold true for division or subtraction? If so, give an example. If it does not, give a counterexample.

Let's look at some other operations and how they affect numbers.

Consider $2 + 4 + 6$. What happens if you put parentheses around any two adjacent numbers? How does it change the sum?

Consider $3 \cdot 6 \cdot 4$. What happens if you put parentheses around any two adjacent numbers? How does it change the product?

This is the ***associative property***.

- The ____________ of the numbers does not change.
- The grouping of the numbers can change and does not affect the ___________ or ______________.
- If a, b and c are real numbers, then $(a + b) + c =$ _______________ and/or $(ab)c =$ __________.

Does the associative property hold true for division or subtraction? If it does not, give a counterexample.

Let's Practice!

1. Name the property (or properties) used to write the equivalent expression.

   a. $[5 + (-3)] + 2 = 5 + [(-3) + 2]$

   b. $(8 \cdot 4) \cdot 6 = 6 \cdot (8 \cdot 4)$

   c. $p + (u + t) = (t + u) + p$

STUDY EDGE TIP

With properties, look closely at each piece of the problem. The changes can be very subtle.

Try It!

2. Identify the property (or properties) used to find the equivalent expression.

   a. $(11 + 4) + 5 = (5 + 11) + 4$

   b. $v \cdot (y \cdot b) = (v \cdot y) \cdot b$

   c. $(8 + 1) + 6 = 8 + (1 + 6)$

   d. $(9 \times 13) \times 14 = (13 \times 9) \times 14$

3. The following is a proof that shows $(3x)(2y)$ is equivalent to $6xy$. Fill in each blank with either "commutative property" or "associative property" to indicate the property being used.

$(3x)(2y) = 3(x \cdot 2)y$ ______________________________

$= 3(2 \cdot x)y$ ______________________________

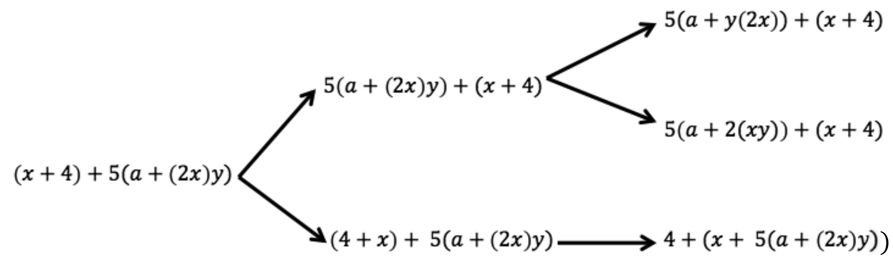
$= (3 \cdot 2)(x \cdot y)$ ______________________________

$= 6xy$

BEAT THE TEST!

1. Underline the differences in each step below. Then, use these abbreviations for the properties of real numbers to justify each step of the flow chart:

   C_+ for the commutative property of addition
   $C_\times$ for the commutative property of multiplication
   A_+ for the associative property of addition
   $A_\times$ for the associative property of multiplication



Section 1 – Topic 5
Properties of Exponents

Let's review the properties of exponents.

$2^4 =$

$2^3 =$

$2^2 =$

$2^1 =$

What pattern do you notice?

Continuing the pattern, what would the following equal?

$2^0 =$

- This is the ***zero exponent property***: $a^0 =$ ___ .

Continuing the pattern, what would the following equal?

$2^{-1} =$

$2^{-2} =$

- This is the ***negative exponent property***: $a^{-n} =$ ______ and $\frac{1}{a^{-n}}=$______ .

Let's explore multiplying expressions with exponents and the same base.

$2^3 \cdot 2^4 =$

$2^5 \cdot 2^{-3} =$

$x^3 \cdot x^2 =$

- This is the ***product property***: $a^m \cdot a^n =$ _______ .

Let's explore dividing expressions with exponents and the same base.

$\frac{4^5}{4^3} =$

$\frac{x^7}{x^8} =$

- This is the ***quotient property***: $\frac{a^m}{a^n} =$ ___________ .

Let's explore raising expressions with exponents to a power.

$(5^3)^2 =$

$(y^4)^3 =$

- This is the ***power of a power property***: $(a^m)^n =$ _______ .

Let's explore raising products to a power.

$(2 \cdot 3)^2 =$

$(4 \cdot x)^3 =$

- This is the ***power of a product property***: $(ab)^n =$ ________.

Let's explore raising quotients to a power.

$\left(\frac{20}{3}\right)^2 =$

$\left(\frac{6}{y}\right)^3 =$

- This is the ***power of a quotient property***: $\left(\frac{a}{b}\right)^n =$ _______.

Let's Practice!

1. Determine if the following equations are true or false. Justify your answer.

   a. $3^3 \cdot 3^4 = \frac{(3^9)}{(3^2)}$

   b. $(5 \cdot 4^2)^3 = 5^4 \cdot 5^0 \cdot \left(\frac{4^6}{5^{-1}}\right)^{-1}$

Try It!

2. Use the properties of exponents to match each of the following with its equivalent expression.

A. $\left(\frac{7}{2}\right)^4$

B. $(7 \cdot 2^2)^3$

C. $(7^2)(7^2)$

D. $(7^2)(7)^0$

E. $\left(\frac{7}{2}\right)^{-4}$

F. $\frac{(7^6)}{(7^3)}$

I. $7^3 \cdot 2^6$

II. $\frac{7^4}{2^4}$

III. $\frac{2^4}{7^4}$

IV. 7^4

V. 7^2

VI. 7^3

BEAT THE TEST!

1. Crosby and Adam were working with exponents.

Part A: Crosby claims that $3^3 \cdot 3^2 = 3^5$. Adam argues that $3^3 \cdot 3^2 = 3^6$. Which one of them is correct? Use the properties of exponents to justify your answer.

Part B: Crosby claims that $\frac{3^8}{3^2} = 3^4$. Adam argues that $\frac{3^8}{3^2} = 3^6$. Which one of them is correct? Use the properties of exponents to justify your answer.

Section 1 – Topic 6
Radical Expressions and Expressions with Rational Exponents

Exponents are not always in the form of integers. Sometimes, you will see them expressed as rational exponents.

Use the properties of exponents to write the following expressions with rational exponents as expressions with integer exponents.

$9^{\frac{1}{2}} =$

$8^{\frac{1}{3}} =$

Do you notice a pattern? If so, what pattern did you notice?

Use this pattern and the properties of exponents to write the following expressions with rational exponents as radical expressions.

$2^{\frac{2}{3}} =$

$5^{\left(\frac{3}{2}\right)} =$

- This is the ***rational exponent property:***
  $\boldsymbol{x^{\frac{a}{b}}} =$ __________ $=$ __________.

Let's Practice!

1. Use the rational exponent property to write an equivalent expression for each of the following radical expressions.

   a. $\sqrt{x+2}$

   b. $\sqrt[3]{x-5}+2$

2. Use the rational exponent property to write each of the following expressions as integers.

   a. $9^{\frac{1}{2}}$

   b. $16^{\frac{1}{2}}$

   c. $8^{\frac{1}{3}}$

   d. $8^{\frac{2}{3}}$

   e. $125^{\frac{2}{3}}$

   f. $16^{\frac{3}{4}}$

Try It!

3. Use the rational exponent property to write an equivalent expression for each of the following radical expressions.

   a. $\sqrt{y}$

   b. $\sqrt[5]{y+6}-3$

4. Use the rational exponent property to write each of the following expressions as integers.

   a. $49^{\frac{1}{2}}$

   b. $27^{\frac{1}{3}}$

   c. $216^{\left(\frac{2}{3}\right)}$

BEAT THE TEST!

1. Match each of the following to its equivalent expression.

A.	$2^{\frac{1}{3}}$	I.	$m^{\frac{1}{2}}-3$
B.	$\sqrt{m-3}$	II.	$(3m)^{\frac{1}{2}}$
C.	$2^{\frac{2}{3}}$	III.	$(m-3)^{\frac{1}{2}}$
D.	$\sqrt{m}-3$	IV.	$\sqrt{2}$
E.	$2^{\frac{1}{2}}$	V.	$\sqrt[3]{4}$
F.	$\sqrt{3m}$	VI.	$\sqrt[3]{2}$

Section 1 – Topic 7
Adding Expressions with Radicals and Rational Exponents

Let's explore operations with radical expressions and expressions with rational exponents.

$\sqrt{5}+\sqrt{2}$

$5^{\frac{1}{2}}+2^{\frac{1}{2}}$

$\sqrt{5}+\sqrt{5}$

$5^{\frac{1}{2}}+5^{\frac{1}{2}}$

$2\sqrt{3}-8\sqrt{3}$

$2\cdot 3^{\frac{1}{2}}-8\cdot 3^{\frac{1}{2}}$

STUDY EDGE TIP

To add radicals, the radicand of both radicals must be the same. To add expressions with rational exponents, the base and the exponent must be the same. In both cases, you simply add the coefficients.

Let's Practice!

1. Perform the following operations.

   a. $\sqrt{12}+\sqrt{3}$

   b. $12^{\frac{1}{2}}+3^{\frac{1}{2}}$

   c. $\sqrt{72}+\sqrt{15}+\sqrt{18}$

   d. $72^{\frac{1}{2}}+15^{\frac{1}{2}}+18^{\frac{1}{2}}$

   e. $\sqrt{32}+\sqrt[3]{16}$

   f. $32^{\frac{1}{2}}+16^{\frac{1}{3}}$

STUDY EDGE TIP

For radicals and expressions with rational exponents, always look for factors that are perfect squares when taking the square root (or perfect cubes when taking the cube root).

Try It!

2. Perform the following operations.

   a. $\sqrt{6} + 3\sqrt{6}$

   b. $6^{\frac{1}{2}} + 3 \cdot 6^{\frac{1}{2}}$

   c. $\sqrt{50} + \sqrt{18} + \sqrt{10}$

   d. $50^{\frac{1}{2}} + 18^{\frac{1}{2}} + 10^{\frac{1}{2}}$

   e. $\sqrt[3]{2} + \sqrt[3]{8} + \sqrt[3]{16}$

   f. $2^{\frac{1}{3}} + 8^{\frac{1}{3}} + 16^{\frac{1}{3}}$

BEAT THE TEST!

1. Which of the following expressions are equivalent to $3\sqrt{2}$? Select all that apply.

- ☐ $3^{\frac{1}{2}} + 2^{\frac{1}{2}}$
- ☐ $8^{\frac{1}{2}} + 2^{\frac{1}{2}}$
- ☐ $3 \cdot 2^{\frac{1}{2}}$
- ☐ $\sqrt{18}$
- ☐ $2\sqrt{18}$
- ☐ $\sqrt{8} + \sqrt{2}$

2. Miguel completed the following proof to show that $\sqrt{27} + \sqrt{3} = 4 \cdot 3^{\frac{1}{2}}$:

$$\sqrt{27} + \sqrt{3}$$
$$= 27^{\frac{1}{2}} + 3^{\frac{1}{2}}$$
$$= \_\_\_\_\_\_\_\_\_\_$$
$$= 3 \cdot 3^{\frac{1}{2}} + 3^{\frac{1}{2}}$$
$$= 4 \cdot 3^{\frac{1}{2}}$$

Which equation can be placed in the blank to correctly complete Miguel's work?

Ⓐ $3^{\frac{1}{2}}\left(9^{\frac{1}{2}} + 3^{\frac{1}{2}}\right) = 3^{\frac{1}{2}}\left(3 + 3^{\frac{1}{2}}\right)$

Ⓑ $(9 \cdot 3)^{\frac{1}{2}} + 3^{\frac{1}{2}} = 9^{\frac{1}{2}} \cdot 3^{\frac{1}{2}} + 3^{\frac{1}{2}}$

Ⓒ $\left(9^{\frac{1}{2}} + 3^{\frac{1}{2}}\right) + 3^{\frac{1}{2}} = \left(3 + 3^{\frac{1}{2}}\right) + 3^{\frac{1}{2}}$

Ⓓ $(3^3)^{\frac{1}{2}} + 3^{\frac{1}{2}} = 9^{\frac{1}{2}} + 3^{\frac{1}{2}}$

Section 1 – Topic 8
More Operations with Radicals and Rational Exponents

Let's explore multiplying and dividing expressions with radicals and rational exponents.

$\sqrt{10} \cdot \sqrt{2}$

$10^{\frac{1}{2}} \cdot 2^{\frac{1}{2}}$

$\sqrt{2} \cdot \sqrt[3]{2}$

$2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}}$

$\frac{\sqrt{10}}{\sqrt{2}}$

$\frac{10^{\frac{1}{2}}}{2^{\frac{1}{2}}}$

STUDY EDGE TIP

The properties of exponents also apply to expressions with rational exponents.

Let's Practice!

1. Use the properties of exponents to perform the following operations.

   a. $\left(x^{\frac{1}{3}}\right)^{\frac{1}{2}}$

   b. $\left(\sqrt{7}\right)^{3}$

   c. $\left(a^{\frac{1}{2}}b^{\frac{2}{5}}\right) \cdot \left(a^{\frac{2}{3}}b^{\frac{1}{2}}\right)$

d. $\frac{\sqrt[4]{8}}{\sqrt{8}}$

Try It!

2. Use the properties of exponents to perform the following operations.

a. $(m^0 n^2)^{\frac{1}{5}}$

b. $\left(\sqrt{8} \cdot \sqrt[3]{3}\right)^{\frac{2}{3}}$

c. $\sqrt[4]{4} \cdot \sqrt[3]{4}$

d. $\left(3 \cdot \sqrt[6]{27}\right)^{\frac{1}{2}}$

BEAT THE TEST!

1. Which of the following expressions are equivalent to $2^{\frac{1}{2}}$? Select all that apply.

- ☐ $\sqrt[3]{4}$
- ☐ $\sqrt[3]{8}$
- ☐ $\sqrt[4]{4}$
- ☐ $\sqrt[6]{8}$
- ☐ $\sqrt[6]{16}$

Section 1 – Topic 9
Operations with Rational and Irrational Numbers

Let's review rational and irrational numbers.

- Numbers that can be represented as $\frac{a}{b}$, where a and b are integers and $b \neq 0$, are called _____________ numbers.
- Numbers that cannot be represented in this form are called _______________numbers.
  - Radicals that are not perfect squares are examples of such numbers.

Determine whether the following numbers are rational or irrational.

	Rational	Irrational
$\sqrt{9}$	○	○
$\sqrt{8}$	○	○
π	○	○
$\frac{22}{7}$	○	○
$9.\overline{48}$	○	○
$\frac{33}{2}$	○	○
$2.23606...$	○	○
-25	○	○

Given two rational numbers, a and b, what can be said about the sum of a and b?

Given two rational numbers, a and b, what can be said about the product of a and b?

Given two irrational numbers, a and b, what can be said about the sum of a and b?

Given two irrational numbers, a and b, what can be said about the product of a and b?

Given a rational number, a, and an irrational number, b, what can be said about the sum of a and b?

Given a non-zero rational number, a, and an irrational number, b, what can be said about the product of a and b?

Let's Practice!

1. Consider the following expression.

   $2 + \sqrt{3}$

   The above expression represents the
   ○ sum
   ○ product

   of a(n)
   ○ rational number
   ○ irrational number
   and a(n)

   ○ rational number
   ○ irrational number
   and is equivalent to a(n)

   ○ rational number.
   ○ irrational number.

Try It!

2. Consider the following expression.

   $7 \cdot \frac{1}{3}$

   The above expression represents the
   ○ sum
   ○ product

   of a(n)
   ○ rational number
   ○ irrational number
   and a(n)

   ○ rational number
   ○ irrational number
   and is equivalent to a(n)

   ○ rational number.
   ○ irrational number.

BEAT THE TEST!

1. Let a and b be non-zero rational numbers and c and d be irrational numbers. Consider the operations below and choose whether the result can be rational, irrational, or both.

	Rational	Irrational
$a + b$	○	○
$a - c$	○	○
$a \cdot b$	○	○
$\frac{c}{d}$	○	○
$c + d$	○	○
$a \cdot b \cdot c$	○	○

2. Consider $x \cdot y = z$. If z is an irrational number, what can be said about x and y?

Section 2 – Equations and Inequalities

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.A-SSE.1.2	Use the structure of an expression to identify ways to rewrite it.
MAFS.912.A-REI.1.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MAFS.912.A-REI.2.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MAFS.912.A-CED.1.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational exponential functions.
MAFS.912.A-CED.1.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MAFS.912.A-CED.1.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MAFS.912.A-REI.4.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Topics in this Section

Section 2 – Topic 1
Equations: True or False?

Consider the statement $4 + 5 = 2 + 7$. This is a grammatically correct sentence.

Is the sentence true or false?

Consider the statement $1 + 3 = 8 + 6$. This statement is also a grammatically correct sentence.

Is the sentence true or false?

The previous statements are examples of ***number sentences***.

- A number sentence is a statement of equality between two ________________ expressions.
- A number sentence is said to be true if both numerical expressions are _________________.
- If both numerical expressions don't equal the same number, we say the number sentence is ___________.
- True and false statements are called ***truth values***.

Let's Practice!

1. Determine whether the following number sentences are true or false. Justify your answer.

   a. $13 + 4 = 7 + 11$

   b. $\frac{1}{2} + \frac{5}{8} = 1.4 - 0.275$

Try It!

2. Determine whether the following number sentences are true or false. Justify your answer.

   a. $(83 \cdot 401) \cdot 638 = 401 \cdot (638 \cdot 83)$

   b. $(6 + 4)^2 = 6^2 + 4^2$

A number sentence is an example of an ***algebraic equation***.

- An algebraic equation is a statement of equality between two __________________.
- Algebraic equations can be number sentences (when both expressions contain only numbers), but often they contain ________________ whose values have not been determined.

Consider the algebraic equation $4(x+2)=4x+8$.

Are the expressions on each side of the equal sign equivalent? Justify your answer.

What does this tell you about the numbers we can substitute for x?

Let's Practice!

3. Consider the algebraic equation $x+3=9$.

   a. What value can we substitute for x to make it a true number sentence?

   b. How many values could we substitute for x and have a true number sentence?

4. Consider the algebraic equation $x+6=x+9$. What values could we substitute for x to make it a true number sentence?

Try It!

5. Complete the following.

   a. $d^2=4$ is true for _________________________.

   b. $2m=m+m$ is true for _________________________.

   c. $d+67=d+68$ is true for _________________________.

BEAT THE TEST!

1. Which of the following have the correct solution? Select all that apply.

   - ☐ $2x + 5 = 19;\ x = 7$
   - ☐ $3 + x + 2 - x = 16;\ x = 3$
   - ☐ $\frac{x+2}{5} = 2;\ x = 8$
   - ☐ $6 = 2x - 8;\ x = 7$
   - ☐ $14 = \frac{1}{3}x + 5; x = 18$

Section 2 – Topic 2
Identifying Properties When Solving Equations

The following equations are equivalent. Describe the operation that occurred in the second equation.

$3 + 5 = 8$ and $3 + 5 - 5 = 8 - 5$

$x - 3 = 7$ and $x - 3 + 3 = 7 + 3$

$2(4) = 8$ and $\frac{2(4)}{2} = \frac{8}{2}$

$\frac{x}{2} = 3$ and $2 \cdot \frac{x}{2} = 2 \cdot 3$

This brings us to some more properties that we can use to write equivalent expressions.

Properties of Equality

If x is a solution to an equation, it will also be a solution to the new equation formed when the same number is added to or subtracted from each side of the original equation.

These are the ***addition and subtraction properties of equality***.

- If $a = b$, then $a + c = b + c$ and $a - c = b - c$.
- Give examples of this property.

If x is a solution to an equation, it will also be a solution to the new equation formed when the same number is multiplied by or divided into each side of the original equation.

These are the ***multiplication and division properties of equality***.

- If $a = b$, then $a \cdot c = b \cdot c$ and $\frac{a}{c} = \frac{b}{c}$.
- Give examples of this property.

Let's Practice!

1. The following equations are equivalent. Determine the property that was used to write the second equation.

   a. $x - 5 = 3x + 7$ and $x - 5 + 5 = 3x + 7 + 5$

   b. $x = 3x + 12$ and $x - 3x = 3x - 3x + 12$

   c. $-2x = 12$ and $\frac{-2x}{-2} = \frac{12}{-2}$

Try It!

2. The following equations are equivalent. Determine the property that was used to write the second equation.

   a. $2(x+4)=14-6x$ and $2x+8=14-6x$

   b. $2x+8=14-6x$ and $2x+8+6x=14-6x+6x$

   c. $2x+8+6x=14$ and $2x+6x+8=14$

   d. $8x+8=14$ and $8x+8-8=14-8$

   e. $8x=6$ and $\frac{1}{8}\cdot 8x=\frac{1}{8}\cdot 6$

BEAT THE TEST!

1. For each algebraic equation, select the property or properties that could be used to solve it.

Algebraic Equation	Addition or Subtraction Property of Equality	Multiplication or Division Property of Equality	Distributive Property	Commutative Property
$\frac{x}{2}=5$	○	○	○	○
$2x+7=13$	○	○	○	○
$4x=23$	○	○	○	○
$x-3=-4$	○	○	○	○
$4(x+5)=40$	○	○	○	○
$10+x=79$	○	○	○	○
$-8-x=19$	○	○	○	○
$2(x-8)+7x=9$	○	○	○	○

Section 2 – Topic 3
Solving Equations

Sometimes, you will be required to justify the steps to solve an equation. The following equation is solved for x. Use the properties to justify the reasons for each step in the chart below.

Statements	Reasons
a. $5(x+3)-2=2-x+9$	a. Given
b. $5x+15-2=2-x+9$	b.
c. $5x+15-2=2+9-x$	c.
d. $5x+13=11-x$	d. Equivalent Equation
e. $5x+13-13=11-13-x$	e.
f. $5x=-2-x$	f. Equivalent Equation
g. $5x+x=-2-x+x$	g.
h. $6x=-2$	h. Equivalent Equation
i. $\frac{6x}{6}=\frac{-2}{6}$	i.
j. $x=-\frac{1}{3}$	j. Equivalent Equation

Other times, you may be required to write and solve an equation for a situation.

Consider the following scenario. Your class is raising funds for an end of the year trip to an amusement park. Your class plans to rent one bus. It costs $\$150.00$ to rent a school bus for the day plus $\$33.00$ per student admission ticket.

What is the variable in the situation?

Write an expression to represent the amount of money the school needs to raise.

Your class raised $\$1000$ for the trip. Write an equation to represent the number of students that can attend the trip.

Solve the equation to determine the number of students who can attend the trip.

Let's Practice!

1. Consider the following equation $2x - 3(2x - 1) = 3 - 4x$. Solve the equation for x. For each step, identify the property used to write an equivalent equation.

STUDY EDGE TIP

Some equations, such as $2x = 2x$ have **all real numbers** as the solution. No matter what number we substitute for x, the equation would still be true.

Try It!

2. Consider the following equation $3(4x + 1) = 3 + 12x - 5$. Solve the equation for x. For each step, identify the property used to convert the equation.

STUDY EDGE TIP

Some equations, such as $2x + 5 = 2x - 1$ have **no solution**. There is **NO** number that we could substitute for x that would make the equation true.

3. A high school surveyed its student population about their favorite sports. The 487 students who listed soccer as their favorite sport represented 17 fewer students than three times the number of students who listed basketball as their favorite sport. Write and solve an equation to determine how many students listed basketball as their favorite sport.

BEAT THE TEST!

1. The following equation is solved for x. Use the properties to justify the reasons for each step in the chart below.

Statements	Reasons
a. $2(x+5)-3=15$	a. Given
b. $2x+10-3=15$	b.
c. $2x+7=15$	c. Equivalent Equation
d. $2x+7-7=15-7$	d.
e. $2x=8$	e. Equivalent Equation
f. $\frac{2x}{2}=\frac{8}{2}$	f.
g. $x=4$	g. Equivalent Equation

Section 2 – Topic 4
Solving Equations Using the Zero Product Property

If someone told you that the product of two numbers is 10, what could you say about the two numbers?

If someone told you that the product of two numbers is zero, what could you say about the two numbers?

This is the ***zero product property***.

- If $ab = 0$, then either $a = 0$ or $b = 0$.

Describe how to use the zero product property to solve the equation $(x-3)(x+9)=0$. Then, identify the solutions.

Let's Practice!

1. Identify the solution(s) to $2x(x+4)(x+5)=0$.

2. Identify the solution(s) to $(2x-5)(x+11)=0$.

Try It!

3. Michael was given the equation $(x+7)(x-11)=0$ and asked to find the zeros. His solution set was $\{-11, 7\}$. Explain whether you agree or disagree with Michael.

4. Identify the solution(s) to $2(y-3)\cdot 6(-y-3)=0$.

BEAT THE TEST!

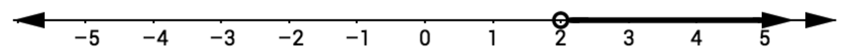
1. Use the values below to determine the solutions for each equation.

0	2	3	$\frac{4}{5}$
$\frac{2}{7}$	$-\frac{1}{2}$	$-\frac{3}{4}$	-14
6	0	$-\frac{1}{4}$	-2

$(2y+1)(y+14)=0$			
$(7n-2)(5n-4)=0$			
$(4x+3)(x-6)=0$			
$x(x+2)(x-3)=0$			
$t(4t+1)(t-2)=0$			

Section 2 – Topic 5
Solving Inequalities – Part 1

Let's start by reviewing how to graph inequalities.



- When the endpoint is an _________ dot or circle, the number represented by the endpoint _____ _________ a part of the solution set.

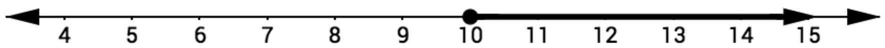
Describe the numbers that are graphed in the example above.

Can you list all the numbers? Explain your answer.

Write an inequality that represents the graph above.

Write the solution set that represents the graph above.

Consider the following graph.



➢ When the endpoint is a ___________ dot or circle, the number represented by the endpoint _______ a part of the solution set.

Write an inequality that represents the graph above.

Write the solution set that represents the graph above.

Why is "or equal to" included in the solution set?

Just like there are Properties of Equality, there are also ***Properties of Inequality***.

If $x > 5$, is $x + 1 > 5 + 1$? Substitute values for x to justify your answer.

Addition and Subtraction Property of Inequality

➢ If $a > b$, then $a + c > b + c$ and $a - c > b - c$ for any real number c.

Consider $(2x - 1) + 2 > x + 1$. Use the addition or subtraction property of inequality to solve for x.

Let's Practice!

1. Consider the inequality $(4 + x) - 5 \geq 10$. Use the addition or subtraction property of inequality to solve for x. Express the solution in set notation and graphically on a number line.

Try It!

2. Consider the following inequality $4x + 8 < 1 + (2x - 5)$. Use the addition or subtraction property of inequality to solve for x. Express the solution in set notation and graphically on a number line.

3. Peter deposited $\$27$ into his savings account, bringing the total to over $\$234$. Write and solve an inequality to represent the amount of money in Peter's account before the $\$27$ deposit.

Section 2 – Topic 6
Solving Inequalities – Part 2

Consider $x > 5$ and $2 \cdot x > 2 \cdot 5$. Identify a solution to the first inequality. Show that this solution also makes the second inequality true.

Consider $x > 5$ and $-2 \cdot x > -2 \cdot 5$. Identify a solution to the first inequality. Show that this solution makes the second inequality false.

How can we change the second inequality so that the solution makes it true?

Consider $-q > 5$. Use the addition and/or subtraction property of inequality to solve.

Multiplication Property of Inequality

- If $a > b$, then for any positive real number k, ak_____bk.
- If $a < b$, then for any positive real number k, ak_____bk.
- If $a > b$, then for any negative real number k, ak_____bk.
- If $a < b$, then for any negative real number k, ak_____bk.

The same property is true when dealing with ≤ or ≥.

Let's Practice!

1. Find the solution set to each inequality. Express the solution in set notation and graphically on a number line.

   a. $-9y + 4 < -7y - 2$

   b. $\frac{m}{3} + 8 \leq 9$

2. At 5:00 PM in Atlanta, Georgia, Ethan noticed the temperature outside was $72°\text{F}$. The temperature decreased at a steady rate of $2°\text{F}$ per hour. At what time was the temperature less than $64°\text{F}$?

Try It!

3. Find the solution set to the inequality. Express the solution in set notation and graphically on a number line.

   a. $-6\,(x-5) > 42$

   b. $4(x+3) \geq 2(2x-2)$

<u>**BEAT THE TEST!**</u>

1. Ulysses is spending his vacation in South Carolina. He rents a car and is offered two different payment options. He can either pay $\$25.00$ each day plus $\$0.15$ per mile (option A) or pay $\$10.00$ each day plus $\$0.40$ per mile (option B). Ulysses rents the car for one day.

   Part A: Write an inequality representing the number of miles where option A will be the cheaper plan.

   Part B: How many miles will Ulysses have to drive for option A to be the cheaper option?

2. Stephanie has just been given a new job in the sales department of Frontier Electric Authority. She has two salary options. She can either receive a fixed salary of $\$500.00$ per week or a salary of $\$200.00$ per week plus a 5% commission on her weekly sales. The variable s represents Stephanie's weekly sales. Which solution set represents the dollar amount of sales that she must generate in a week in order for the option with commission to be the better choice?

Ⓐ $\{s|s > \$300.00\}$
Ⓑ $\{s|s > \$700.00\}$
Ⓒ $\{s|s > \$3,000.00\}$
Ⓓ $\{s|s > \$6,000.00\}$

Section 2 – Topic 7
Solving Compound Inequalities

Consider the following options:

Option A: You get to play Call of Duty after you clean your room and do the dishes.

Option B: You get to play Call of Duty after you clean your room or do the dishes.

What is the difference in Option A and B?

Consider the following. Circle the statements that are true.

$2 + 9 = 11$ and $10 < 5 + 6$

$4 + 5 \neq 9$ and $2 + 3 > 0$

$0 > 4 - 6$ or $3 + 2 = 6$

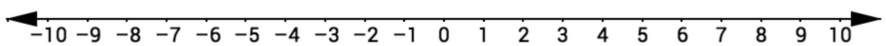
$15 - 20 > 0$ or $2.5 + 3.5 = 7$

These are called ***compound equations or inequalities***.

- When the two statements in the previous sentences were joined by the word *AND*, the compound equation or inequality is true only if __________ statements are true.

- When the two statements in the previous sentences were joined by the word *OR*, the compound equation or inequality is true if at least _________ of the statements is true. Therefore, it is also considered true if _________ statements are true.

Let's graph $x < 6$ and $x > 1$.



This is the __________________ ______________ to the compound inequality.

How many solutions does this inequality have?

Many times this is written as $1 < x < 6$. This notation denotes the conjunction "and."

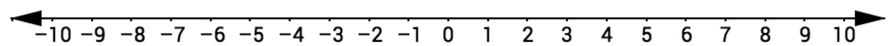
We read this as "x is greater than one ________ less than six."

Let's Practice!

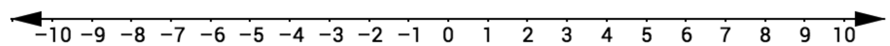
1. Consider $x < 1$ or $x > 6$. Could we write the inequalities above as $1 > x > 6$? Explain your answer.

2. Graph the solution set to each compound inequality on a number line.

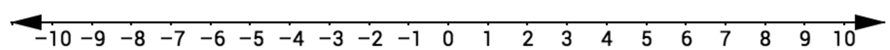
   a. $x = 2$ or $x > 5$



   b. $x > 6$ or $x < 6$



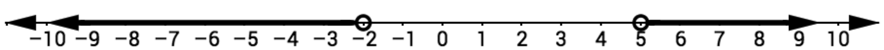
   c. $1 \leq -x \leq 7$



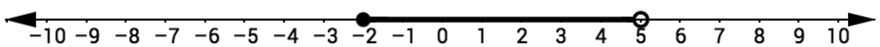
STUDY EDGE TIP

Be on the lookout for negative coefficients. When solving inequalities, you will need to reverse the inequality symbol when you multiply or divide by a negative value.

3. Write a compound inequality for the following graphs.



a. Compound inequality:

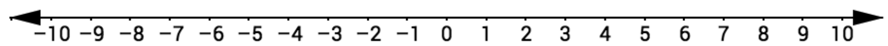


b. Compound inequality:

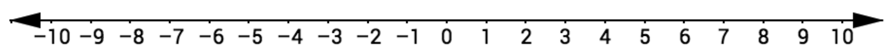
Try It!

4. Graph the solution set to each compound inequality on a number line.

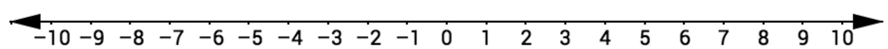
a. $x < 1$ or $x > 8$



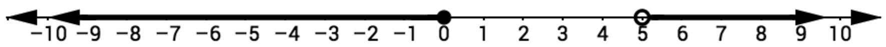
b. $x \geq 6$ or $x < 4$



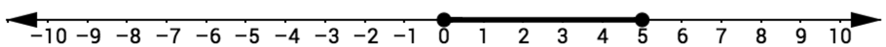
c. $-6 \leq x \leq 4$



5. Write a compound inequality for the following graphs.



a. Compound inequality:

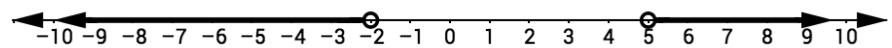


b. Compound inequality:

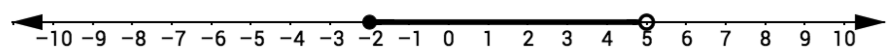
BEAT THE TEST!

1. Use the terms and symbols in the bank to write a compound inequality for each of the following graphs. You may only use each term once, but you do not have to use all of them.

$3x$	-14	-6	$\geq$	$-$	17	15	$<$
$7x$	$<$	2	or	$\leq$	$3x$	$+$	$>$



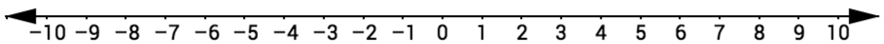
Compound Inequality:



Compound Inequality:

Section 2 – Topic 8
Solving Absolute Value Equations and Inequalities

Absolute value represents the distance of a number from zero on a number line.



How far away is "9" from zero on the number line?

This is written as ______________.

How far away is "-9" from zero on the number line?

This is written as ______________.

This is the ***absolute value*** of a number.

- For any real numbers c and d, if $|c| = d$, then $c = d$ or $c = -d$.

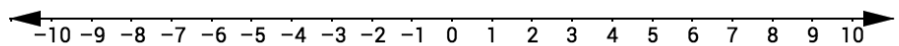
For example, $|f| = 5$, so $f =$ _____ or $f =$ _____.

Consider $|c| < 5$.

Using our definition of absolute value, this is saying that c represents all the numbers ____________ ____________ five units from zero on the number line.

What are some numbers that could be represented by c?

Graph all the numbers represented by c on a number line.



What is the solution set for c?

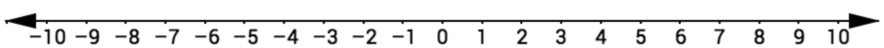
- For any real numbers c and d, if $|c| < d$ or if $|c| \leq d$, then $-d < c < d$ or $-d \leq c \leq d$.

Consider $|c| > 5$.

Using our definition of absolute value, this is saying that c represents all the numbers ____________ ________ five units from zero on the number line.

What are some numbers that could be represented by c?

Graph all the numbers represented by c on a number line.



What is the solution set for c?

- For any real numbers c and d, if $|c| > d$, then $c > d$ or $c < -d$.
- For any real numbers c and d, if $|c| \geq d$, then $c \geq d$ or $c \leq -d$.

Let's Practice!

1. Solve each absolute value inequality and graph the solution set.

   a. $|n + 5| < 7$

   b. $|a| + 3 > 9$

2. Tammy purchased a pH meter to measure the acidity of her freshwater aquarium. The ideal pH level for a freshwater aquarium is between 6.5 and 7.5 inclusive.

   a. Graph an inequality that represents the possible pH levels needed for Tammy's aquarium.

   b. Define the variable and write an absolute value inequality that represents the possible pH levels needed for Tammy's aquarium.

Try It!

3. Solve each equation or inequality and graph the solution set.

   a. $|p + 7| = -13$

   b. $2|x| - 4 < 14$

   c. $|2m + 4| \geq 12$

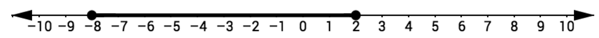
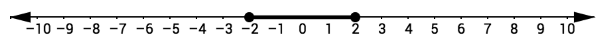
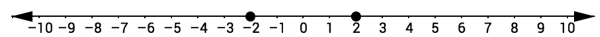
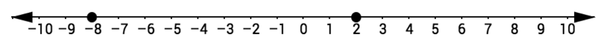
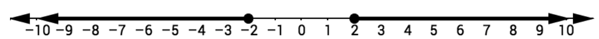
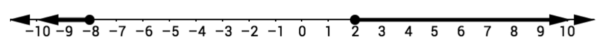
4. Baseball fans often leave a baseball game if their team is ahead or behind by five runs or more. Toronto Blue Jays fans follow this pattern, and the Blue Jays have scored eight runs in a particular game.

   a. Graph an inequality that represents the possible runs, r, scored by the opposing team if Toronto fans are leaving the game.

   b. Write an absolute value inequality that represents the possible runs, r, scored by the opposing team if Toronto fans are leaving the game.

BEAT THE TEST!

1. Match the following absolute value equations and inequalities to the graph that represents their solution.

		A. $\lvert x\rvert = 2$
		B. $\lvert x\rvert \geq 2$
		C. $\lvert x\rvert \leq 2$
		D. $\lvert x+3\rvert \leq 5$
		E. $\lvert x+3\rvert \geq 5$
		F. $\lvert x+3\rvert = 5$

Section 2 – Topic 9
Rearranging Formulas

Solve each equation for x.

$2x + 4 = 12$

$2x + y = z$

Did we use different properties when we solved the two equations?

Consider the formula for the perimeter of a rectangle: $P = 2l + 2w$.

Sometimes, we might need the formula solved for length.

STUDY EDGE TIP

It is helpful to circle the variable that you are solving for.

Let's Practice!

1. Consider the equation $rx - sx + y = z$; solve for x.

Try It!

2. Consider the equation $8c + 6j = 5p$; solve for c.

3. Consider the equation $\frac{x-c}{2}=d$; solve for c.

BEAT THE TEST!

1. Isaiah planted a seedling in his garden and recorded its height every week. The equation shown can be used to estimate the height, h, of the seedling after w weeks since he planted the seedling.

$$h=\frac{3}{4}w+\frac{9}{4}$$

Solve the formula for w, the number of weeks since he planted the seedling.

2. Shoe size and foot length for women are related by the formula $S = 3F - 24$, where S represents the shoe size and F represents the length of the foot in inches. Solve the formula for F.

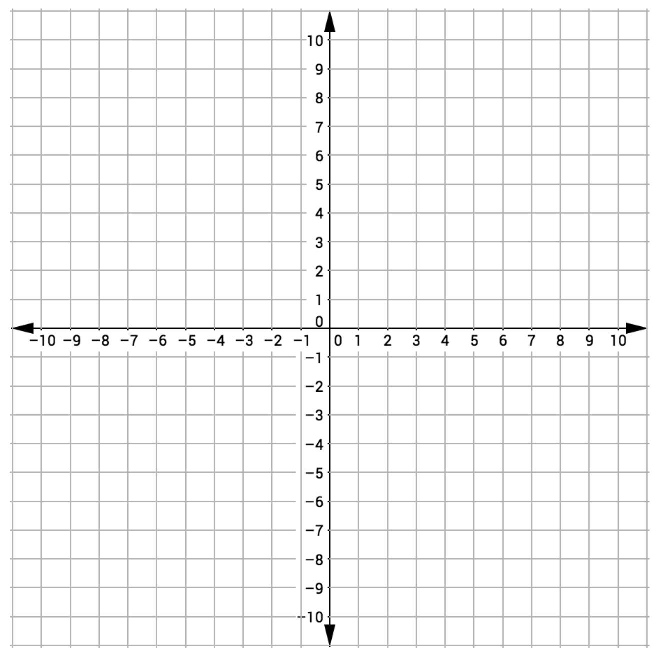
Section 2 – Topic 10
Solution Sets to Equations with Two Variables

Consider $x + 2 = 5$. What is the only possible value of x that makes the equation a true statement?

Now consider $x + y = 5$. What are some solutions for x and y that would make the equation true?

Possible solutions can be listed as ***ordered pairs***.

Graph each of the ordered pairs from the previous problem on the graph below.



What do you notice about the points you graphed?

How many solutions are there to the equation $x + y = 5$?

Let's Practice!

1. Tammy has 10 songs on her phone's playlist. The playlist features songs from her two favorite artists, Beyoncé and Pharrell.

   a. Create an equation using two variables to represent this situation.

   b. List at least three solutions to the equation that you created.

   c. Do we have infinitely many solutions to this equation? Why or why not?

STUDY EDGE TIP

In this case, our solutions must be natural numbers. Notice that the solutions follow a linear pattern. However, they do not form a line. This is called a ***discrete function***.

   d. Create a graph that represents the solution set to your equation.

   e. Why are there only positive values on this graph?

Try It!

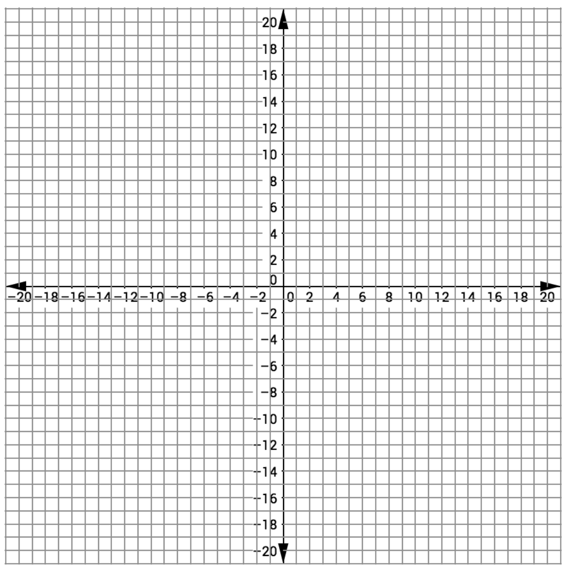
2. The sum of two numbers is 15.

   a. Create an equation using two variables to represent this situation.

   b. List at least three possible solutions.

   c. How many solutions are there to this equation?

   d. Create a visual representation of all the possible solutions on the graph.



STUDY EDGE TIP

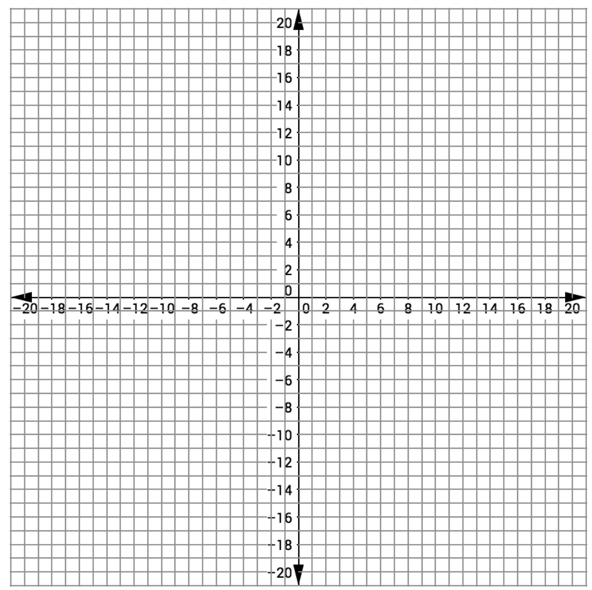
In this case, our solutions are rational numbers. Notice that the solutions form a line. This is called a ***continuous function***.

3. What if we changed the problem to say the sum of two integers is 15?

   a. Create an equation using two variables to represent this situation.

   b. Is this function discrete or continuous? Explain your answer.

   c. Represent the solution on the graph below.



BEAT THE TEST!

1. Elizabeth's tablet has a combined total of 20 apps and movies. Let x represent the number of apps and y represent the number of movies. Which of the following could represent the number of apps and movies on Elizabeth's tablet? Select all that apply.

- ☐ $x + y = 20$
- ☐ 7 apps and 14 movies
- ☐ $x - y = 20$
- ☐ $y = -x + 20$
- ☐ 8 apps and 12 movies
- ☐ $xy = 20$

Section 3 – Introduction to Functions

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.A-APR.1.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MAFS.912.A-SSE.1.1	Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.
MAFS.912.A-SSE.1.2	Use the structure of an expression to identify ways to rewrite it.
MAFS.912.A-CED.1.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
MAFS.912.A-REI.2.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MAFS.912.F-IF.1.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MAFS.912.F-IF.1.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MAFS.912.F-IF.2.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
MAFS.912.F-IF.2.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MAFS.912.F-IF.3.7.b	Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions
MAFS.912.F-BF.1.1.b.c.	Write a function that describes a relationship between two quantities. b. Combine standard function types using arithmetic operations. For example, build a function that

	models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. Compose functions.
MAFS.912.F-BF.2.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k, kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Topics in this Section

Section 3 – Topic 1
Input and Output Values

A function is a relationship between input and output.

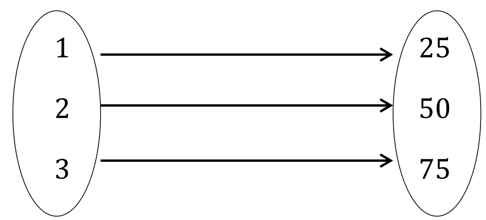
- ***Domain*** is the set of values of x used for the ___________ of the function.
- ***Range*** is the set of values of y calculated from the domain for the __________ of the function.

In a function, every x corresponds to only one y.

- y can also be written as $f(x)$.

Consider the following function:

For every x input domain there is a unique y output range



We also refer to the variables as independent and dependent. The dependent variable _____________ ____ the independent variable.

Refer to the mapping diagram on the previous page.

Which variable is independent?

Which variable is dependent?

Consider a square whose perimeter depends on the length of its sides.

What is the independent variable?

What is the dependent variable?

How would you represent this situation using function notation?

STUDY EDGE TIP

We can choose any letter to represent a function, such as $f(x)$ or $g(x)$. By using different letters, we show that we are talking about different functions.

Let's Practice!

1. You earn $\$10.00$ per hour babysitting. Your total earnings depend on the amount of hours you spend babysitting.

    a. What is the independent variable?

    b. What is the dependent variable?

    c. How would you represent this situation using function notation?

2. The table below represents a relation.

x	y
-3	5
0	4
2	6
-3	8

    a. Is the relation also a function? Justify your answer.

    b. If the relation is not a function, what number could be changed to make it a function?

Try It!

3. Mrs. Krabappel is buying composition books for her classroom. Each composition book costs $1.25.

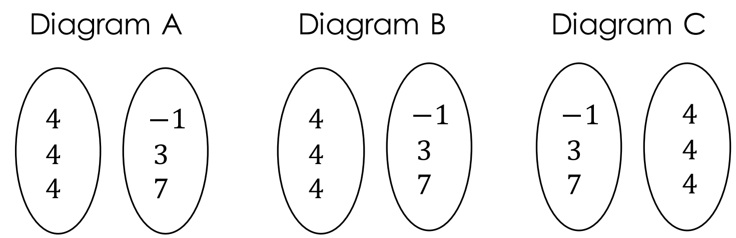
   a. What does her total cost depend upon?

   b. What is the input and output?

   c. Write a function to describe the situation.

   d. If Mrs. Krabappel buys 24 composition books, it will cost $30. Write this function using function notation.

4. Consider the following incomplete mapping diagrams.



   a. Complete Diagram A so that it is a function.

   b. Complete Diagram B so that it is NOT a function.

   c. Is it possible to complete the mapping diagram for Diagram C so it represents a function? If so, complete the diagram to show a function. If not, justify your reasoning.

BEAT THE TEST!

1. Isaac Messi is disorganized. To encourage Isaac to be more organized, his father promised to give him three dollars for every day that his room is clean and his schoolwork is organized.

   Part A: Define the input and output in the given scenario.

   Input:

   Output:

   Part B: Write a function to model this situation.

2. The cost to manufacture x pairs of shoes can be represented by the function $C(x) = 63x$. Complete the statement about the function.

   If $C(6) = 378$, then [0 / 6 / 63 / 378] pairs of shoes cost [\$6. / \$189. / \$378. / \$2,268.]

3. Which of the following relations is NOT a function?

   Ⓐ $\{(0,5),(2,3),(5,8),(3,8)\}$
   Ⓑ $\{(4,2),(-4,5),(0,0)\}$
   Ⓒ $\{(6,5),(4,1),(-3,2),(4,2)\}$
   Ⓓ $\{(-3,-3),(2,1),(5,-2)\}$

Section 3 – Topic 2
Representing, Naming, and Evaluating Functions

Consider the function $f(x) = 2^{x+1}$.

What are the values of the domain of $f(x)$?

What are the values of the range of $f(x)$?

Evaluate $f(x)$ to find the range for the domain $\{-2, 0, 2\}$.

Determine whether the following values are true for the given function.

$f(5) = 32$ $f(-1) = 1$

Let's Practice!

1. You make trips to the grocery store to purchase doughnuts. Every time you go, you take a taxi. The round-trip taxi ride costs $\$7.00$ and each doughnut costs $\$0.50$.

   a. Write a function to describe the cost of any given trip to buy doughnuts.

   b. What are the values of your domain?

   c. You take the taxi to the store and buy four dozen doughnuts. Represent this situation using function notation and find the total cost.

   d. Your dad gave you $\$30.00$ to buy doughnuts for your friends. You brought him back $\$5.00$. How many dozens of doughnuts did you buy?

Try It!

2. Consider the function $g(x) = 2^{3x+1}$. Evaluate $g(x)$ to find the range for the domain $\{-2, -1, 0\}$.

3. Your sister is using your credit card to buy tickets on TicketBoss for a Taylor Quick concert. There is a $\$5.00$ transaction fee with each order, and tickets cost $\$55.00$.

   a. Write a function to describe the situation.

   b. Evaluate the total cost function if your sister decides to buy seven tickets in a single transaction.

   c. Your credit card statement shows a $\$225.00$ transaction from TicketBoss. How many tickets did your sister buy?

BEAT THE TEST!

1. Match the functions in the left column with the values in the right column.

A. $f(x) = 2^{x+1}$	I. $f(5) = 36$
B. $f(x) = 3x - 2$	II. $f(5) = 64$
C. $f(x) = (x+1)^2$	III. $f(-7) = 6$
D. $f(x) = \lvert x+1 \rvert$	IV. $f(-1) = 2$
E. $f(x) = \sqrt[3]{x}$	V. $f(2) = 4$
F. $f(x) = x(2x+5)$	VI. $f(2) = 18$
G. $f(x) = \frac{x}{2}$	VII. $f(8) = 2$
H. $f(x) = \frac{8}{\lvert x \rvert + 3}$	VIII. $f(8) = 4$

Section 3 – Topic 3
Adding and Subtracting Functions

Let $h(x) = 2x^2 + x - 5$ and $g(x) = -3x^2 + 4x + 1$.

Find $h(x) + g(x)$.

Find $h(x) - g(x)$.

Let's Practice!

1. Consider the following functions.

$$f(x) = 3x^2 + x + 2$$
$$g(x) = 4x^2 + 2(3x - 4)$$
$$h(x) = 5(x^2 - 1)$$

   a. Find $f(x) - g(x)$.

   b. Find $g(x) - h(x)$.

Try it!

2. Recall the functions we used earlier:

$$f(x) = 3x^2 + x + 2$$
$$g(x) = 4x^2 + 2(3x - 4)$$
$$h(x) = 5(x^2 - 1)$$

   a. Let $m(x)$ be $f(x) + g(x)$. Find $m(x)$.

   b. Find $h(x) - m(x)$.

BEAT THE TEST!

1. Consider the functions below.

$$f(x) = (2x^2 + 3x - 5)$$
$$g(x) = (5x^2 + 4x - 1)$$

Which of the following is the resulting polynomial when $f(x)$ is subtracted from $g(x)$?

Ⓐ $-3x^2 - x - 4$
Ⓑ $-3x^2 + 7x - 6$
Ⓒ $3x^2 + x + 4$
Ⓓ $3x^2 + 7x - 6$

Section 3 – Topic 4
Multiplying Functions

Use the distributive property and modeling to perform the following function operations.

Let $f(x) = 3x^2 + 4x + 2$ and $g(x) = 2x + 3$.

Find $f(x) \cdot g(x)$.

Let $m(y) = 3y^5 - 2y^2 + 8$ and $p(y) = y^2 - 2$.

Find $m(y) \cdot p(y)$.

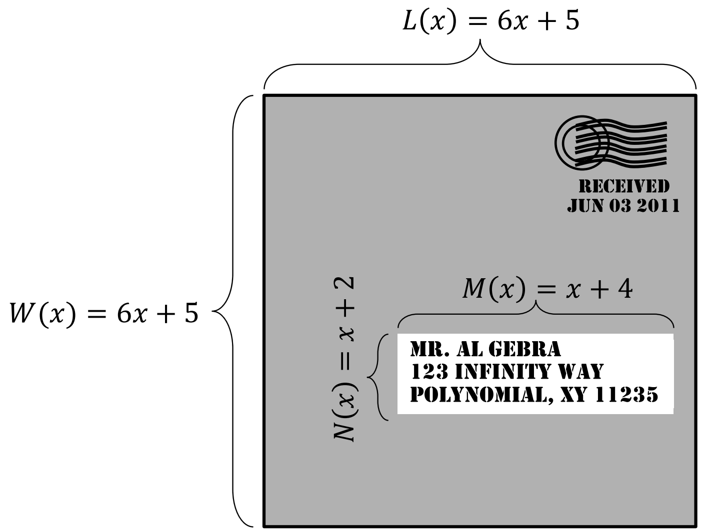
Let's Practice!

1. Let $h(x) = x - 1$ and $g(x) = x^3 + 6x^2 - 5$.

   Find $h(x) \cdot g(x)$.

Try it!

2. The envelope below has a mailing label:



a. Let $A(x) = L(x) \cdot W(x) - M(x) \cdot N(x)$. Find $A(x)$.

b. What does the function $A(x)$ represent in this problem?

BEAT THE TEST!

1. A square has sides of length s. A rectangle is six inches shorter and eight inches wider than the square.

   Part A: Express both the length and the width of the rectangle as a function of the sides of the square.

   Part B: Write a function to represent the area of the rectangle in terms of the sides of the square.

2. Felicia needs to find the area of a rectangular field in her backyard. The length is represented by the function $L(x) = 4x^4 - 3x^2 + 6$ and the width is represented by the function $W(x) = x + 1$. Which of the following statements is correct about the area, $A(x)$, of the rectangular field in Felicia's backyard? Select all that apply.

   - ☐ $A(x) = 2[L(x) + W(x)]$
   - ☐ The resulting expression for $A(x)$ is a fifth-degree polynomial.
   - ☐ The resulting expression for $A(x)$ is a polynomial with leading coefficient of 5.
   - ☐ The resulting expression for $A(x)$ is a binomial with constant of 6.
   - ☐ $W(x) = \frac{A(x)}{L(x)}$

Section 3 – Topic 5
Dividing Functions

Let's discuss an important fact that we need to be aware of when dividing.

Consider $\frac{10}{2} = 5$.

What is another way of expressing this equation?

Consider $\frac{0}{2} = 0$.

What is another way of expressing this equation?

Consider $\frac{2}{0}$.

Can you write an equivalent expression?

The quotient of any real number and zero is always ***undefined.***

Consider $\frac{f(x)}{g(x)}$.

What can be said of $g(x)$?

Therefore, when we are dividing functions or polynomials and we have expressions with variables in the denominator, we need to:

- Determine what value(s) of the variable would make the denominator equal zero.
- Rewrite the equation or expression stating the constraints.

Let $a(x) = 2$, $b(x) = x - 5$, $m(x) = 9$, and $n(x) = x^2 - 16$.

Rewrite the following as an equation and state the constraints.

$\frac{a(x)}{b(x)} = 7$

$\frac{m(x)}{n(x)} = 9$

Let $p(x) = \frac{x+3}{x-2}$ and $q(x) = \frac{5}{x-2}$.

For what value of x does $p(x) = q(x)$?

Let's Practice!

1. Let $f(x) = \frac{2}{x}$, $g(x) = x + 5$, and $h(x) = \frac{4}{x-3}$.

   a. Find the value of x for which $f(x) = h(x)$.

   b. Find the value of x for which $\frac{g(x)}{g(x)} = 1$.

   c. Find the value of x for which $h(x) = \frac{1}{g(x)}$.

Try It!

2. Let $f(x) = \frac{4}{x}$, $g(x) = x - 5$, $h(x) = x + 8$, and $p(x) = \frac{6}{x-3}$.

   a. Find the value of x for which $\frac{h(x)}{h(x)} = 10$.

   b. Find the value of x for which $f(x) = p(x)$.

   c. What is the difference between the domain of $g(x)$ and the domain of $\frac{1}{g(x)}$?

BEAT THE TEST!

1. Consider the following function.

$$f(x) = \frac{x-5}{x-13}$$

Part A: Write an equation satisfying $f(x) = 3$ and state the constraint(s).

Part B: For what value of x does $f(x) = 3$?

2. Consider the following function.

$$h(t) = \frac{8(t^2 - 16)}{2t(t^2 - 9)(t + 2)}$$

Which of the following values of t are excluded? Select all that apply.

- ☐ -9
- ☐ -3
- ☐ -2
- ☐ 0
- ☐ 2
- ☐ 3
- ☐ 9

Section 3 – Topic 6
Closure Property

When we add two integers, what type of number is the sum?

When we multiply two irrational numbers, what type of number is the product?

A set is ___________ for a specific operation if and only if the operation on two elements of the set ***always*** produces an element of the same set.

Are integers closed under addition? Justify your answer.

Are irrational numbers closed under multiplication? Justify your answer.

Would integers be closed under division?

Let's apply the closure property to polynomials.

Are the following statements true or false? If false, give a counterexample.

Polynomials are closed under addition.

Polynomials are closed under subtraction.

Polynomials are closed under multiplication.

Polynomials are closed under division.

Let's Practice!

1. Check the boxes for the following sets that are closed under the given operations.

Set	$+$	$-$	$\times$	$\div$
$\{0, 1, 2, 3, 4, \dots\}$	☐	☐	☐	☐
$\{\dots, -4, -3, -2, -1\}$	☐	☐	☐	☐
$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$	☐	☐	☐	☐
{rational numbers}	☐	☐	☐	☐
{polynomials}	☐	☐	☐	☐

Try It!

2. Ms. Sanabria claims that the closure properties for polynomials are analogous to integers. Mr. Roberts claims that the closure properties for polynomials are analogous to rational numbers. Who is correct? Explain your answer.

BEAT THE TEST!

1. Choose from the following words and expressions to complete the statement below.

$2x^5 + (3y)^{-2} - 2$	$(5y)^2 + 4x + 3y^3$	$5y^{-1} + 7x^2 + 8y^2$
integers	variables	whole numbers
coefficients	rational numbers	exponents

The product of $5x^4 - 3x^2 + 2$ and ______________________ illustrates the closure property because the _______________ of the product are ____________________ and the product is a polynomial.

Section 3 – Topic 7
Real-World Combinations and Compositions of Functions

There are many times in real-world situations when we must combine functions. Profit and revenue functions are a great example of this.

Let's Practice!

1. At the fall festival, the senior class sponsors hayrides to raise money for the senior trip. The ticket price is $5.00 and each hayride carries an average of 15 people. They consider raising the ticket price in order to earn more money. For each $0.50 increase in price, an average of 2 fewer seats will be sold. Let x represent the number of $0.50 increases.

    a. Write a function, $T(x)$, to represent the cost of one ticket based on the number of increases.

    b. Write a function, $R(x)$, to represent the number of riders based on the number of increases.

    c. Write a revenue function for the hayride that could be used to maximize revenue.

Try It!

2. The freshman class is selling t-shirts to raise money for a field trip. The cost of the t-shirt and design is $8, with a $45 setup fee. The class plans to sell the shirts for $12.

    a. Define the variable.

    b. Write a cost function.

    c. Write a revenue function.

    d. Write a profit function.

Let's Practice!

3. Priscilla works at a cosmetics store. She receives a weekly salary of $\$350$ and is paid a 3% commission on weekly sales over $\$1500$.

   a. Let x represent Priscilla's weekly sales. Write a function, $f(x)$, to represent Priscilla's weekly sales over $\$1500$.

   b. Let x represent Priscilla's weekly sales that she is paid commission on. Write a function, $g(x)$, to represent Priscilla's commission.

   c. Write a composite function, $(g \circ f)(x)$ to represent the amount of money Priscilla earns on commissions.

Try It!

4. A landscaping company installed a circular sprinkler system. The water reaches its maximum radius of 10 feet after 30 seconds. The company wants to know the area that the sprinkler is covering at any given time after the sprinklers are turned on.

   a. Let t represent the time in seconds after the sprinkler is turned on. Write a function, $r(t)$, to represent the size of the growing radius based on time after the sprinkler is turned on.

   b. Let r represent the size of the radius at any given time. Write a function, $A(r)$, to represent the area that the sprinkler covers at any given time, in seconds.

   c. Write a composite function, $A(r(t))$ to represent the area based on the time, in seconds, after the sprinkler is turned on.

BEAT THE TEST!

1. A furniture store charges 6.5% sales tax on the cost of the furniture and a $20 delivery fee. (The delivery fee is not subject to sales tax.)

   The following functions represent the situation:

   $$f(a) = 1.065a$$
   $$g(b) = b + 20$$

   Part A: Write the function $g(f(a))$.

   Part B: Match each of the following to what they represent. Some letters will be used twice.

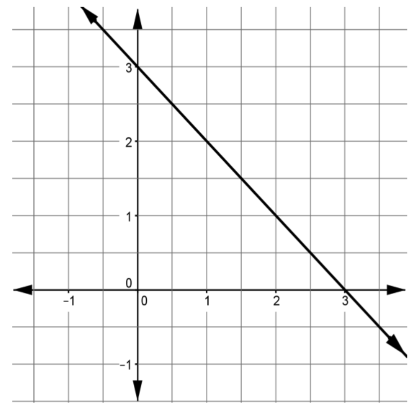
a	A. The cost of the furniture, sales tax, and delivery fee.
b	B. The cost of the furniture and sales tax.
$f(a)$	C. The cost of the furniture.
$g(b)$	
$g(f(a))$	

Section 3 – Topic 8
Key Features of Graphs of Functions – Part 1

Let's review the definition of a function.

Every input value (x) corresponds to ___________ ______ output value (y).

Consider the following graph.



How can a vertical line help us quickly determine if a graph represents a function?

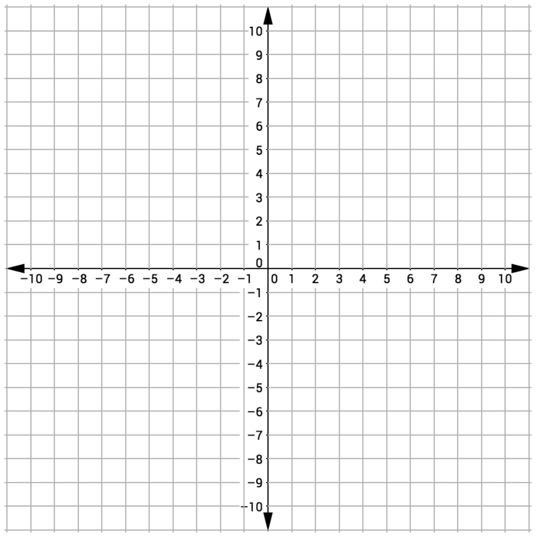
We call this the ***vertical line test***. Use the vertical line test to determine if the graph above represents a function.

Important facts:

- Graphs of lines are not always functions. Can you describe a graph of a line that is not a function?

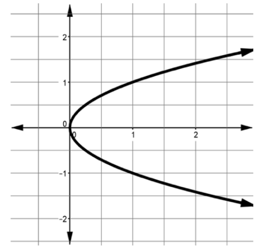
- Functions are not always linear.

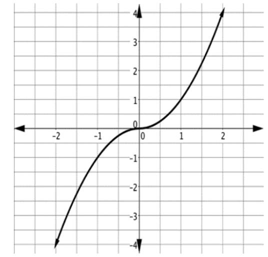
Sketch a graph of a function that is not linear.



Let's Practice!

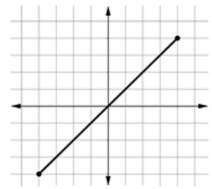
1. Use the vertical line test to determine if the following graphs are functions.





Try It!

2. Which of the following represent a function? Select all that apply.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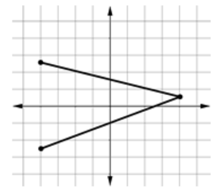
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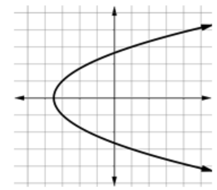
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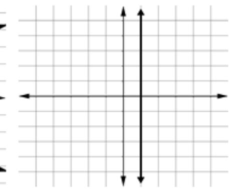
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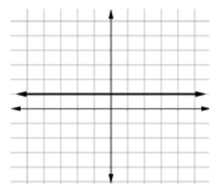
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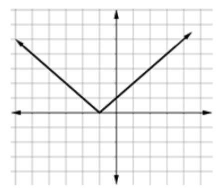
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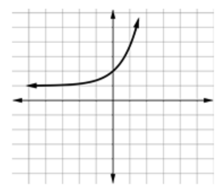
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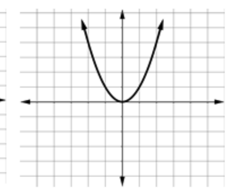

○

3. Consider the following scenarios. Determine if each one represents a function or not.

   a. An analyst takes a survey of people about their height, in inches, and their ages and then relates their heights to their ages.

   b. A Geometry student is dilating a circle and analyzes the area of the circle as it relates to the radius.

   c. A teacher has a roster of 32 students and relates the students' letter grades to the percent earned.

   d. A boy throws a tennis ball in the air and lets it fall to the ground. The boy relates the time passed to the height of the ball.

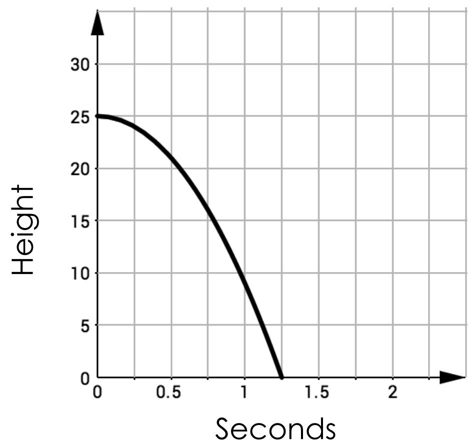
It's important to understand key features of graphs.

- The $\boldsymbol{x}$ –***intercept*** of a graph is the location where the graph crosses the ________________.
- The y –coordinate of the x –intercept is always ________.
- The $\boldsymbol{y}$ –***intercept*** of a graph is the location where the graph crosses the ________________.
- The x –coordinate of the y –intercept is always ____________.
- The x –intercept is the ________________ to a function or graph.

Each of these features are very helpful in understanding real world context.

Let's Practice!

4. Consider the following graph that represents the height, in feet, of a water balloon dropped from a 2nd story window after a given number of seconds.



   a. What is the x –intercept?

   b. What is the y –intercept?

   c. Label the intercepts on the graph.

Try It!

5. Refer to the previous problem for the following questions.

   a. What does the y –intercept represent in this real world context?

   b. What does the x –intercept represent in this real world context?

   c. What is the solution to this situation?

Section 3 – Topic 9
Key Features of Graphs of Functions – Part 2

Let's discuss other key features of graphs of functions.

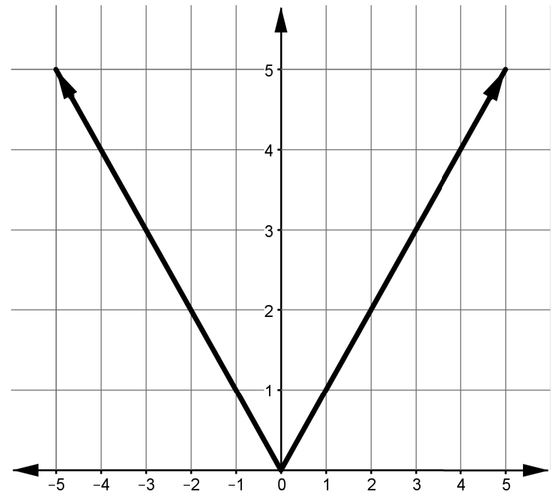
- ***Domain***: the input or the _____ values.
- ***Range***: the ____________________ or the y-values.
- ***Increasing intervals***: as the x-values _________________, the y-values _________________.
- ***Decreasing intervals***: as the x-values _________________, the y-values _________________.
- ***Relative maximum***: the point on a graph where the interval changes from __________________ to __________________.
- ***Relative minimum***: the point on a graph where the interval changes from __________________ to __________________.

STUDY EDGE TIP

We read a graph from left to right to determine if it is increasing or decreasing, like reading a book.

Let's Practice!

1. Use the following graph of an ***absolute value function*** to answer the questions below.



   a. Define the domain.

   b. Define the range.

   c. Where is the graph increasing?

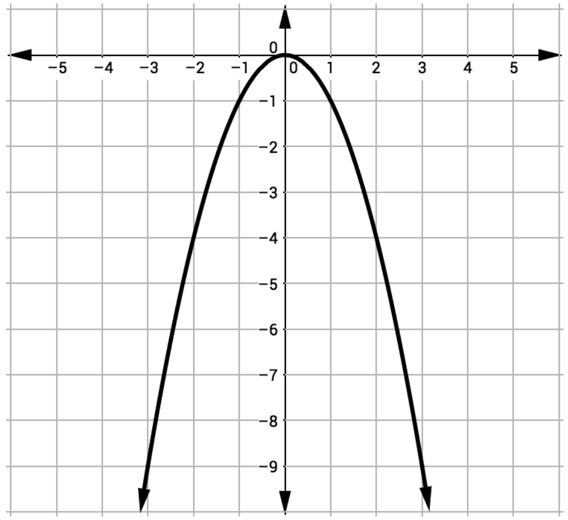
   d. Where is the graph decreasing?

   e. Identify any relative maximums.

   f. Identify any relative minimums.

Try It!

2. Use the graph of the following ***quadratic function*** to answer the questions below.



   a. Define the domain.

   b. Define the range.

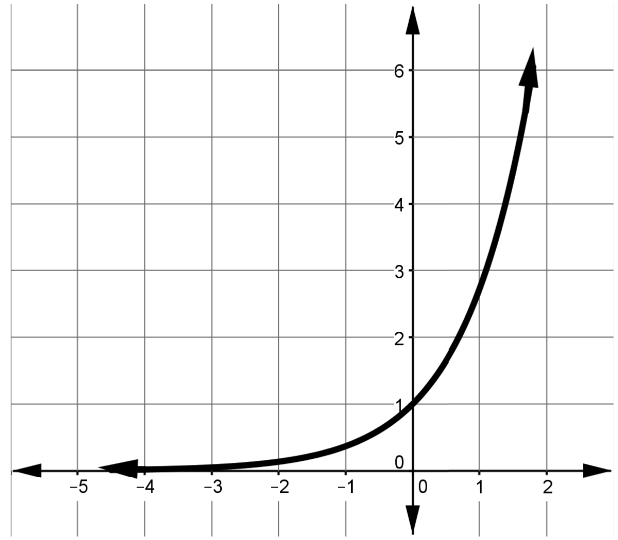
   c. Where is the graph increasing?

   d. Where is the graph decreasing?

   e. Identify any relative maximums.

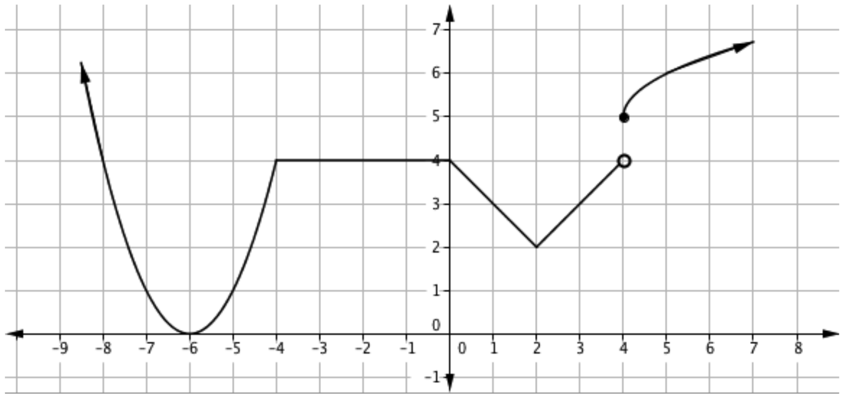
   f. Identify any relative minimums.

3. Describe everything you know about the following graph of an ***exponential function***.



BEAT THE TEST!

1. The following graph is a piecewise function.



Which of the following statements are true about the graph? Select all that apply.

- ☐ The graph is increasing when the domain is $-6 < x < -4$.
- ☐ The graph has one relative minimum.
- ☐ The graph is increasing when $-4 \leq x \leq 0$.
- ☐ The graph is increasing when $x > 4$.
- ☐ The graph is decreasing when the domain is $\{x|x < -6 \cup x > 2\}$.
- ☐ The range is $\{y|0 \leq y < 4 \cup y \geq 5\}$.
- ☐ There is a relative minimum at $(2, 2)$.

Section 3 – Topic 10
Understanding Piecewise-Defined Functions

What is a ***piecewise function***?

- ➢ A function made up of distinct "________________" based on different rules for the _________________.
- ➢ The pieces of a piecewise function are graphed together on the same coordinate plane.
- ➢ The ***domain*** is the ______________________, or the x-values.
- ➢ The ***range*** is the ____-values, or output.
- ➢ Since it is a function, it will pass the vertical line test.

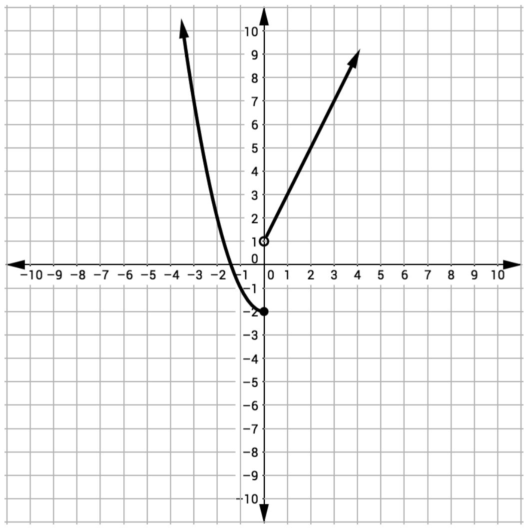
Describe an example of a piecewise function used in our daily lives.

Consider the following piecewise-defined function:

$$f(x) = \begin{cases} x^2 - 2, \text{when } x \le 0 \\ 2x + 1, \text{when } x > 0 \end{cases}$$

- Each function has a defined __________ value, or rule.
  - ○ x is less than or equal to 0 for the first function.
  - ○ x is greater than zero for the second function.

- Both of these functions will be on the same graph. They are the "pieces" of this completed piecewise-defined function.



Label the "pieces" of $f(x)$ above.

Let's note some of the features of the graph:

- The domain of the piecewise graph can be represented with intervals. If we define the first interval as $x \le 0$, the second interval would be ________________.

- The graph is nonlinear (curved) when the domain is ________________.

- The graph is linear when the domain is ______________.

- There is one closed endpoint on the graph, which means that the particular domain value, zero, is ______________ in that piece of the function. This illustrates the inclusion of zero in the function ________________.

- There is one open circle on graph, which means that the particular value, zero, is _______ ________________ in that piece of the function. This illustrates the constraint that $x > 0$ for the function ______________.

Let's Practice!

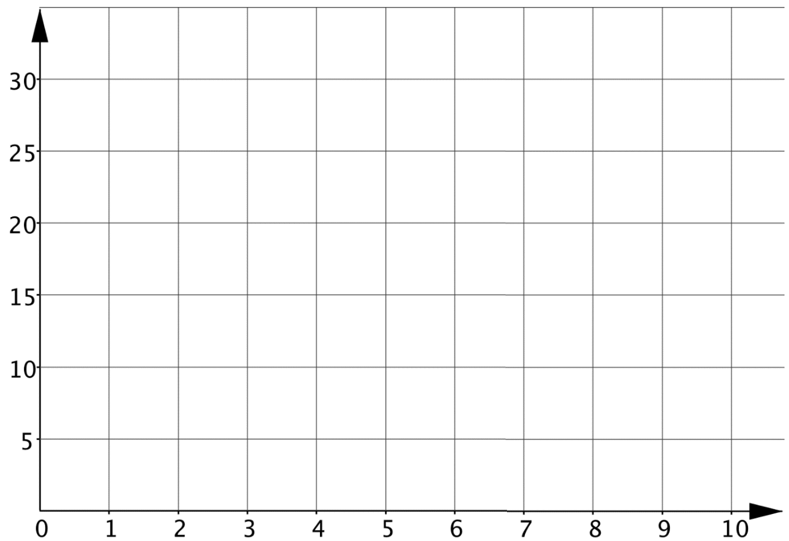
1. Airheadz, a trampoline gym, is open seven days a week for 10 hours a day. Their prices are listed below:

   Two hours or less: $\$15.00$
   Between two and five hours: $\$25.00$
   Five or more hours: $\$30.00$

   The following piecewise function represents their prices:

$$f(x) = \begin{cases} 15, \text{when } 0 < x \leq 2 \\ 25, \text{when } 2 < x < 5 \\ 30, \text{when } \quad x \geq 5 \end{cases}$$

   Graph the above function in the following grid.



- $f(x)$ is a special type of piecewise function known as a ____________ function, which resembles a series of steps.
- Step functions pair every x-value in a given interval (particular section of the _______________) with a single value in the range (______-value).

Try It!

2. Consider the previous graph in exercise 1.
   a. How many pieces are in the step function? Are the pieces linear or nonlinear?
   b. How many intervals make up the step function? What are the interval values?
   c. Why are open circles used in some situations and closed circles in others?
   d. How do you know this is a function?
   e. What is the range of this piecewise function?

BEAT THE TEST!

1. Evaluate the piecewise-defined function for the given values of x by matching the domain values with the range values.

$$f(x)=\begin{cases} x-1, & x\leq -2 \\ 2x-1, & -2<x\leq 4 \\ -3x+8, & x>4 \end{cases}$$

x	$f(x)$
8	7
-2	3
4	-3
2	-16
-5	-6
0	-1

2. Complete the following sentences by choosing the correct answer from each box.

Part A: Piecewise-defined functions are represented by

- o one function
- o at least one function
- o at least two functions

that must correspond to

- o different domain values
- o different range values
- o real numbers

.

Part B: When evaluating piecewise-defined functions, choose which equation to use based on the

- o constant
- o x-value
- o slope

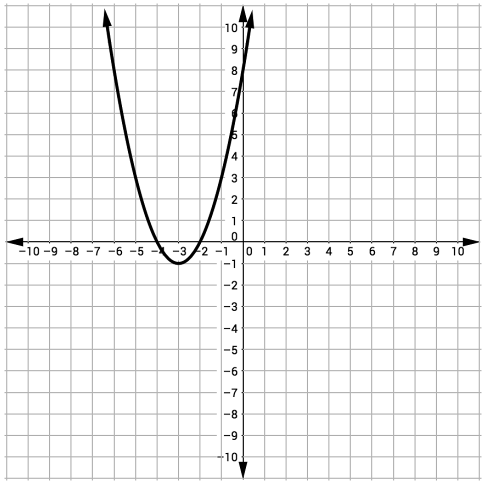
and then substitute and evaluate using

- o exponent rules
- o order of operations
- o your instincts

.

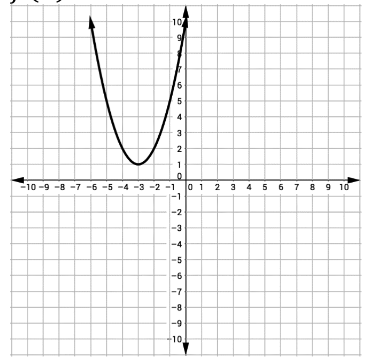
Section 3 – Topic 11
Transformations of Functions

The graph of $f(x)$ is shown below.

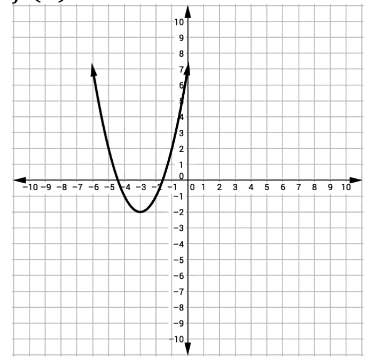


The graphs on the following page are transformations of $f(x)$. Describe what happened in each graph.

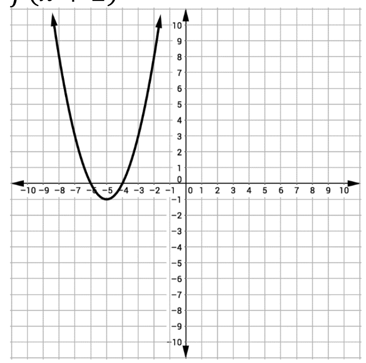
$f(x)+2$



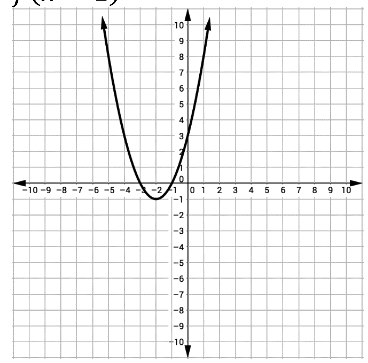
$f(x)-1$



$f(x+2)$



$f(x-1)$



Which graphs transformed the independent variable?

Which graphs transformed the dependent variable?

Let's Practice!

1. In the following functions, state whether the independent or dependent variable is being transformed and describe the transformation (assume $k > 0$).

   a. $f(x) + k$

   b. $f(x) - k$

   c. $f(x + k)$

   d. $f(x - k)$

2. The following table represents the function $g(x)$.

x	$g(x)$
-2	0.25
-1	0.5
0	1
1	2
2	4

The function $h(x) = g(2x)$. Complete the table for $h(x)$.

x	$g(2x)$	$h(x)$
-1	$g(2(-1))$	
-0.5	$g(2(-0.5))$	
0		
0.5		
1		

Try It!

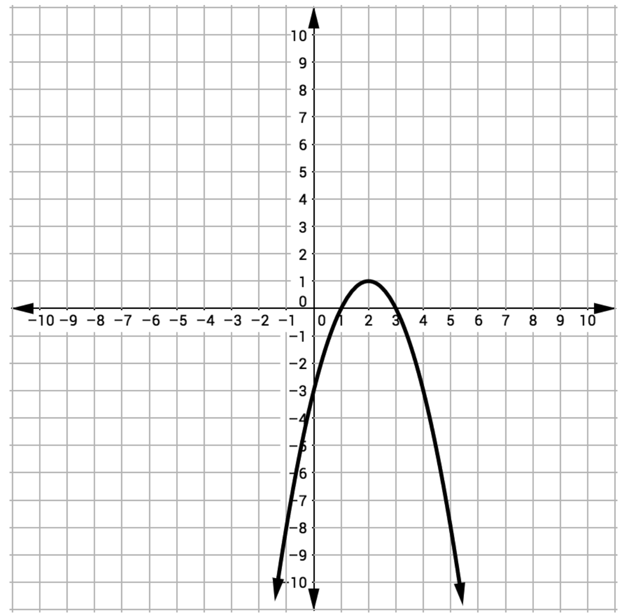
3. The table below shows the values for the function $f(x)$.

x	-2	-1	0	1	2
$f(x)$	4	2	0	2	4

Complete the table for the function $-\frac{1}{2}f(x)$.

x	$-\frac{1}{2}f(x)$
-2	
-1	
0	
1	
2	

4. The graph of $f(x)$ is shown below.

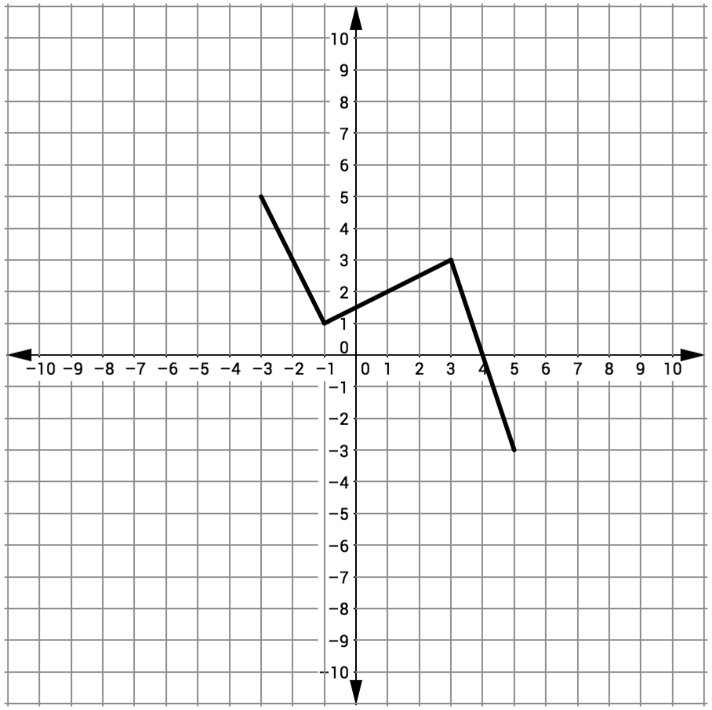


Let $g(x) = f(x + 3) - 2$.

Graph $g(x)$ on the coordinate plane with $f(x)$.

BEAT THE TEST!

1. The graph of $f(x)$ is shown below.



Let $g(x) = f(x-3)$ and $h(x) = f(x) - 3$.

Graph $g(x)$ and $h(x)$ on the coordinate plane with $f(x)$.

2. The table below shows the values for the function $p(x)$.

x	-4	-1	0	2	3
$p(x)$	12	6	4	8	10

Complete the table for the function $\frac{1}{2}p(x) - 3$.

x	$\frac{1}{2}p(x) - 3$

Section 4: Linear Functions

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.F-IF.1.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MAFS.912.F-IF.2.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MAFS.912.F-LE.1.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input/output pairs (including reading these from a table). This section focuses on linear functions and arithmetic sequences.
MAFS.912.F-BF.1.1	Write a function that describes a relationship between two quantities. Determine an explicit expression, a recursive process, or steps for calculation from a context.
MAFS.912.A-CED.1.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MAFS.912.A-CED.1.3	Represent constraints by equations or inequalities and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
MAFS.912.A-REI.3.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
MAFS.912.A-REI.3.6	Solve systems of linear equations exactly and approximately (*e.g., with graphs*), focusing on pairs of linear equations in two variables.
MAFS.912.A-REI.4.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
MAFS.912.A-REI.4.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (*e.g., using technology to graph the functions, make tables of values, or find successive approximations*). *Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* ***This section only covers linear cases.***

MAFS.912.A-REI.4.12	Graph the solutions to a linear inequality in two variables as a half-plane (*excluding the boundary in the case of a strict inequality*), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
MAFS.912.S-ID.3.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Topics in this Section

Topic 1: Arithmetic Sequences
Topic 2: Rate of Change of Linear Functions
Topic 3: Interpreting Rate of Change and y-intercept in a Real-World Context – Part 1
Topic 4: Interpreting Rate of Change and y-intercept in a Real-World Context – Part 2
Topic 5: Introduction to Systems of Equations
Topic 6: Finding Solution Sets to Systems of Equations Using Substitution and Graphing
Topic 7: Using Equivalent Systems of Equations
Topic 8: Finding Solution Sets to Systems of Equations Using Elimination
Topic 9: Solution Sets to Inequalities with Two Variables
Topic 10: Finding Solution Sets to Systems of Linear Inequalities

Section 4 – Topic 1
Arithmetic Sequences

Let's look at the following sequence of numbers:
$3, 8, 13, 18, 23, \dots$.

- The "…" at the end means that this ______________ goes on forever.
- $3, 8, 13, 18$, and 23 are the actual __________ of this sequence.
- There are 5 terms in this sequence so far:
  - 3 is the 1^{st} term.
  - 8 is the 2^{nd} term.
  - 13 is the _____ term.
  - 18 is the _____ term.
  - 23 is the _____ term.

This is an example of an ***arithmetic sequence***.

- This is a sequence where each term is the _________ of the previous term and a common difference, d.

We can represent this sequence in a table:

Term Number	Sequence Term	Term
1	a_1	3
2	a_2	8
3	a_3	13
4	a_4	
5	a_5	
⋮	⋮	⋮
n	a_n	

Function Notation	
$f(1)$	a formula to find the 1st term
	a formula to find the 2nd term
$f(3)$	a formula to find the ____ term
$f(4)$	a formula to find the ____ term
	a formula to find the ____ term
⋮	⋮
$f(n)$	a formula to find the ____

How can we find the 9th term of this sequence?

One way is to start by finding the previous term:

Term Number	Sequence Term	Term
1	a_1	3
2	a_2	$8 = 3 +$ ____
3	a_3	$13 = 8 +$ ____
4	a_4	$18 = 13 +$ ____
5	a_5	$23 =$ ____ $+ 5$
6	a_6	
7	a_7	
8	a_8	
9	a_9	

Function Notation	
$f(1)$	3
$f(2)$	$3 + 5$
$f(3)$	$8 + 5$
$f(4)$	$13 + 5$
$f(5)$	$18 + 5$
$f(6)$	$23 + 5$
$f(7)$	$28 + 5$
$f(8)$	$33 + 5$
$f(9)$	$38 + 5$

Write a general equation that we could use to find *any* term in the sequence.

This is a ***recursive formula***.

- In order to solve for a term, you must know the value of its preceding term.

Can you think of a situation where the recursive formula would take a long time to use?

Let's look at another way to find unknown terms:

Term Number	Sequence Term	Term	Function Notation	
1	a_1	3	$f(1)$	3
2	a_2	$8 = 3 + 5$	$f(2)$	$3 + 5(1)$
3	a_3	$13 = 8 + 5 = 3 + 5 + 5$	$f(3)$	$a_1 + 5(2)$
4	a_4	$18 = 13 + 5 = 3 + 5 + 5 + 5$	$f(4)$	$a_1 + 5(3)$
5	a_5	$23 = 18 + 5 = 3 + 5 + 5 + 5 + 5$	$f(5)$	$a_1 + 5(4)$
6	a_6	$28 = 23 + 5 = 3 + 5 + 5 + 5 + 5 + 5$	$f(6)$	$a_1 + 5(5)$
7	a_7	$33 = 28 + 5 = 3 + 5 + 5 + 5 + 5 + 5 + 5$	$f(7)$	$a_1 + 5(6)$
8	a_8	$38 = 33 + 5 = 3 + 5 + 5 + 5 + 5 + 5 + 5 + 5$	$f(8)$	
9	a_9	$43 = 38 + 5 = 3 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5$	$f(9)$	

Write a general equation that we could use to find *any* term in the sequence.

This is an ***explicit formula***.

- To solve for a term, you need to know the first term of the sequence and the difference by which the sequence is increasing or decreasing.

Let's Practice!

1. Consider the sequence $10, 4, -2, -8, \ldots$.

   a. Write a recursive formula for the sequence.

   b. Write an explicit formula for the sequence.

   c. Find the 42^{nd} term of the sequence.

Try It!

2. Consider the sequence $7, 17, 27, 37, \ldots$.

   a. Find the next three terms of the sequence.

   b. Write a recursive formula for the sequence.

   c. Write an explicit formula for the sequence.

   d. Find the 33^{rd} term of the sequence.

BEAT THE TEST!

1. Yohanna is conditioning all summer to prepare for her high school's varsity soccer team tryouts. She is incorporating walking planks into her daily workout training plan. Every day, she will complete four more walking planks than the day before.

   Part A: If she starts with five walking planks on the first day, write an explicit formula that can be used to find the number of walking planks Yohanna completes on any given day.

   Part B: How many walking planks will Yohanna do on the 12^{th} day?

   Ⓐ 49
   Ⓑ 53
   Ⓒ 59
   Ⓓ 64

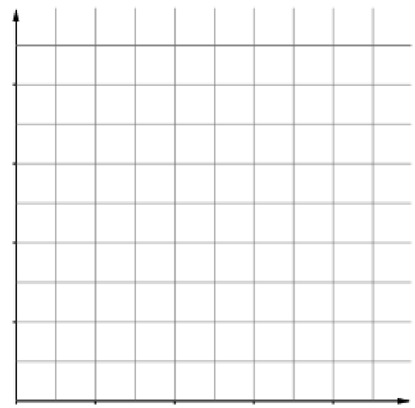
Section 4 – Topic 2
Rate of Change of Linear Functions

Génesis reads 16 pages of *The Fault in Our Stars* every day.

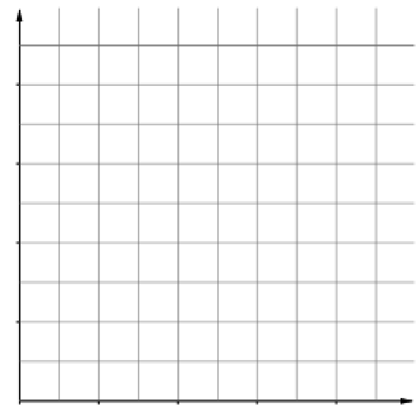
Zully reads 8 pages every day of the same book.

Represent both situations on the graphs below using the same scales on the axes for both graphs.

Graph 1: Génesis' Reading Speed



Graph 2: Zully's Reading Speed

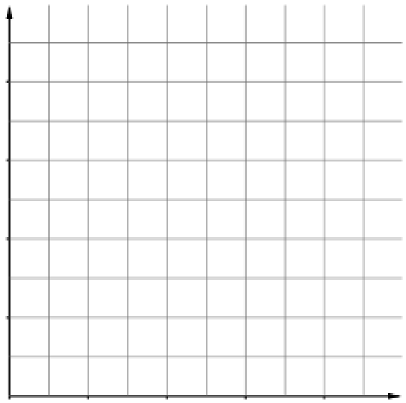


Aaron loves Cherry Coke. Each mini-can contains 100 calories.

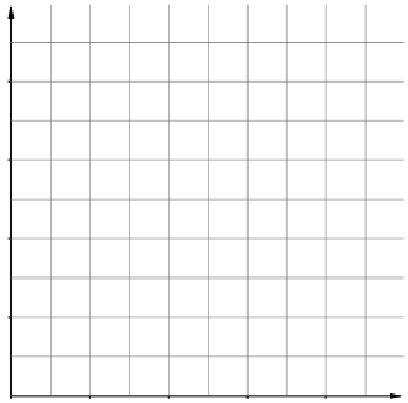
Jacobe likes to munch on carrot snack packs. Each snack pack contains 40 calories.

Represent both situations on the graphs below using the same scales for both graphs.

Graph 3: Aaron's Calorie Intake



Graph 4: Jacobe's Calorie Intake



In each of the graphs, we were finding the ***rate of change*** in the given situation.

What is the rate of change for each of the graphs?

Graph 1: ____________________ per ______________________

Graph 2: ____________________ per ______________________

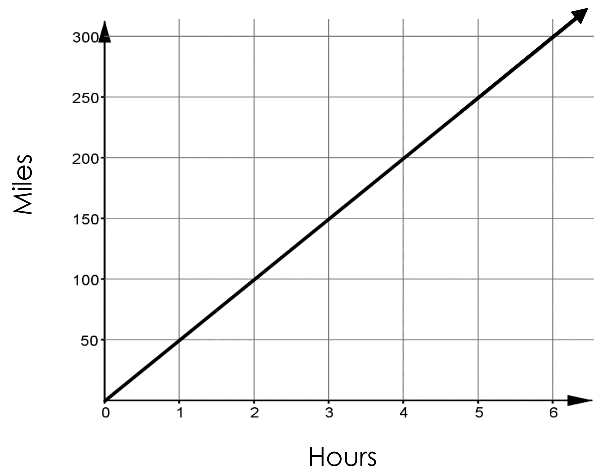
Graph 3: ____________________ per ______________________

Graph 4: ____________________ per ______________________

This is also called the ________________________ of the line.

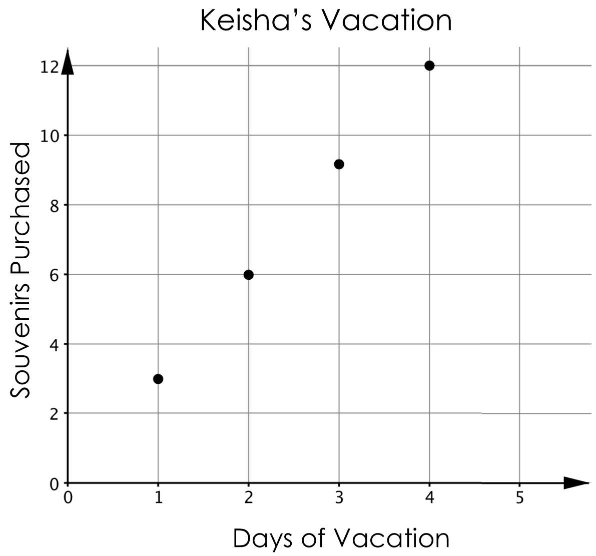
We can also find slope by looking at the $\frac{\textbf{change in } y}{\textbf{change in } x}$ or $\frac{\textbf{rise}}{\textbf{run}}$.

What is the slope of the following graph? What does the slope represent?



Let's Practice!

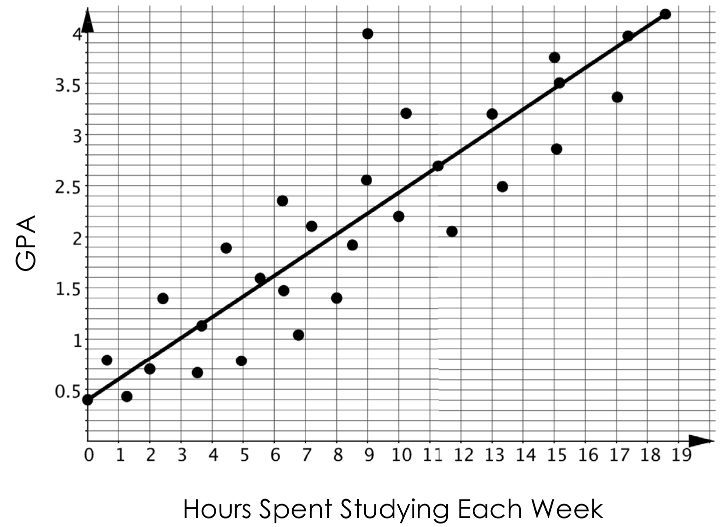
1. Consider the following graph.



a. What is the rate of change of the graph?

b. What does the rate of change represent?

2. Freedom High School collected data on the GPA of various students and the number of hours they spend studying each week. A scatterplot of the data is shown below with the line of best fit.

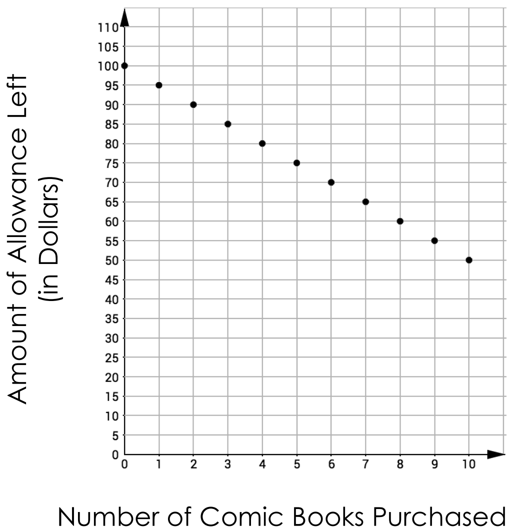


a. What is the slope of the line of best fit?

b. What does the slope represent?

Try It!

3. Sarah's parents give her $100.00 allowance at the beginning of each month. Sarah spends her allowance on comic books. The graph below represents the amount of money Sara spent on comic books last month.



a. What is the rate of change?

b. What does the rate of change represent?

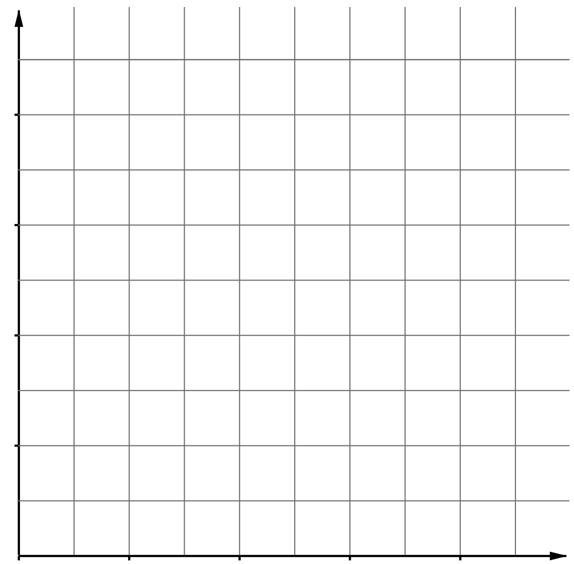
BEAT THE TEST!

1. A cleaning service cleans many apartments each day. The following table shows the number of hours the cleaners spend cleaning and the number of apartments they clean during that time.

Apartment Cleaning

Time (Hours)	1	2	3	4
Apartments Cleaned	2	4	6	8

Part A: Represent the situation on the graph below.



Part B: The data suggest a linear relationship between the number of hours spent cleaning and the number of apartments cleaned. Assuming the relationship is linear, what does the rate of change represent in the context of this relationship?

Ⓐ The number of apartments cleaned after one hour.
Ⓑ The number of hours it took to clean one apartment.
Ⓒ The number of apartments cleaned each hour.
Ⓓ The number of apartments cleaned before the company started cleaning.

Part C: Which equation describes the relationship between the time elapsed and the number of apartments cleaned?

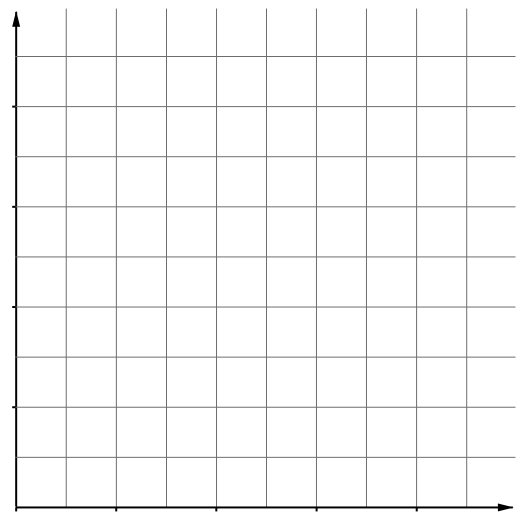
Ⓐ $y = x$
Ⓑ $y = x + 2$
Ⓒ $y = 2x$
Ⓓ $y = 2x + 2$

Section 4 – Topic 3
Interpreting Rate of Change and y-Intercept in a Real-World Context – Part 1

Cab fare includes an initial fee of $2.00 plus $3.00 for every mile traveled.

Define the variable and write a function that represents this situation.

Represent the situation on a graph.



What is the slope of the line? What does the slope represent?

At what point does the line intersect the y-axis? What does this point represent?

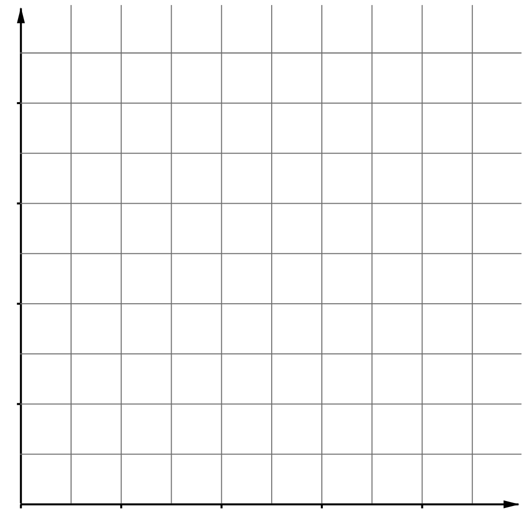
This point is the ***y*-intercept** of a line.

Let's Practice!

1. You saved $250.00 to spend over the summer. You decide to budget $25.00 to spend each week.

   a. Define the variable and write a function that represents this situation.

   b. Represent the situation on a graph.



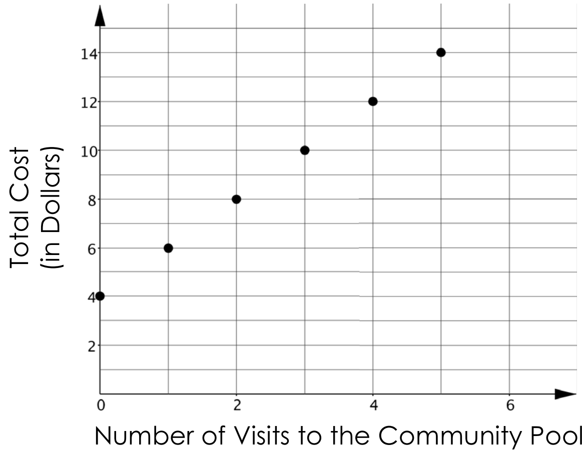
   c. What is the slope of the line? What does the slope represent?

   d. What is different about the slope of this line compared to our earlier problem? Why is it different?

   e. What is the y-intercept? What does this point represent?

Try It!

2. Consider the following graph:



   a. What is the slope of the line? What does the slope represent?

   b. What is the y-intercept? What does the y-intercept represent?

   c. Define the variables and write a function that represents this situation.

   d. What does each point represent?

Consider the three functions that you wrote regarding the cab ride, summer spending habits, and the community pool membership. What do you notice about the constant term and the coefficient of the x term?

- The constant term is the ______________________.
- The coefficient of the x is the __________ or _________________________.

These functions are written in ***slope-intercept*** form.

We can use slope-intercept form to graph any linear equation.

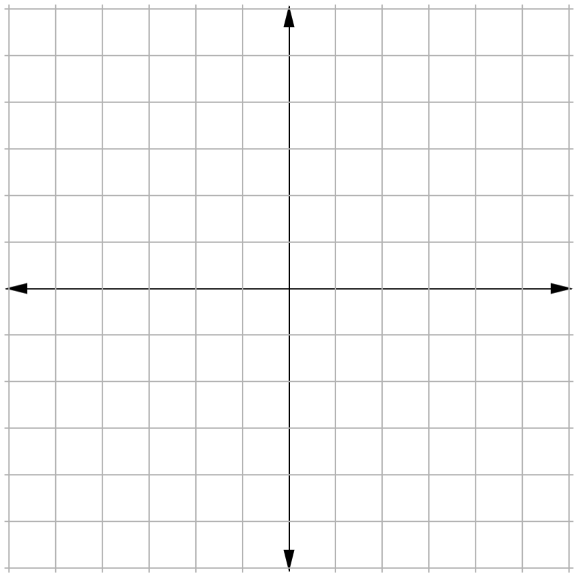
STUDY EDGE TIP

The coefficient of x is the slope and the constant term is the y-intercept ONLY if the equation is in slope-intercept form, $y = mx + b$.

Section 4 – Topic 4
Interpreting Rate of Change and y-Intercept in a Real-World Context – Part 2

Let's Practice!

1. Graph $y = 2x + 3$.



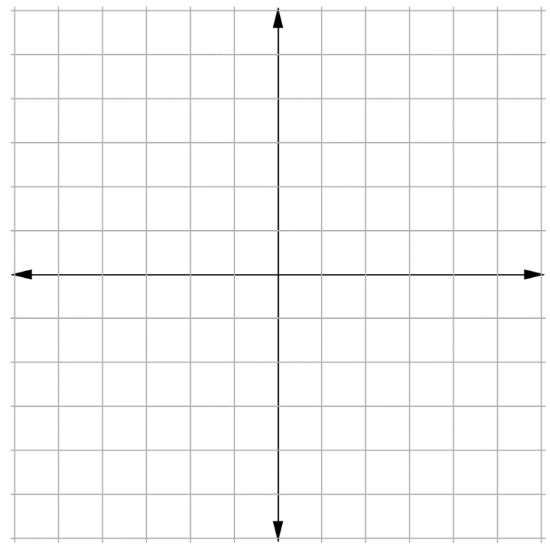
2. Consider the equation $2x + 5y = 1$.

   a. How does this equation look different from $0.$ slope-intercept form of an equation?

   b. Rewrite the equation in slope-intercept form.

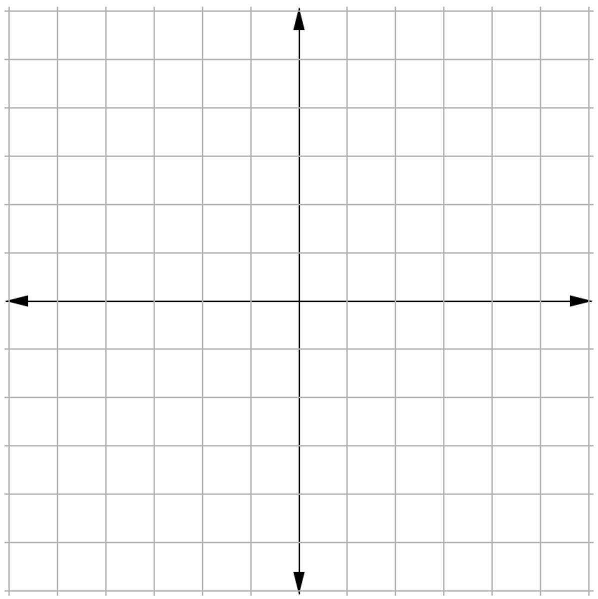
   c. Identify the slope and y-intercept.

   d. Graph the equation.



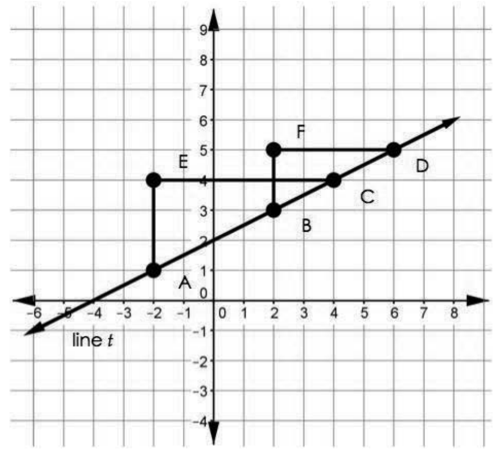
Try It!

3. Graph the equation $-4x - 5y = -10$.



BEAT THE TEST!

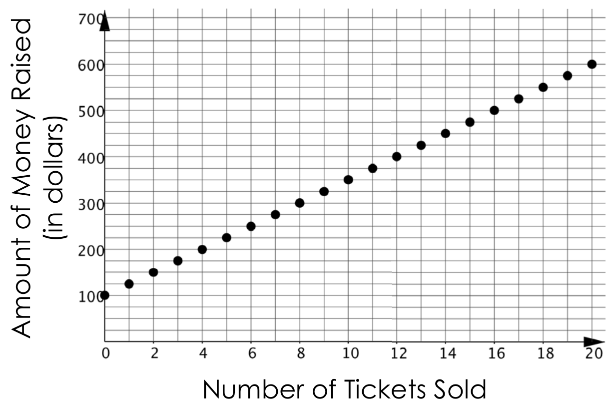
1. Line t, ΔECA, and ΔFDB are shown on the coordinate grid below.



Which of the following statements are true? Select all that apply.

- ☐ The slope of $\overline{AC}$ is equal to the slope of $\overline{BD}$.
- ☐ The slope of $\overline{AC}$ is equal to the slope of line t.
- ☐ The slope of line t is equal to $\frac{EC}{AE}$.
- ☐ The slope of line t is equal to $\frac{BF}{FD}$.
- ☐ The y-intercept of line t is 2.
- ☐ Line t represents a discrete function.

2. The senior class at Elizabeth High School was selling tickets to raise money for prom. The graph below represents the situation.



Part A: How much does one ticket cost?

Part B: How much money did the senior class have at the start of the fundraiser?

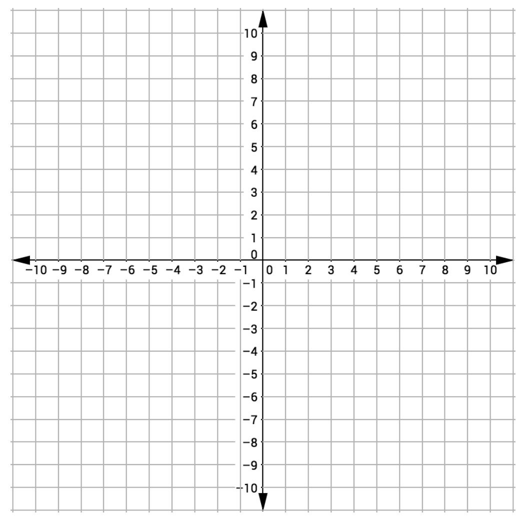
Section 4 – Topic 5
Introduction to Systems of Equations

A system of equations is a set of 2 or more equations.

Consider the following systems of equations.

Line 1: $2x - y = -5$
Line 2: $2x + y = 1$

Graph the system of equations on the coordinate plane below.



Recall that a solution to a linear equation is any ordered pair that makes that equation a true statement.

What do you notice about the point $(-2,5)$?

What do you notice about the point $(1,7)$?

What do you notice about the point $(-1,3)$?

What do you notice about the point $(1,1)$?

Let's Practice!

1. Consider the following system of equations made up of Line 1 and Line 2.

$$\text{Line } 1: \; 5x + 2y = 8$$
$$\text{Line } 2: -3x - 2y = -4$$

Complete the following sentences.

a. The ordered pair $(-2,5)$ is a solution to

- ○ Line 1
- ○ Line 2
- ○ The system of equations

.

b. The ordered pair $(2,-1)$ is a solution to

- ○ Line 1
- ○ Line 2
- ○ The system of equations

.

c. The ordered pair $(0,4)$ is a solution to

- ○ Line 1
- ○ Line 2
- ○ The system of equations

.

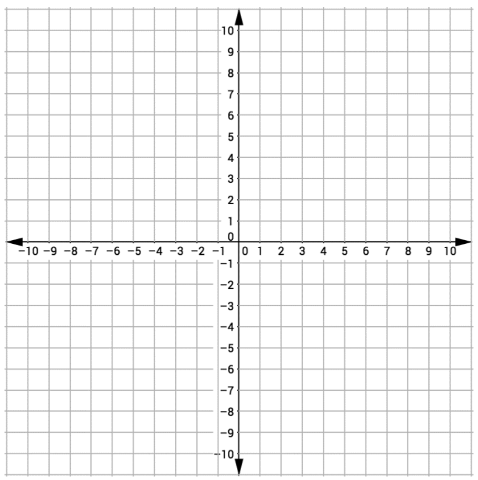
2. Is there ever a time when a system of equations will not have a solution? If so, sketch an example.

Try It!

3. Consider the following system of equations.

$$x - y = 3$$
$$-2x + 2y = -6$$

   a. Sketch the graph of the system of equations.

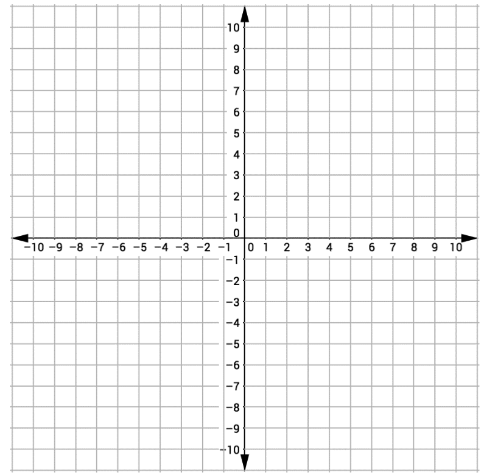


   b. What can be said about the solution to this system of equations?

4. Consider the following system of equations.

$$4x + 3y = 3$$
$$2x - 5y = -5$$

   a. Graph the system of equations.



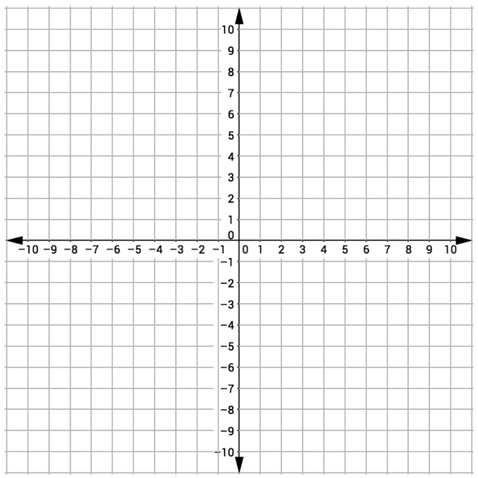
   b. What is the solution to the system?

BEAT THE TEST!

1. Consider the following system of equations.

$$x + y = 5$$
$$2x - y = -2$$

Part A: Sketch the graph of the system of equations.



Part B: Determine the solution to the system of equations.

Part C: Create a third equation that could be added to the system so that the solution does not change. Graph the line on the coordinate plane above.

Section 4 – Topic 6
Finding Solution Sets to Systems of Equations Using Substitution and Graphing

There are many times that we are able to use systems of equations to solve real world problems.

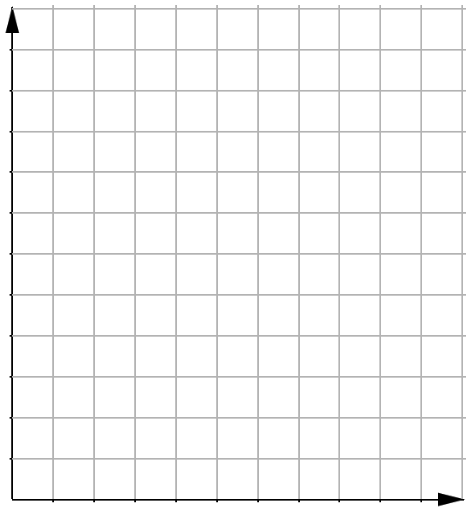
One method of solving systems of equations is by graphing like we did in the previous video.

Let's Practice!

1. Brianna's lacrosse coach suggested that she practices yoga to improve her flexibility. "Yoga-ta Try This!" Yoga Studio has two membership plans. Plan A costs $20.00 per month plus $10.00 per class. Plan B costs $100.00 per month for unlimited classes.

   a. Define a variable and write two functions to represent the monthly cost of each plan.

b. Represent the two situations on the graph below.



c. What is the rate of change for each plan?

d. What does the rate of change represent in this situation?

e. What do the y-intercepts of the graphs represent?

2. Brianna is trying to determine which plan is more appropriate for the number of classes she wants to attend.

   a. When will the two plans cost exactly the same?

   b. When is plan A the better deal?

   c. When is plan B the better deal?

We can also help Brianna determine the best plan for her without graphing. Consider our two equations again.

We simply want to know when the total costs would be equal.

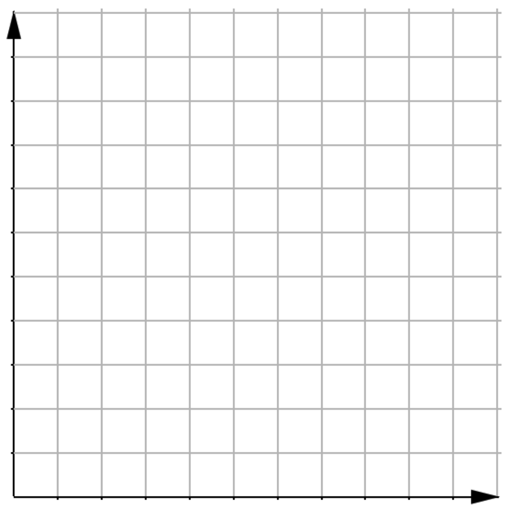
- Set the two plans equal to each other and solve for the number of visits.
- This method is called solving by ____________________.

Try It!

3. Vespa Scooter Rental rents scooters for $45.00 and $0.25 per mile. Scottie's Scooter Rental rents scooters for $35.00 and $0.30 per mile.

   a. Define a variable and write two functions to represent the situation.

   b. Represent the two situations on the graph below.



   c. What is the rate of change of each line? What do they represent?

   d. What do the y-intercepts of each line represent?

It's difficult to find the solution by looking at the graph. In such cases, it's better to use substitution to solve the problem.

4. Use the substitution method to help the renter determine when the two scooter rentals will cost the same amount.

   a. When will renting a scooter from Vespa Scooter Rental cost the same as renting a scooter from Scottie's Scooter Rental?

   b. Describe a situation when renting from Vespa Scooter Rental would be a better deal than renting from Scottie's Scooter Rental.

BEAT THE TEST!

1. Lyle and Shaun open a savings account at the same time. Lyle deposits $\$100$ initially and adds $\$20$ per week. Shaun deposits $\$500$ initially and adds $\$10$ per week. Lyle wants to know when he will have the same amount in his savings account as Shaun.

   Part A: Write two equations to represent the amount of money Lyle and Shaun have in their accounts.

   Part B: Which method would you use to solve the problem, substitution or graphing? Explain your answer.

   Part C: After how many weeks of making the additional deposits will Lyle have the same amount of money as Shaun?

Section 4 – Topic 7
Using Equivalent Systems of Equations

An ordered pair that satisfies all equations in a system is called the ________________ to that system.

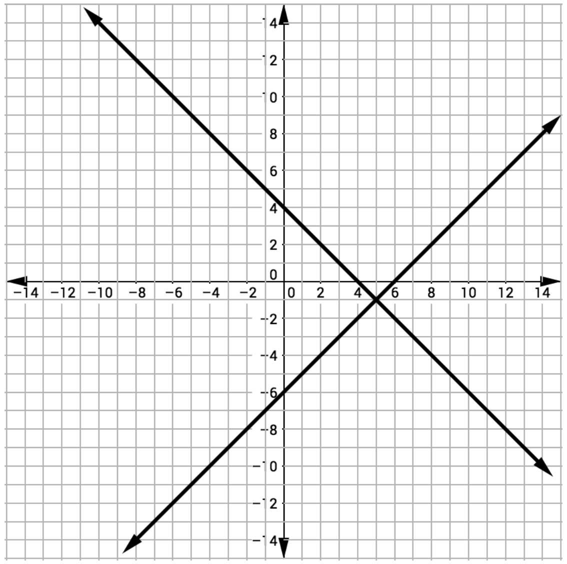
If two systems of equations have the same solution, they are called ___________________ systems.

Let's explore how to write equivalent systems of equations.

Consider the following system of equations:

$$x + y = 4$$
$$x - y = 6$$

The solution to this system is $(5, -1)$. We can also see this when we graph the lines.



Describe the result when we multiply either of the equations by some factor.

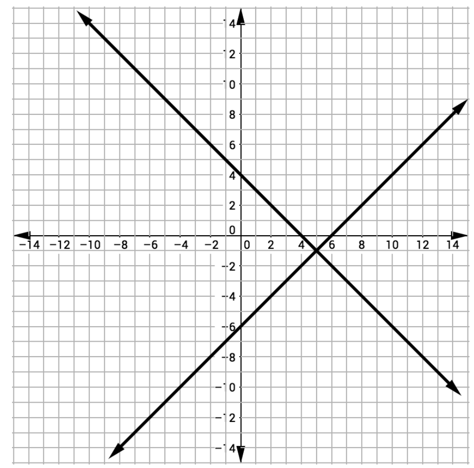
Use this process to write an equivalent system.

Consider the original system of equations again.

$$x + y = 4$$
$$x - y = 6$$

What is the resulting equation when we add the two equations in the system together?

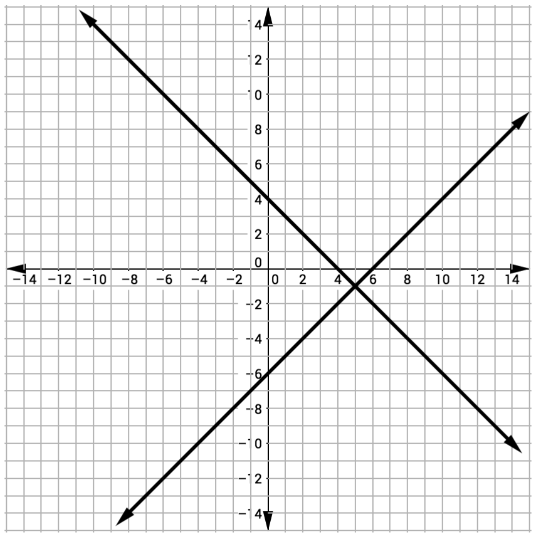
Graph the new equation on the same coordinate plane with our original system.



Algebraically, show that $(5, -1)$ is also a solution to the sum of the two lines.

What is the resulting equation when we subtract the second equation from the first equation?

Graph the new equation on the same coordinate plane with our original system.



Algebraically, show that $(5, -1)$ is also a solution to the difference of the two lines.

Let's revisit the original system:

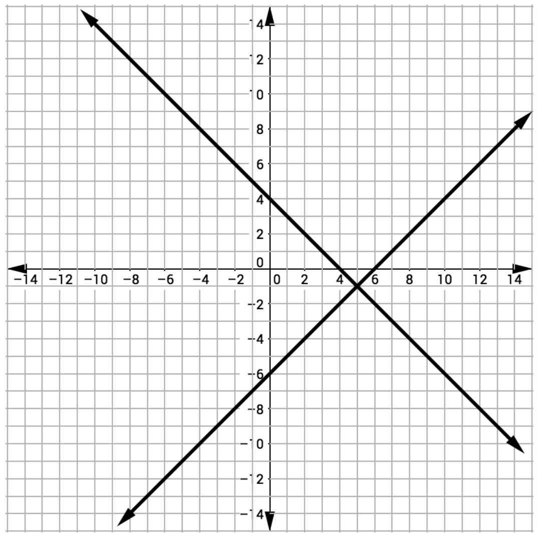
Equation 1: $x + y = 4$
Equation 2: $x - y = 6$

Complete the following steps to show that replacing one equation by the sum of that equation and a multiple of the other equation produces a system with the same solutions.

Create a third equation by multiplying Equation 1 by two.

Create a fourth equation by finding the sum of the third equation and Equation 2.

Graph the fourth equation on the same coordinate plane with our original system.



Algebraically, show that (5,-1) is a solution to the fourth equation.

Let's Practice!

1. Consider the following system, which has a solution of $(2, 5)$ and $M, N, P, R, S,$ and T are non-zero real numbers:

$$Mx + Ny = P$$
$$Rx + Sy = T$$

   Write two new equations that could be used to create an equivalent system of equations.

Try It!

2. List three ways that we can write new equations that can be used to create equivalent systems.

BEAT THE TEST!

1. The system $\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$ has the solution $(1, -3)$, where A, B, C, D, E, and F are non-zero real numbers. Select all the systems of equations with the same solution.

- ☐ $(A - D)x + (B - E)y = C - F$
  $Dx + Ey = F$
- ☐ $(2A + D)x + (2B + E)y = C + 2F$
  $Dx + Ey = F$
- ☐ $Ax + By = C$
  $-3Dx - 3Ey = -3F$
- ☐ $(A - 5D)x + (B - 5E)y = C - 5F$
  $Dx + Ey = F$
- ☐ $Ax + (B + E)y = C$
  $(A + D)x + Ey = C + F$

Section 4 – Topic 8
Finding Solution Sets to Systems of Equations Using Elimination

Consider the following system of equations:

$$2x + y = 8$$
$$x - 2y = -1$$

Write an equivalent system that will eliminate one of the variables when you add the equations.

Determine the solution to the system of equations.

Describe what the graph of the two systems would look like.

This method of solving a system is called ____________________.

Let's Practice!

1. Ruxin and Andre were invited to a Super Bowl party. They were asked to bring pizzas and sodas. Ruxin brought three pizzas and four bottles of soda and spent $48.05. Andre brought five pizzas and two bottles of soda and spent $67.25.

   a. Write a system of equations to represent the situation.

   b. Write an equivalent system that will eliminate one of the variables when you add the equations.

   c. Solve the system to determine the cost of one pizza and one soda.

Try It!

2. Jazmin and Justine went shopping for back to school clothes. Jazmin purchased three shirts and one pair of shorts and spent $\$38.00$. Justine bought four shirts and three pairs of shorts and spent $\$71.50$.

   a. Assuming all the shirts cost the same amount and all the shorts cost the same amount, write a system of equations to represent each girl's shopping spree.

   b. Use the elimination method to solve for the price of one pair of shorts.

BEAT THE TEST!

1. Complete the following table.

Solve by Elimination: $\begin{cases} 2x - 3y = 8 \\ 3x + 4y = 46 \end{cases}$

Operations	Equations	Labels
	$2x - 3y = 8$ $3x + 4y = 46$	Equation 1 Equation 2
☐	$-6x + 9y = -24$	New equation 1
Multiply equation 2 by 2.	☐	New equation 2
☐	$-6x + 9y = -24$ $6x + 8y = 92$ $17y = 68$	
Divide by 17.	☐	
Solve for x.	☐	
Write x and y as coordinates.	(☐ , ☐)	Solution to the system

2. Which of the systems of equations below could not be used to solve the following system for x and y?

$$6x + 4y = 24$$
$$-2x + 4y = -10$$

Ⓐ $6x + 4y = 24$
$2x - 4y = 10$

Ⓑ $6x + 4y = 24$
$-4x + 8y = -20$

Ⓒ $18x + 12y = 72$
$-6x + 12y = -30$

Ⓓ $12x + 8y = 48$
$-4x + 8y = -10$

Section 4 – Topic 9
Solution Sets to Inequalities with Two Variables

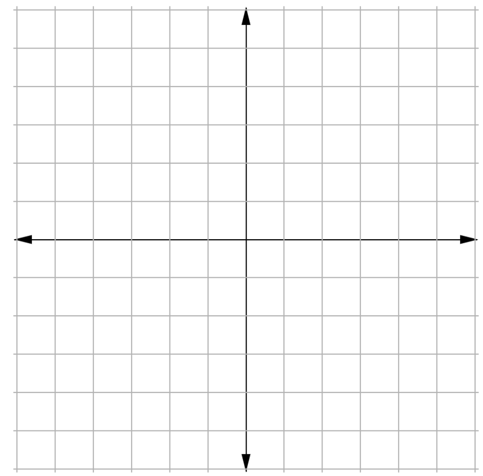
Consider the following linear inequality.

$$y \geq 2x - 1$$

Underline each ordered pair (x, y) that is a solution to the above inequality.

$(0,5)$ $(4,1)$ $(-1,-1)$ $(1,1)$ $(3,0)$ $(-2,3)$ $(4,3)$ $(-1,-3)$

Plot each solution as a point (x, y) in the coordinate plane.



Graph the line $y = 2x - 1$ in the same coordinate plane. What do you notice about the solutions to the inequality $y \geq 2x - 1$ and the graph of the line $y = 2x - 1$?

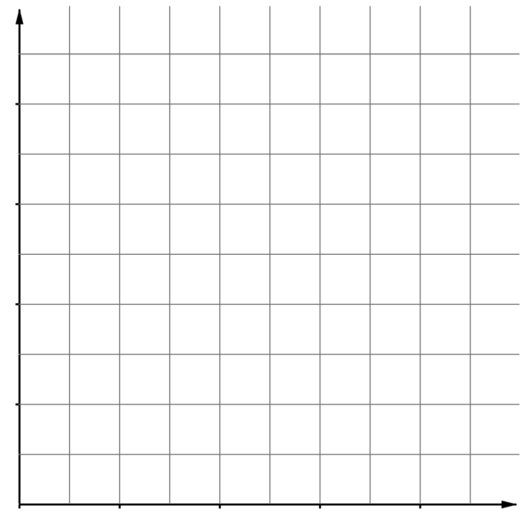
Let's Practice!

1. The senior class is raising money for Grad Bash. The students' parents are donating cakes. The students plan to sell entire cakes for $20.00 each and slices of cake for $3.00 each. If they want to raise at least $500.00, how many of each could they sell?

   a. List two possibilities for the number of whole cakes and cake slices students could sell to reach their goal of raising at least $500.00.

   b. Write an inequality to represent the situation.

   c. Graph the region where the solutions to the inequality would lie.



   d. What is the difference between the ordered pairs that fall on the line and the ones that fall in the shaded area?

   e. What does the x-intercept represent?

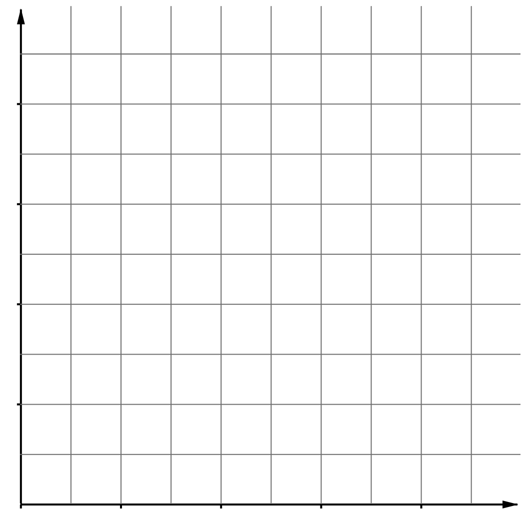
Try It!

2. The freshman class wants to include at least 120 people in the pep rally. Each skit will have 15 people, and the dance routines will feature 12 people.

   a. List two possible combinations of skits and dance routines.

   b. Write an inequality to represent the situation.

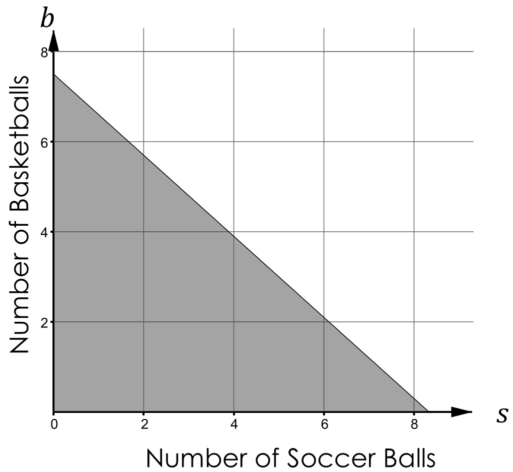
   c. Graph the region where the solutions to the inequality lie.



   d. What does the y-intercept represent?

BEAT THE TEST!

1. Coach De Leon purchases sports equipment. Basketballs cost $20.00 each, and soccer balls cost $18.00 each. He had a budget of $150.00. The graph shown below represents the number of basketballs and soccer balls he can buy given his budget constraint.



Part A: Write an inequality to represent the situation.

Part B: Determine whether these combinations of basketballs, b, and soccer balls, s, can be purchased.

	$b=5$ $s=3$	$b=2$ $s=4$	$b=7$ $s=3$	$b=0$ $s=8$	$b=8$ $s=0$	$b=6$ $s=3$	$b=4$ $s=7$
Yes	○	○	○	○	○	○	○
No	○	○	○	○	○	○	○

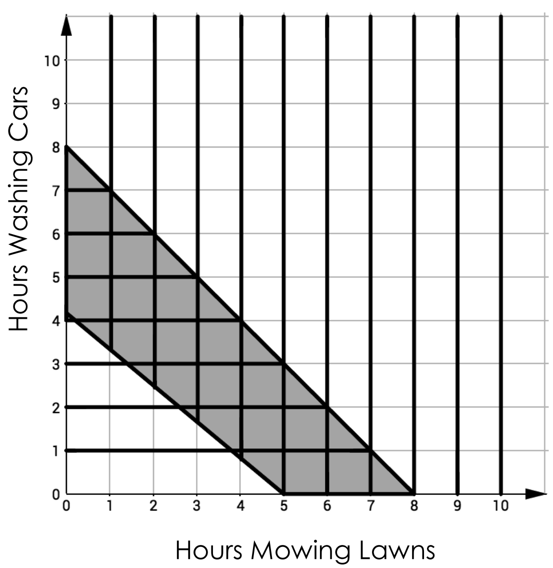
Section 4 – Topic 10
Finding Solution Sets to Systems of Linear Inequalities

Juan must purchase car insurance. He needs to earn at least $50.00 a week to cover the payments. The most he can work each week is 8 hours because of football practice. Juan can earn $10.00 per hour mowing yards and $12.00 per hour washing cars.

The system $\begin{cases} 10x + 12y \geq 50 \\ x + y \leq 8 \end{cases}$ represents Juan's situation.

Define the variables.

The graph below depicts Juan's situation. Interpret the graph and identify two different solutions for Juan's situation.

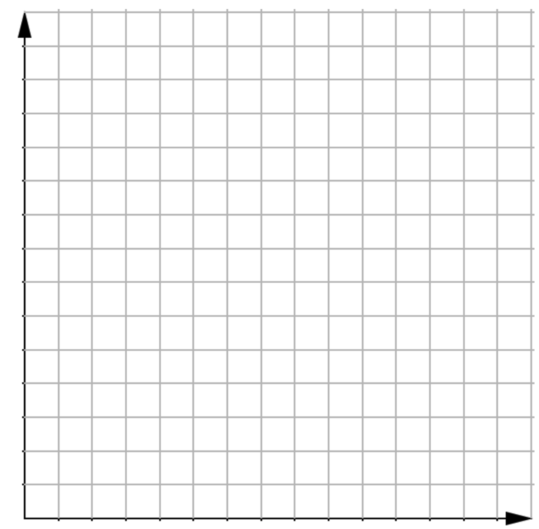


Let's Practice!

1. Bristol is having a party and has invited 24 friends. She plans to purchase sodas that cost $5.00 for a 12-pack and chips that cost $3.00 per bag. She wants each friend to have at least two sodas. Bristol's budget is $35.00.

   a. Write a system of inequalities to represent the situation.

   b. Graph the region of the solutions to the inequality.



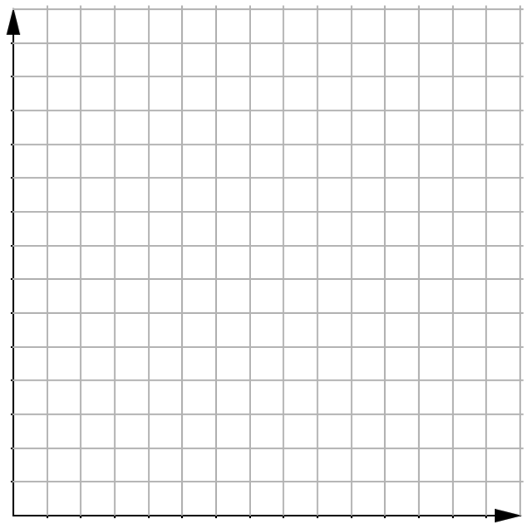
c. Name two different solutions for Bristol's situation.

Try It!

2. Anna is an avid reader. Her generous grandparents gave her money for her birthday, and she decided to spend at most $\$150.00$ on books. *Reading Spot* is running a special: all paperback books are $\$8.00$ and hardback books are $\$12.00$. Anna wants to purchase at least 12 books.

   a. Write a system of inequalities to represent the situation.

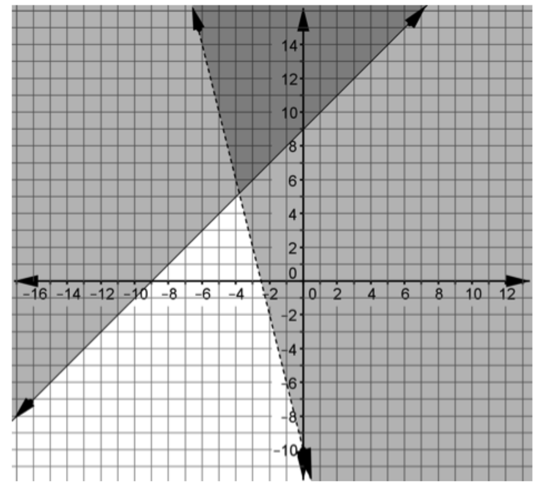
   b. Graph the region of the solutions to the inequality.



   c. Name two different solutions for Anna's situation.

BEAT THE TEST!

1. Tatiana is reviewing for the Algebra 1 Final exam. She made this graph representing a system of inequalities:



Part A: Underline the ordered pairs below that represent solutions to the system of inequalities.

$(-8,3)$	$(-3,8)$	$(-1,9)$	$(-4,9)$	$(9,6)$	$(0,9)$
$(5,5)$	$(-5,10)$	$(-9,1)$	$(-2,7)$	$(1,6)$	$(0,0)$

Part B: Derive the system of inequalities that describes the region of the graph Tatiana drew.

Section 5 – Quadratic Functions – Part 1

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.A-SSE.1.2	Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored* $(x^2 - y^2)(x^2 + y^2)$.
MAFS.912.A-SSE.2.3.a.b	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MAFS.912.F-IF.3.8.a	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MAFS.912.A-REI.2.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (*e.g., for* $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions.
MAFS.912.F-IF.2.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

Topics in this Section

Section 5 – Topic 1
Real-World Examples of Quadratic Functions

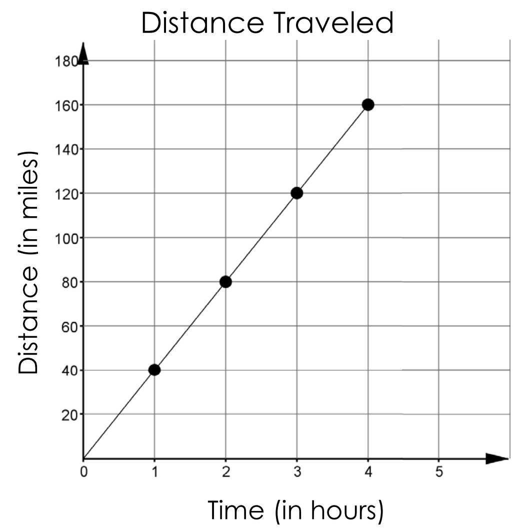
Let's revisit linear functions.

Imagine that you are driving down the road at a constant speed of 40 mph. This is a linear function.

We can represent the distance traveled versus time on a table (to the right).

Time (in hours)	Distance Traveled (in miles)
1	40
2	80
3	120
4	160

We can represent the scenario on a graph:



We can represent the distance traveled $d(t)$, in terms of time, t hours, with the equation $d(t) = 40t$.

Linear functions **always** have a constant rate of change. In this section, we are going to discover a type of non-linear function.

Consider the following situation:

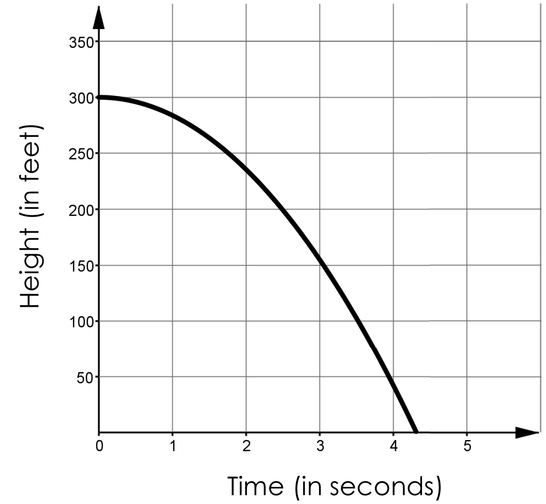
Liam dropped a watermelon from the top of a $300\ ft$ tall building. He wanted to know if the watermelon was falling at a constant rate over time. He filmed the watermelon's fall and then recorded his observations in the following table.

Time (in seconds)	Height (in feet)
0	300.0
1	283.9
2	235.6
3	155.1
4	42.4

What do you notice about the rate of change?

Why do you think that the rate of change is not constant?

Liam entered the data of the falling watermelon into his graphing calculator. The graph below displays the first quadrant of the graph.



What is the independent variable?

What is the dependent variable?

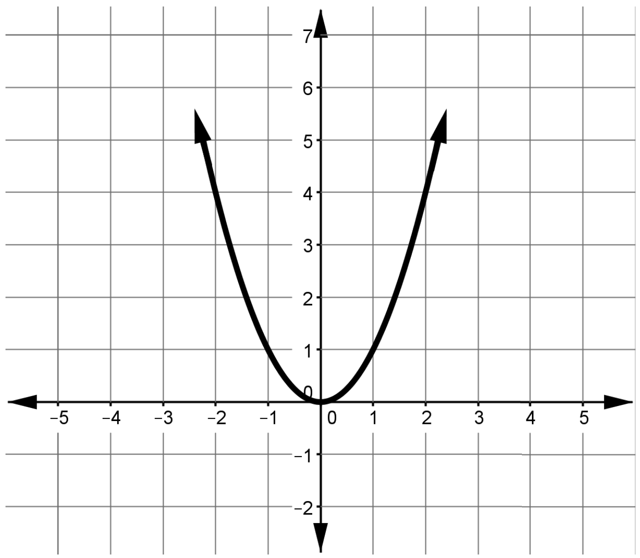
Liam then used his calculator to find the equation of the function:

$$h(t) = -16t^2 + 300$$

Important facts:

- We call this non-linear function a ____________________.
- The general form of the equation is _________________________.

The graph of $f(x) = x^2$ is shown below:



- This graph is called a ________________________.

Why did we only consider the first quadrant of Liam's graph?

In Liam's graph, what was the watermelon's height when it hit the ground?

The time when the watermelon's height was at zero is called the solution to this quadratic equation. We also call these the ____________ of the equation.

There was only one solution to Liam's equation. Describe a situation where there could be two solutions.

What about no solutions?

To solve a quadratic equation using a graph:

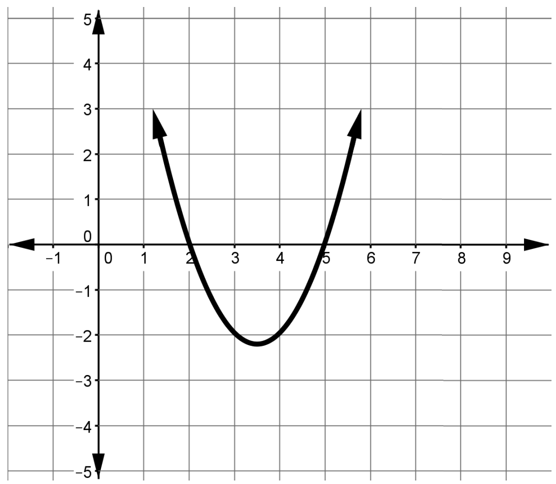
- look for the ________________________ of the graph.
- the solutions are the values where the graph intercepts the ________________________.

STUDY EDGE TIP

Zeros = x-intercepts = Solutions

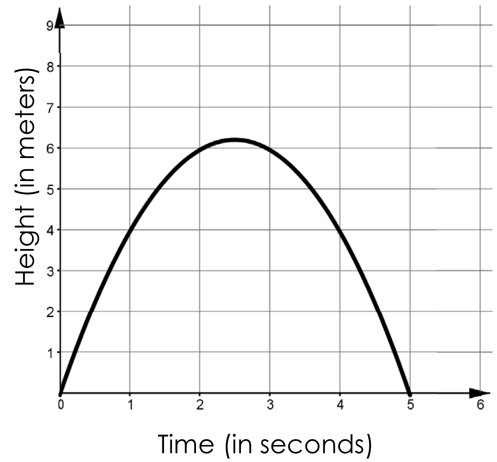
Let's Practice!

1. What are the solutions to the quadratic equation graphed below?



Try It!

2. Aaron shoots a water bottle rocket from the ground. A graph of height over time is shown below:



   a. What type of function best models the rocket's motion?

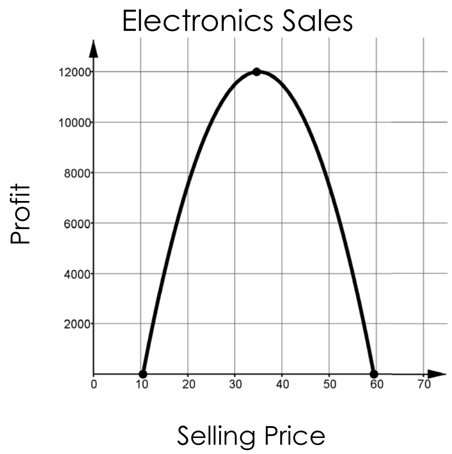
   b. After how many seconds did the rocket hit the ground?

   c. Estimate the maximum height of the rocket.

The maximum or minimum point of the parabola is called the ________________.

BEAT THE TEST!

1. Jordan owns an electronics business. During her first year in the business, she collected data and created the following graph showing the relationship between the selling price of an item and the profit.



Part A: Circle the solutions to the quadratic function graphed above.

Part B: What do the solutions represent?

Part C: Box the vertex of the graph.

Part D: What does the vertex represent?

Section 5 – Topic 2
Factoring Quadratic Expressions

Let's review the two methods we used for multiplying polynomials.

Area Model:

	x	$2y$	$-7z$
3			

Distributive Property:

$3(x + 2y - 7z)$

We can use these same methods to factor out the greatest common factor of an expression.

Area Model:

$10x^3$	$-14x^2$	$12x$

Distributive Property:

$10x^3 - 14x^2 + 12x$

Use the area model to write an equivalent expression for $(2x + 5)(x + 3)$.

	x	3
$2x$		
5		

We can use this same area model to factor a quadratic expression. Look at the resulting trinomial and notice the following four patterns.

- The first term of the trinomial can always be found in the ________________ ______________ rectangle.
- The last term of the trinomial can always be found in the ________________ ______________ rectangle.
- The second term of the trinomial is the ____________ of the ________________ _____________ and ____________ ______________ rectangles.
- The ________________ of the __________________ are always equal.

Use the distributive property to write an equivalent expression for $(2x + 5)(x + 3)$.

We can also use the distributive property to factor a quadratic expression.

What are the two middle terms of the expanded form?

Consider the resulting trinomial: $2x^2 + 11x + 15$

Notice that the product of the two middle terms of expanded form are equal to the product of the first and last term of the trinomial. The middle terms also sum to the middle term of the trinomial.

Let's consider how we can use this and the distributive property to factor a quadratic expression.

Factor $2x^2 + 3x - 5$ using the distributive property:

- Multiply the first term by the last term.

- Find two factors whose product is equal to $-10x^2$ and whose sum is equal to $3x$.

- Replace the middle term with these two factors.

- Factor the polynomial by grouping the first 2 terms and the last 2 terms.

Let's Practice!

1. Consider the quadratic expression $3x^2 + 4x - 4$.

   a. Factor using the area model.

   b. Factor using the distributive property.

STUDY EDGE TIP

You can check your answer to every factor by using the distributive property. The product of the factors should always result in the original trinomial.

Try It!

2. Consider the quadratic expression $4w^2 - 21w + 20$.

   a. Factor using the area model.

   b. Factor using the distributive property.

<u>**BEAT THE TEST!**</u>

1. Identify all factors of the expression $18x^2 - 9x - 5$.

   - ☐ $2x + 5$
   - ☐ $6x - 5$
   - ☐ $18x - 5$
   - ☐ $3x + 5$
   - ☐ $3x + 1$

Section 5 – Topic 3
Solving Quadratics by Factoring

Solving a quadratic equation by factoring:

- Once a quadratic equation is factored, we can use the ***zero product property*** to solve the equation.

- The zero product property states that if the product of two factors is zero, then one (or both) of the factors must be ________________.

  - If $ab = 0$, then either $a = 0, b = 0$, or $a = b = 0$.

To solve a quadratic equation by factoring:

Step 1: Set the equation equal to zero.
Step 2: Factor the quadratic.
Step 3: Set each factor equal to zero and solve.
Step 4: Write the solution set.

Let's Practice!

1. Solve for b by factoring $b^2 + 8b + 15 = 0$.

2. Solve for f by factoring $10f^2 + 17f + 3 = 0$.

Try It!

3. Solve for j by factoring $6j^2 - 19j + 14 = 0$.

BEAT THE TEST!

1. Tyra solved the quadratic equation $x^2 - 10x - 24 = 0$ by factoring. Her work is shown below:

   Step 1: $x^2 - 10x - 24 = 0$
   Step 2: $x^2 - 4x - 6x - 24 = 0$
   Step 3: $(x^2 - 4x) + (-6x - 24) = 0$
   Step 4: $x(x - 4) - 6(x - 4) = 0$
   Step 5: $(x - 4)(x - 6) = 0$
   Step 6: $x - 4 = 0, x - 6 = 0$
   Step 7: $x = 4 \text{ or } x = 6$
   Step 8: $\{4, 6\}$

   Tyra did not find the correct solutions. Investigate the steps, decipher her mistakes, and explain how to correct Tyra's work.

Section 5 – Topic 4
Solving Other Quadratics by Factoring

Many quadratic equations will not be in standard form:

- The equation won't always equal zero.
- There may be a greatest common factor (GCF) within all of the terms.

Let's Practice!

1. Solve for m: $3m^2 + 30m - 168 = 0$

2. Solve for x: $(x + 4)(x - 5) = -8$

Try It!

3. Solve for d: $6d^2 + 5d = 1$

4. Solve for p: $p^2 + 36 = 13p$

BEAT THE TEST!

1. What are the solutions to $40x^2 - 30x = 135$? Select all that apply.

   - ☐ $-\frac{9}{2}$
   - ☐ $-\frac{9}{4}$
   - ☐ $-\frac{3}{2}$
   - ☐ $-\frac{3}{4}$
   - ☐ $\frac{3}{4}$
   - ☐ $\frac{3}{2}$
   - ☐ $\frac{9}{4}$

Section 5 – Topic 5
Solving Quadratics by Factoring – Special Cases

There are a few special cases when solving quadratics by factoring.

Perfect Square Trinomials

- $x^2 + 6x + 9$ is an example of ***perfect square trinomial***. We see this when we factor.
- A perfect square trinomial is created when you square a ________________________.

Recognizing a Perfect Square Trinomial

A quadratic expression can be factored as a perfect square trinomial if it can be re-written in the form $a^2 + 2ab + b^2$.

Factoring a Perfect Square Trinomial

- If $a^2 + 2ab + b^2$ is a perfect square trinomial, then $a^2 + 2ab + b^2 = (a + b)^2$.
- If $a^2 - 2ab + b^2$ is a perfect square trinomial, then $a^2 - 2ab + b^2 = (a - b)^2$.

Let's Practice!

1. Determine whether $16x^2 + 88x + 121$ is a perfect square trinomial. Justify your answer.

2. Solve for q: $q^2 - 10q + 25 = 0$

Try It!

3. Determine whether $x^2 - 8x + 64$ is a perfect square trinomial. Justify your answer.

4. Solve for w: $4w^2 + 49 = -28w$

5. What do you notice about the number of solutions to perfect square quadratic equations?

6. Sketch the graph of a quadratic equation that is a perfect square trinomial.

Difference of Squares

Use the distributive property to multiply the following binomials.

$(x+5)(x-5)$

$(5x+3)(5x-3)$

Describe any patterns you notice.

- When we have a binomial in the form $a^2 - b^2$, it is called the ***difference of two squares***. We can factor this as $(a+b)(a-b)$.

Let's Practice!

7. Solve the equation $49k^2 = 64$ by factoring.

Try It!

8. Solve the equation $0 = 121p^2 - 100$.

BEAT THE TEST!

1. Which of the following expressions are equivalent to $8a^3 - 98a$? Select all that apply.

   - ☐ $2(4a^3 - 49a)$
   - ☐ $2a(4a^2 - 49)$
   - ☐ $2a(4a^3 - 49a)$
   - ☐ $(2a - 7)(2a + 7)$
   - ☐ $2(2a - 7)(2a + 7)$
   - ☐ $2a(2a - 7)(2a + 7)$

Section 5 – Topic 6
Solving Quadratics by Taking Square Roots

Consider the following quadratic equation.

$$2x^2 - 36 = 0$$

When quadratic equations are in the form $ax^2 + c = 0$, solve by taking the square root.

Step 1: Get the variable on the left and the constant on the right.

Step 2: Then take the square root of both sides of the equation. (Don't forget the negative root!)

Solve for x by taking the square root.

$2x^2 - 36 = 0$

Let's Practice!

1. Solve $x^2 - 121 = 0$.

Try It!

2. Solve $-5x^2 + 80 = 0$.

BEAT THE TEST!

1. What is the smallest solution to the equation $2x^2 + 17 = 179$?

   Ⓐ -9
   Ⓑ -3
   Ⓒ 3
   Ⓓ 9

2. A rescuer on a helicopter that is 50 feet above the sea drops a lifebelt. The distance from the lifebelt to the sea can be modeled by the equation $h(t) = -16t^2 + s$, where $h(t)$ represents the lifebelt's height from the sea at any given time, t is the time in seconds, and s is the initial height from the sea, in feet.

   How long will it take for the lifebelt to reach the sea? Round your answer to the nearest tenth of a second.

Section 5 – Topic 7
Solving Quadratics by Completing the Square

Sometimes you won't be able to solve a quadratic equation by factoring. However, you can rewrite the quadratic equation so that you can ***complete the square*** to factor and solve.

Let's start by determining what number we can add to a quadratic expression to make it a perfect square trinomial.

What value could be added to the quadratic to make it a perfect square trinomial?

$x^2 + 6x +$ ____

$x^2 + 8x + 3 +$ ____

$x^2 - 22x - 71 +$ ____

Let's see how this can be used to solve quadratic equations.

Recall how we factored perfect square trinomials.
If $a^2 + 2ab + b^2$ is a perfect square trinomial, then $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$.

Solve $2x^2 + 12x + 2 = 3$ by completing the square.

Step 1: Write the equation in standard form.

Step 2: Move the constant term to the right side of the equation.

Step 3: If the coefficient of the x^2 term does not equal 1, then factor out the coefficient.

Step 4: Divide the coefficient of the middle term by two and square the result. Use the addition property of equality to make the trinomial a perfect square.

Step 5: Factor and solve the perfect square trinomial.

Let's Practice!

1. Complete the square to solve $2x^2 - 8x = -5$.

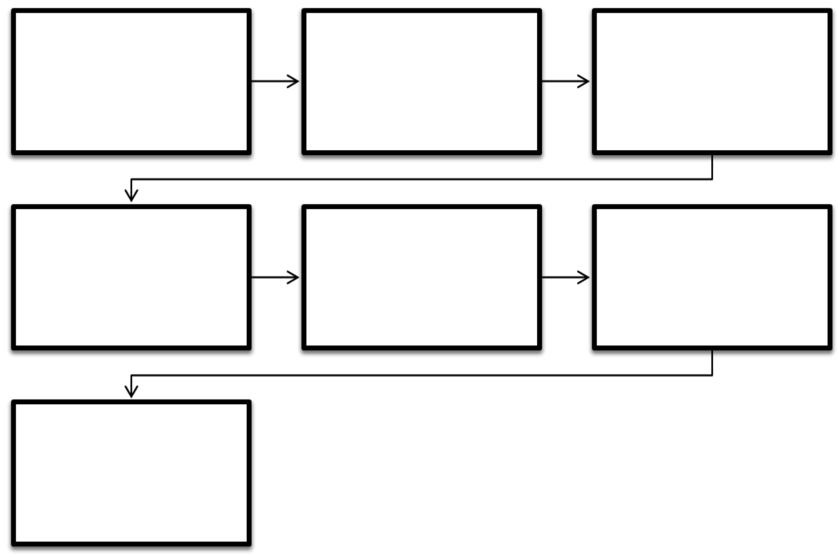
Try It!

2. Complete the square to solve $3x^2 + 12x = -31$.

BEAT THE TEST!

1. Demonstrate how to solve $2x^2 + 24x - 29 = 0$ by completing the square. Place the equations in the correct order.

A. $2(x^2 + 12x + 36) = 29 + 72$	E. $2(x + 6)^2 = 101$
B. $x + 6 = \pm\sqrt{50.5}$	F. $(x + 6)^2 = 50.5$
C. $2(x^2 + 12x + \_\_\_) = 29$	G. $\sqrt{(x + 6)^2} = \pm\sqrt{50.5}$
D. $x = -6 \pm \sqrt{50.5}$	



Section 5 – Topic 8
Deriving the Quadratic Formula

We can use the process of completing the square to derive a formula to solve any quadratic equation.

Consider the equation, $ax^2 + bx + c = 0$, where $a \neq 0$. Recall our steps for completing the square as a method for solving for x:

Subtract the constant term from both sides.

Factor out the coefficient of the x^2 term.

Divide the coefficient of the x term by two and square the result. Determine what you should add to create a perfect square trinomial.

Use the addition property of equality to write a perfect square trinomial.

Factor the trinomial.

Take the square root of both sides.

Solve for x.

BEAT THE TEST!

1. Complete the missing steps in the derivation of the quadratic formula:

$ax^2 + bx + c = 0$

$a\left(x^2 + \frac{b}{a}x + \quad\right) = -c$

$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) = -c + \frac{b^2}{4a}$

$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a}$

$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$

$\left(x + \frac{b}{2a}\right) = \pm\frac{\sqrt{b^2-4ac}}{2a}$

Section 5 – Topic 9
Solving Quadratics Using the Quadratic Formula

For any quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To use the quadratic formula:

Step 1: Set the quadratic equation equal to zero.

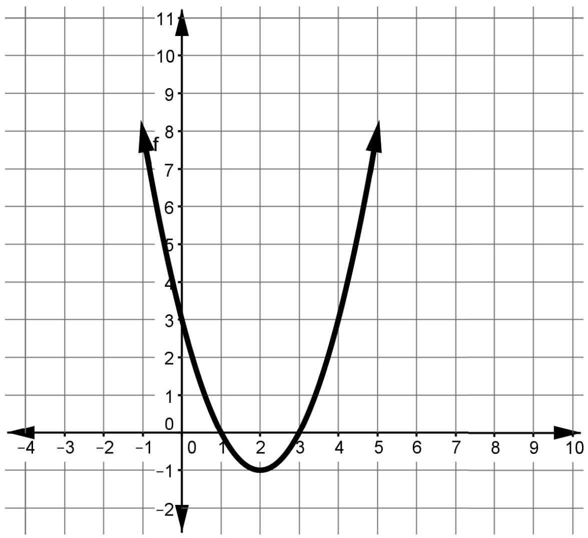
Step 2: Identify a, b, and c.

Step 3: Substitute a, b, and c into the quadratic formula and evaluate to find the zeros.

Let's Practice!

1. Use the quadratic formula to solve $x^2 - 4x + 3 = 0$.

2. Consider the graph of the quadratic equation $x^2 - 4x + 3 = 0$.



Does the graph verify the solutions we found using the quadratic formula?

3. Use the quadratic formula to solve $2w^2 + w = 5$.

Try It!

4. Use the quadratic formula to solve $3q^2 - 11 = 20q$.

BEAT THE TEST!

1. Your neighbor's garden measures 12 meters by 16 meters. He plans to install a pedestrian pathway all around it, increasing the total area to 285 square meters. The new area can be represented by $4w^2 + 56w + 192$. Use the quadratic formula to find the width, w, of the pathway.

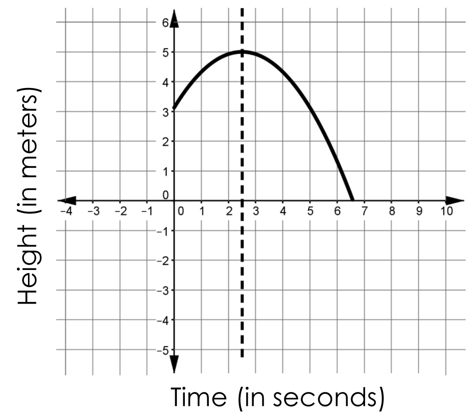
   Part A: Write an equation that can be used to solve for the width of the pathway.

   Part B: Use the quadratic formula to solve for the width of the pathway.

Section 5 – Topic 10
Quadratics in Action

Let's consider solving real-world situations that involve quadratic functions.

Consider an object being launched into the air. We compare the height versus time elapsed.



From what height was the object launched?

How long does it take the object to reach its maximum height?

What is the maximum height?

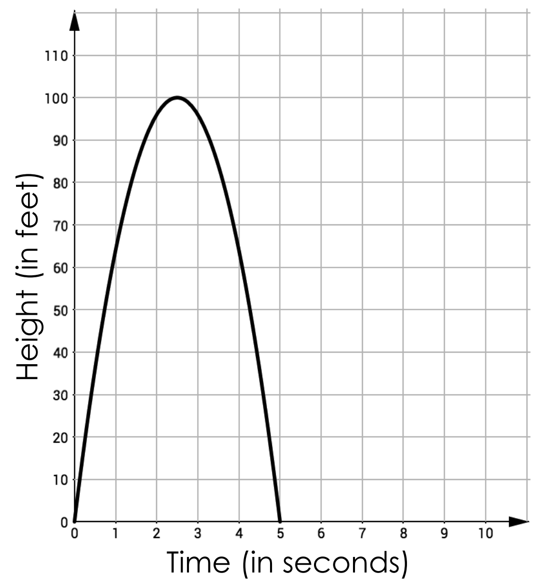
How long does it take until the object hits the ground?

After how many seconds will the object reach 3 meters again?

Question		How to Answer it
1.	From what height was the object launched?	This is the y-intercept. In the standard form, $ax^2 + bx + c$, c is the y-intercept.
2.	How long did it take the object to reach its maximum height?	This is the x-coordinate of the vertex, $x = \frac{-b}{2a}$, where values of a and b come from the standard form of a quadratic equation. $x = \frac{-b}{2a}$ is also the equation that represents the axis of symmetry.
3.	What was the maximum height?	This is the y-coordinate of the vertex. Substitute the x-coordinate from the step above and evaluate to find y. In vertex form, the height is k and the vertex is (h, k).
4.	At what time(s) was the object on the ground?	The x-intercept(s) are the solution(s), or zero(s), of the quadratic function. Solve by factoring, using the quadratic formula, or by completing the square. In a graph, look at the x-intercept(s).
5.	At what time did the object reach a certain height or how high was the object after a certain time?	In function $H(t) = at^2 + bt + c$, if height is given, then substitute the value for $H(t)$. If time is given, then substitute for t.

Let's Practice!

1. Ferdinand is playing golf. He hits a shot off the tee box that has a height modeled by the function $h(t) = -16t^2 + 80t$, where $h(t)$ is the height of the ball, in feet, and t is the time in seconds it has been in the air. The graph that models the golf ball's height over time is shown below.



   a. When does the ball reach its maximum height?

   b. What is the maximum height of the ball?

c. What is the height of the ball at 3 seconds? When is the ball at the same height?

Try It!

d. When is the ball 65 feet in the air? Explain.

e. How long does it take until the golf ball hits the ground?

BEAT THE TEST!

1. A neighborhood is throwing a fireworks celebration for the 4^{th} of July. A bottle rocket was launched upward from the ground with an initial velocity of 160 feet per second. The formula for vertical motion of an object is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a, is -32 feet per square second, v is the initial velocity, s is the initial height, and $h(t)$ is the height in feet modeled as a function of time, t.

   Part A: What function describes the height, h, of the bottle rocket after t seconds have elapsed?

   Part B: What is the maximum height of the bottle rocket?

Section 6 – Quadratic Functions – Part 2

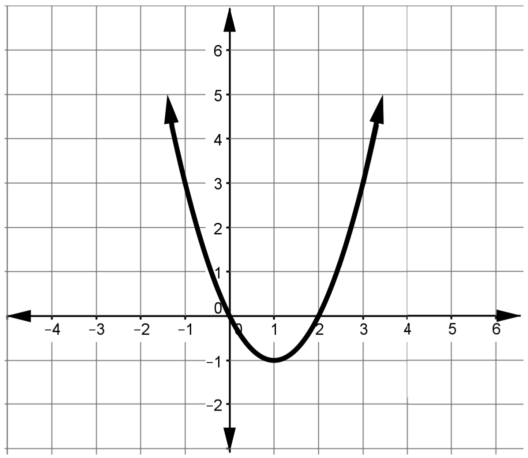
The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.A-CED.1.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MAFS.912.F-IF.2.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MAFS.912.F-IF.3.7.a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. *This section focuses on quadratic functions.*
MAFS.912.F-IF.3.8.a	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MAFS.912.F-IF.3.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MAFS.912.A.REI.2.4.a.b	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

MAFS.912.A-REI.4.11	Explain why the x –coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
MAFS.912.F-BF.2.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Topics in this Section

Section 6 – Topic 1
Observations from a Graph of a Quadratic Function

Let's review some things we learned earlier about the information we can gather from the graph of a quadratic.

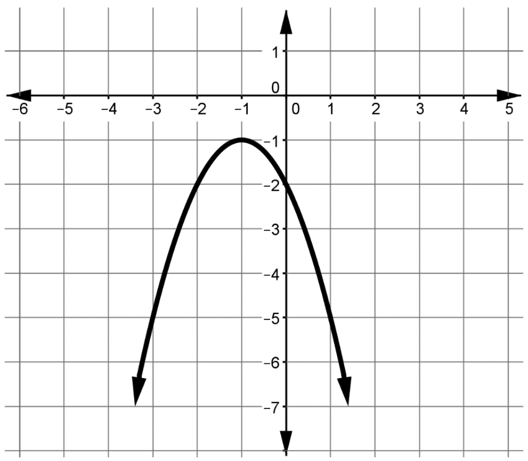


Vertex:

Axis of symmetry:

x-intercept(s):

y-intercept:



Vertex:

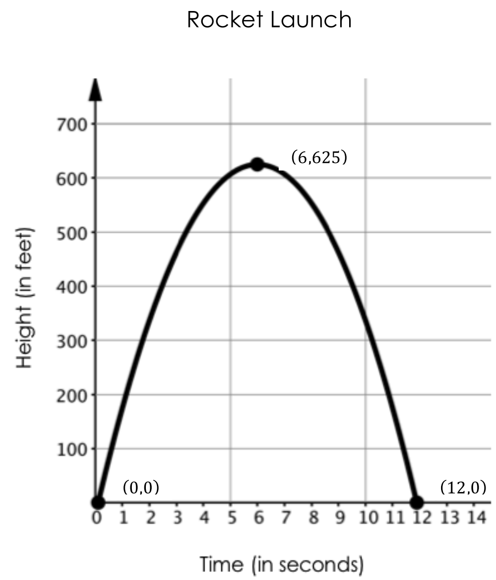
Axis of symmetry:

x-intercept(s):

y-intercept:

Let's Practice!

1. The graph shows the height of a rocket from the time it was launched from the ground. Use the graph to answer the questions below.



a. What is the y-intercept?

b. What does the y-intercept represent?

c. What are the x-intercepts?

d. What do the x-intercepts represent?

e. What is the maximum height of the rocket?

f. When will the rocket reach its maximum height?

g. When is the graph increasing?

h. When is the graph decreasing?

i. What is the domain of the graph?

j. What is the range of the graph?

We can also use the graph to write the equation of the quadratic function.

Recall the standard form of a quadratic equation:

$$f(x) = ax^2 + bx + c$$

There is another form of the quadratic equation called ***vertex form***.

Vertex Form: $f(x) = a(x - h)^2 + k$

- (h, k) is the vertex of the graph.
- a determines if the graph opens up or down.
- a also determines if the parabola is vertically compressed or stretched.

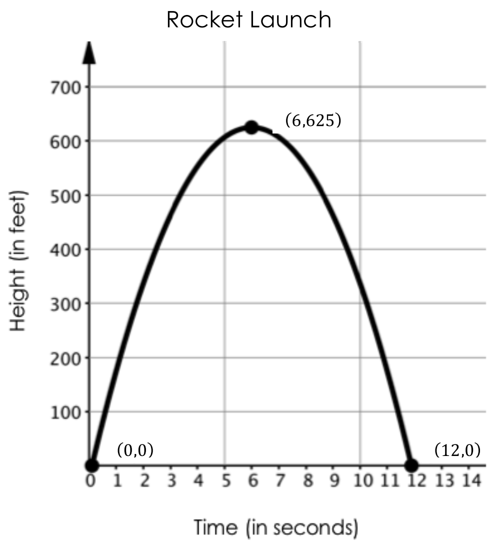
To write an equation in vertex form from a graph, follow these steps:

Step 1: Substitute the vertex, (h, k), and the coordinates of another point on the graph, $(x, f(x))$, into $f(x) = a(x - h)^2 + k$.

Step 2: Solve for a.

Step 3: Substitute (h, k) and a into vertex form.

2. Recall our graph from exercise #1.

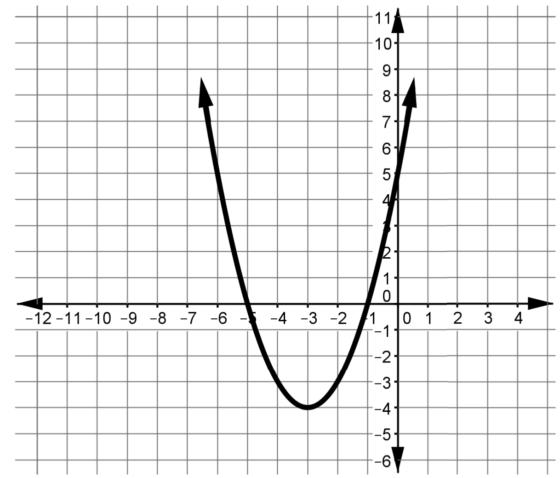


a. Substitute the vertex, (h, k), and the coordinates of another point on the graph, $\left(x, f(x)\right)$, into $f(x) = a(x - h)^2 + k$ and solve for a.

b. Write the function for the graph in vertex form.

Try It!

3. Consider the graph below.

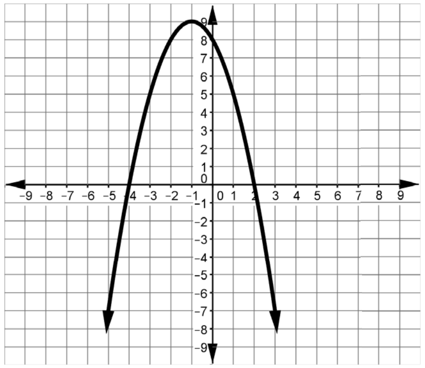


a. State five observations about the graph.

b. Write the equation of the graph.

BEAT THE TEST!

1. The graph of a quadratic function is shown below:



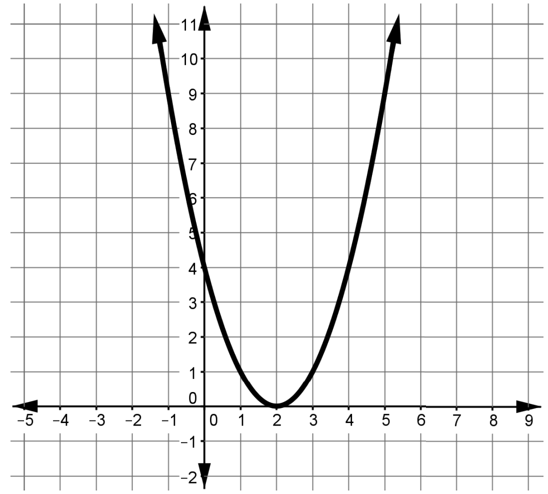
Which statements about this graph are true? Select all that apply.

- ☐ The graph has a y-intercept at $(0, 8)$.
- ☐ The graph has a maximum point at $(-1, 9)$.
- ☐ The graph has an x-intercept at $(2, 0)$.
- ☐ The graph's line of symmetry is the y-axis.
- ☐ The graph has zeros of -4 and 2.
- ☐ The graph represents the function $f(x) = -(x - 1)^2 + 9$.

Section 6 – Topic 2
Nature of the Solutions of Quadratics

Let's use the quadratic formula to discuss the nature of the solutions.

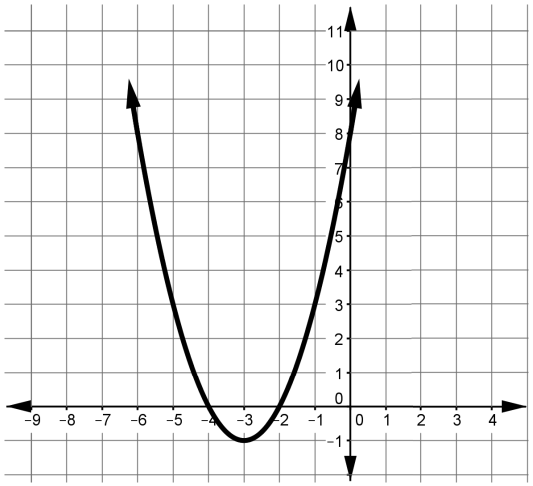
Consider the graph of the function $f(x) = x^2 - 4x + 4$.



Where does the parabola intersect the x-axis?

Use the quadratic formula to find the zero(s) of the function.

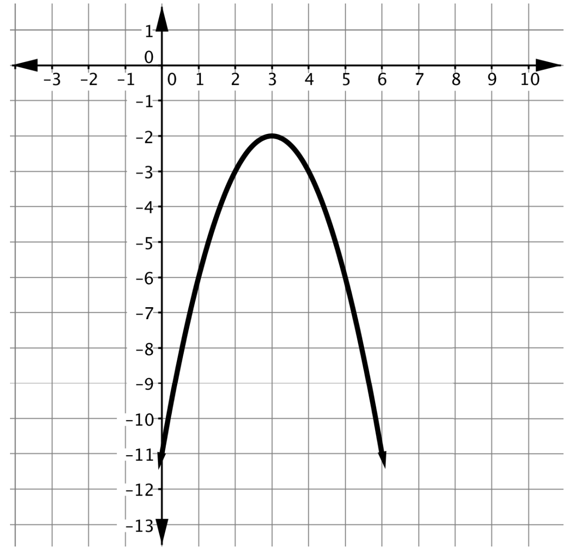
Consider the graph of the function $f(x) = x^2 + 6x + 8$.



Where does the parabola intersect the x-axis?

Use the quadratic formula to find the zero(s) of the function.

Consider the graph of the function $f(x) = -x^2 + 6x - 11$.



Where does the parabola intersect the x-axis?

Use the quadratic formula to find the zero(s) of the function.

STUDY EDGE TIP

When using the quadratic formula, if the discriminant of the quadratic (part under the radical) results in a negative number, then the solutions are non-real, complex solutions.

Let's Practice!

1. Use the discriminant to determine if the following quadratic equations have complex or real solution(s).

   a. $2x^2 - 3x - 10 = 0$

   b. $x^2 - 6x + 9 = 0$

   c. $g(x) = x^2 - 8x + 20$

Try It!

2. Create a quadratic equation that has complex solutions. Justify your answer.

3. Create a quadratic equation that has one real solution.

BEAT THE TEST!

1. Which of the following quadratic equations have real solutions? Select all that apply.

   - ☐ $-3x^2 + 5x = 11$
   - ☐ $-x^2 - 12x + 6 = 0$
   - ☐ $2x^2 + x + 6 = 0$
   - ☐ $5x^2 - 10x = 3$
   - ☐ $x^2 - 2x = -8$

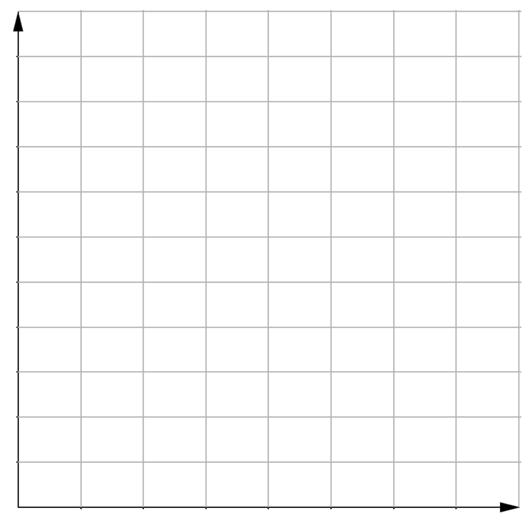
Section 6 – Topic 3
Graphing Quadratics Using a Table

Suppose you jump into a deep pool of water from a diving platform that is 25 feet above the ground. Your height with respect to time can be modeled by the function $H(t) = 25 - 16t^2$, where t is time in seconds.

Complete the table below.

Time (seconds)	0	0.25	0.5	0.75	1	1.25
Elevation (feet)						

Graph function $H(t)$ on the following coordinate grid.

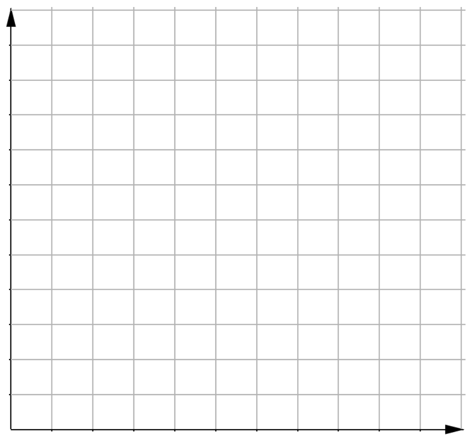


Let's Practice!

1. A construction company uses square-shaped lots of various sizes to build houses on. The CEO of the company decided to diversify her lots and now has houses built on rectangular-shaped lots that are 6 feet longer and 4 feet narrower than her square-shaped lots.

   a. What is the function that models the size of the rectangular lots relative to the size of the square lots?

   b. Complete the table below and graph function.

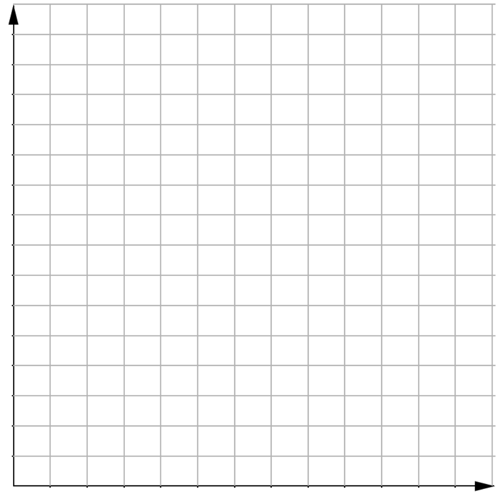


Try It!

2. A business owner recorded the following data for an entire year of sales.

Month	Sales (thousands)
Jan	22
Feb	45
Mar	54
April	63
May	70
June	71
July	70
Aug	64
Sept	54
Oct	38
Nov	24
Dec	5

a. Plot the data on the graph below.



b. What type of business might be represented by this graph?

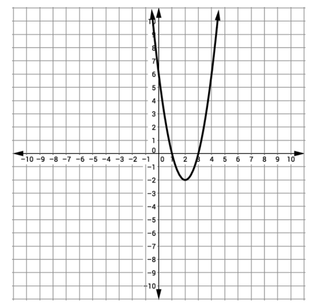
c. Would the quadratic model be an appropriate way to model data for this business going forward? Justify your answer.

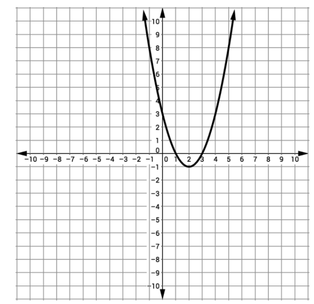
BEAT THE TEST!

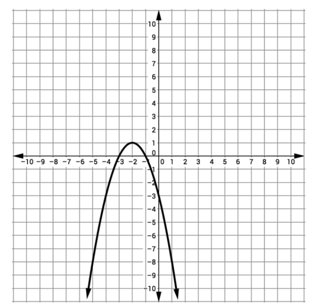
1. Consider the following table of values.

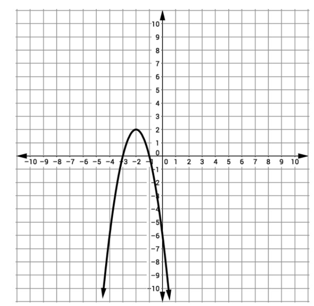
x	-5	-4	-3	-1	2	4
$f(x)$	-16	-6	0	0	-30	-70

Which of the following is the graph corresponding to the table of values?

Ⓐ


Ⓑ


Ⓒ


Ⓓ


Section 6 – Topic 4
Graphing Quadratics Using the Vertex and Intercepts

Given a quadratic equation in standard form, $f(x) = x^2 - 4x - 12$, use the following steps to graph on the following page:

Step 1. Use the a-value to determine if the graph should open upward (positive a) or downward (negative a).

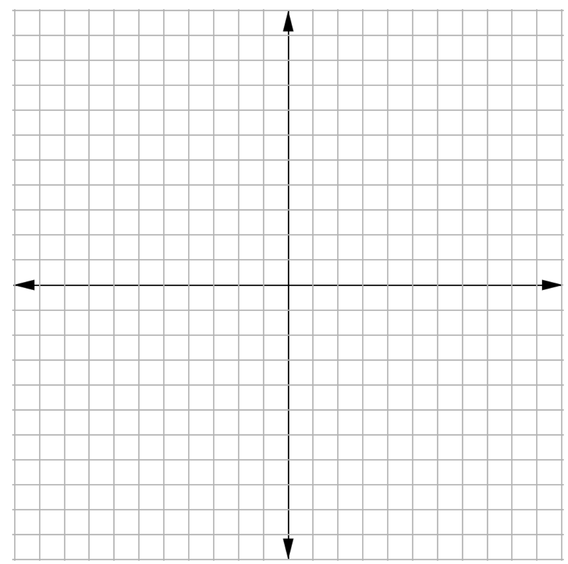
Step 2. Find and graph the axis of symmetry using the formula $x = -\frac{b}{2a}$. This is also the h coordinate of the vertex.

Step 3. Find $f(h)$, the k coordinate of the vertex, by substituting h into the equation. Plot the vertex, (h, k), on the graph.

Step 4. Find and plot the y-intercept, which is the constant c in $f(x) = ax^2 + bx + c$. If needed, use the axis of symmetry to find a reflection point.

Step 5. Find and plot the x-intercepts of the function. Factoring is one option, but you can always use the quadratic formula.

Graph of $f(x) = x^2 - 4x - 12$



Let's Practice!

1. Consider the function $f(x) = -x^2 + 4x + 21$.

   a. Use the a-value to determine if the graph should open upward (positive a) or downward (negative a).

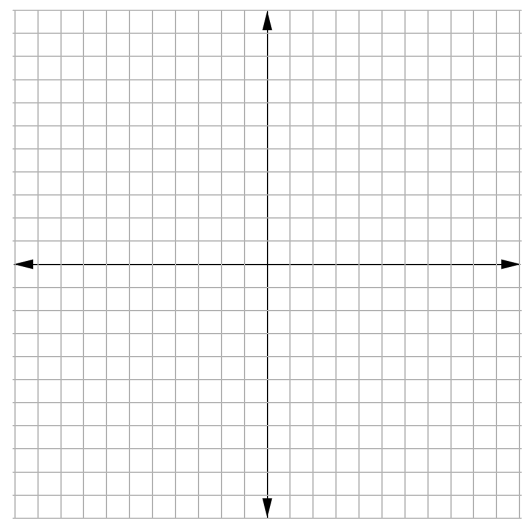
   b. Find and graph the axis of symmetry (the line that cuts the parabola into two equal halves) using the formula $x = \frac{-b}{2a}$. This is also the h coordinate of the vertex.

   c. Find $f(h)$, the k coordinate of the vertex, by substituting h into the equation. Plot the vertex, (h, k), on the graph.

   d. Find and plot the y-intercept, which is the constant c in $f(x) = ax^2 + bx + c$. If possible, use the axis of symmetry to find a reflection point.

   e. Find and plot the x-intercepts of the function. Factoring is one option, but you can always use the quadratic formula.

Graph of $f(x) = -x^2 + 4x + 21$



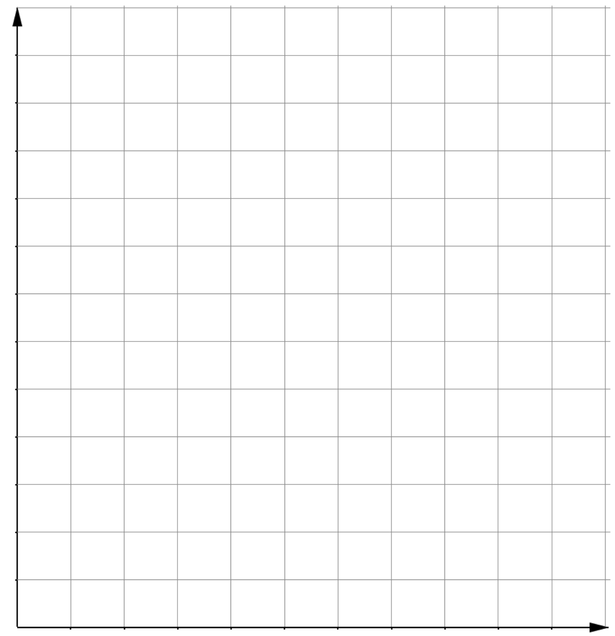
Try It!!

2. Jorah starts at the top of SlotZilla Zipline™ in Las Vegas and rides down Fremont Street. The equation $h(t) = -2.3t^2 + 114$ models Jorah's height, in feet, above the ground over time, t seconds, spent riding the zip line.

   a. What is the vertex of the function $h(t)$?

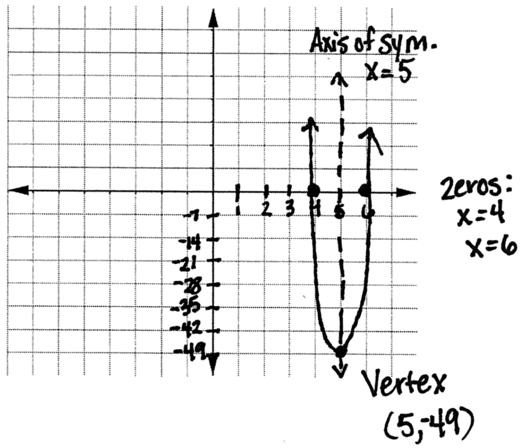
   b. When will Jorah reach the ground?

   c. Sketch the graph that models Jorah's height over the time spent riding the zip line.



BEAT THE TEST!

2. On a test, Mia graphed the quadratic function $f(x) = x^2 - 10x - 24$. The problem was marked incorrect. Identity Mia's mistake.



Section 6 – Topic 5
Graphing Quadratics Using Vertex Form – Part 1

Let's review vertex form.

Vertex Form: $f(x) = a(x - h)^2 + k$

- Point (h, k) is the vertex of the graph.
- Coefficient a determines if the graph opens up or down.
- Coefficient a also determines if the parabola is vertically stretched or compressed when compared to $f(x) = x^2$.

For example, function $s(t) = -16\left(t - \frac{3}{2}\right)^2 + 136$ models the height of a ball that is launched upward from a balcony of a residential building.

What is the vertex of the function?

Does the graph of the function open upward or downward?

Is the function vertically stretched or compressed in comparison to $s(t) = t^2$?

Let's Practice!

1. Graph the function $f(x) = (x - 3)^2 + 4$.

   To graph a quadratic in vertex form, follow these steps:

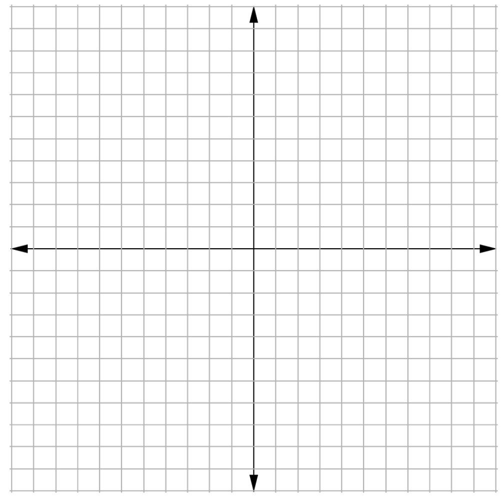
   a. Use the a-value to determine if the graph should open upward (positive a) or downward (negative a).

   b. Find and graph the vertex, (h, k), and axis of symmetry, $x = h$.

   c. Find the y-intercept by substituting zero for x. Plot the y-intercept. If possible, use the axis of symmetry to plot a reflection point.

   d. Find the x-intercepts, or zeros, by substituting zero for $f(x)$ and solving for x using square roots. Plot the x-intercepts.

   e. Use the key features to sketch the graph.



Try It!

2. Graph the function $f(x) = -4(x + 1)^2 + 9$.

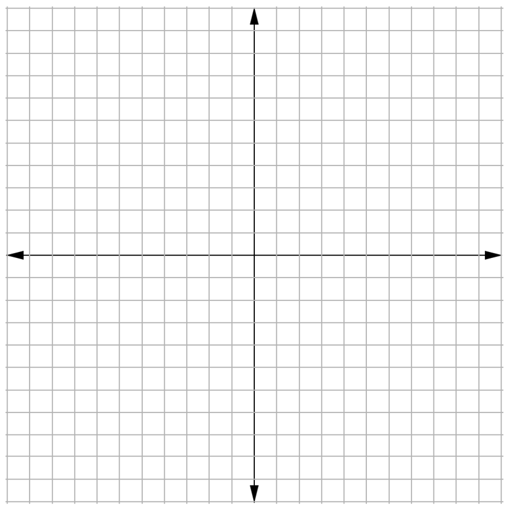
   a. Use the a-value to determine if the graph should open upward (positive a) or downward (negative a).

   b. Find and graph the vertex, (h, k), and axis of symmetry, $x = h$.

   c. Find the y-intercept by substituting zero for x. Plot the y-intercept. If possible, use the axis of symmetry to plot a reflection point.

d. Find the x-intercepts, or zeros, by substituting zero for $f(x)$ and solving for x using square roots. Plot the x-intercepts.

e. Use the key features to sketch the graph.



Section 6 – Topic 6
Graphing Quadratics Using Vertex Form – Part 2

Many times quadratic equations are not written in vertex form. You can always use the process of completing the square to rewrite it in vertex form.

Let's Practice!

1. Write the function $f(x) = x^2 - 4x - 2$ in vertex form. Then, graph the function.

   a. Write the function in standard form.

   b. Group the quadratic and linear term together.

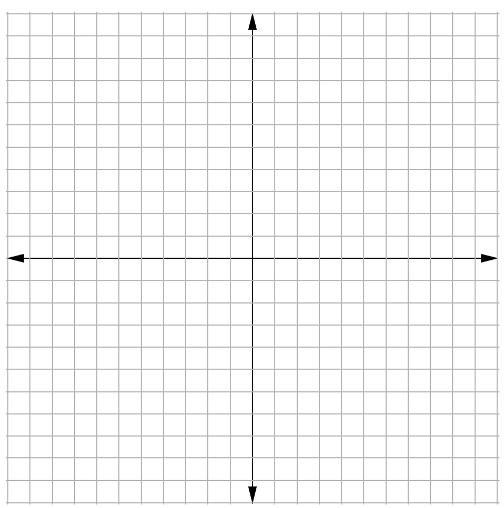
   c. If a does not equal one, factor a out of the equation.

   d. Complete the square.

e. Write the function in vertex form.

f. Determine the key features and graph the quadratic on the coordinate plane below.

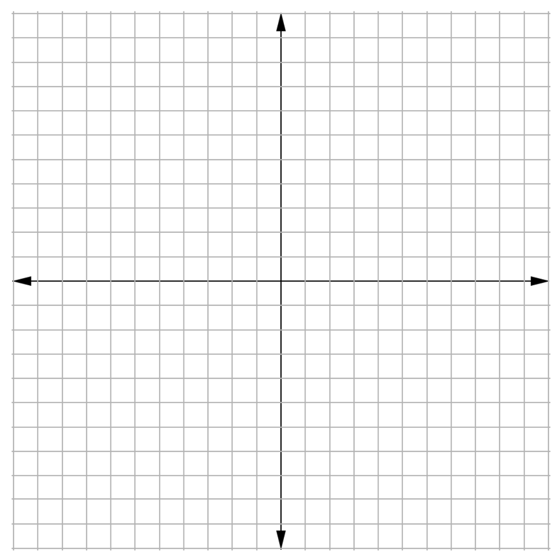
Graph of $f(x) = x^2 - 4x - 2$



Try It!

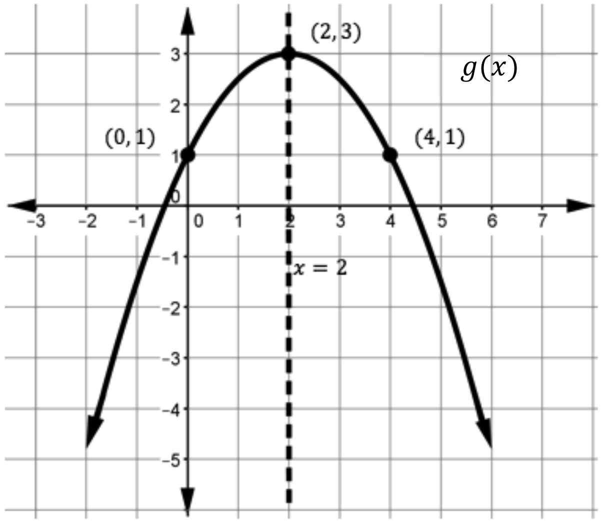
2. Write the function $g(x) = 2x^2 - 12x + 17$ in vertex form. Then, graph the function.

Graph of $g(x) = 2x^2 - 12x + 17$



BEAT THE TEST!

1. The graph of $g(x)$ is shown below.



Which function has a maximum that is greater than the maximum of the graph of $g(x)$?

Ⓐ $y = (x - 2)^2 + 4$

Ⓑ $y = (x + 3)^2 + 2$

Ⓒ $y = -\frac{1}{2}(x - 2)^2 + 3$

Ⓓ $y = -5(x + 3)^2 + 4$

2. Velma rewrote a quadratic function in vertex form.

$h(x) = 4x^2 + 16x + 5$

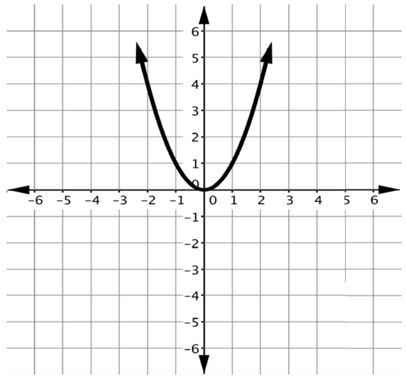
Step 1: $h(x) = 4(x^2 + 4x + \quad\quad) + 5$
Step 2: $h(x) = 4(x^2 + 4x + 4) + 5 - 4$
Step 3: $h(x) = 4(x + 2)^2 + 1$

Velma said that the vertex is $(-2, 1)$. Is Velma correct? If not, identify the step in which Velma made the mistake and correct her work.

Section 6 – Topic 7
Transformations of the Dependent Variable of Quadratic Functions

Consider the graph and table for the function $f(x) = x^2$.

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4



Consider the following transformations on the dependent variable $f(x)$.

$$g(x) = f(x) + 2$$

$$h(x) = f(x) - 2$$

$$m(x) = 2f(x)$$

$$n(x) = \frac{1}{2}f(x)$$

$$p(x) = -f(x)$$

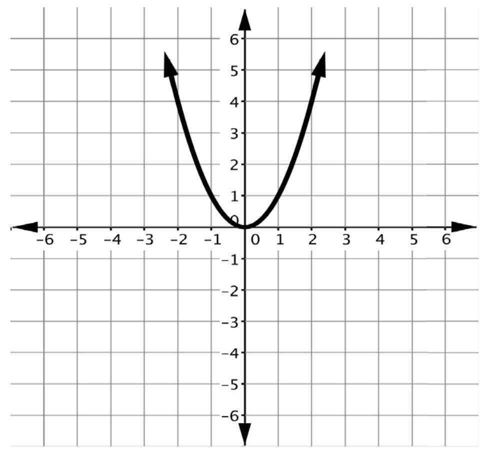
Why do you think these are called transformations on the dependent variable?

Let's Practice!

1. Complete the table to explore what happens when we add a constant to $f(x)$.

x	$f(x)$	$g(x) = f(x) + 2$	$h(x) = f(x) - 2$
-2	4		
-1	1		
0	0		
1	1		
2	4		

2. Sketch the graphs of each function on the same coordinate plane with the graph of $f(x)$.

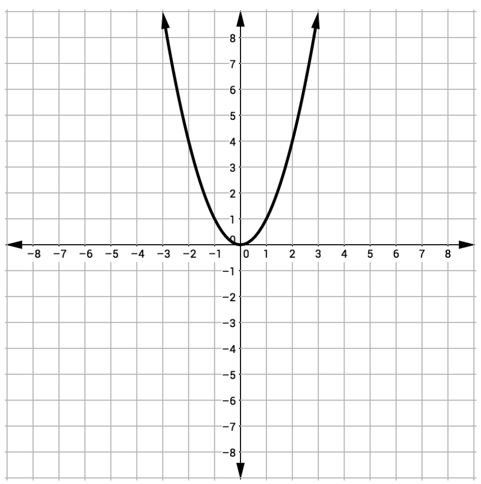


Try It!

3. Complete the table to determine what happens when we multiply $f(x)$ by a constant.

x	$f(x)$	$m(x) = 2f(x)$	$n(x) = \frac{1}{2}f(x)$	$p(x) = -f(x)$
-2	4			
-1	1			
0	0			
1	1			
2	4			

4. Sketch the graphs of each function on the same coordinate plane with the graph of $f(x)$.

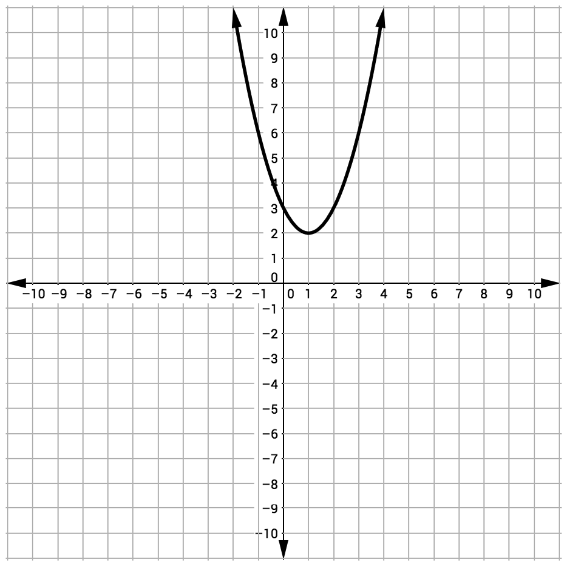


BEAT THE TEST!

1. Given the function $f(x) = x^2 + 3$, identify the effect on the graph by replacing $f(x)$ with the following.

$f(x) + k$, where $k > 0$	A. Vertically compressed $f(x)$ by a factor of k.
$f(x) + k$, where $k < 0$	B. Shifted $f(x)$ down $\lvert k \rvert$ units.
$kf(x)$, where $k > 1$	C. Reflected $f(x)$ about the x-axis.
$kf(x)$, where $0 < k < 1$	D. Vertically stretched $f(x)$ by a factor of k.
$kf(x)$, where $k = -1$	E. Shifted $f(x)$ up k units.

2. The graph of $g(x)$ is shown below.

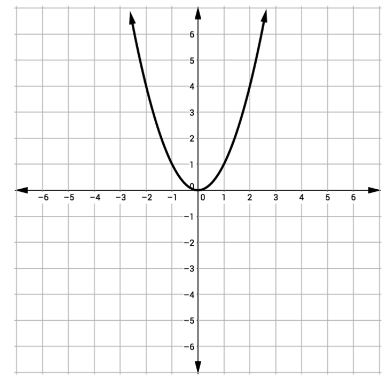


If $f(x) = 3g(x) + 2$, identify three ordered pairs that lie on $f(x)$.

Section 6 – Topic 8
Transformations of the Independent Variable of Quadratic Functions

Consider the graph and table for the function $f(x) = x^2$.

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4



Consider the following transformations on the independent variable x.

$$g(x) = f(x + 2)$$

$$h(x) = f(x - 2)$$

$$m(x) = f(2x)$$

$$n(x) = f\left(\frac{1}{2}x\right)$$

Why do you think these are called transformations on the independent variable?

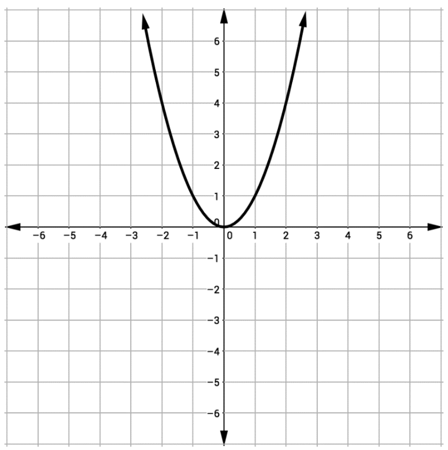
Let's Practice!

1. Complete the table to determine what happens when you add a positive constant to x.

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

x	$g(x) = f(x+2)$	$g(x)$
-4	$g(-4) = f(-4+2) = f(-2)$	4
-3	$g(-3) = f(-3+2) = f(-1)$	1

2. Sketch the graph of $g(x)$ on the same coordinate plane with the graph of $f(x)$.



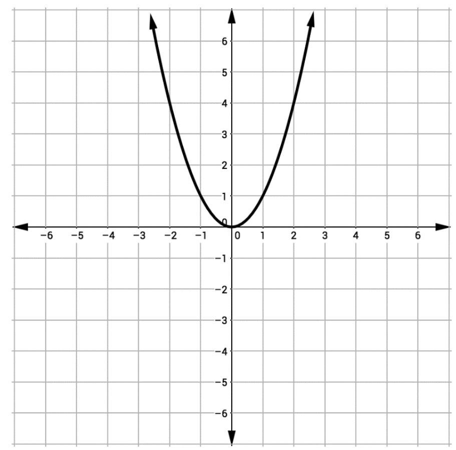
Try It!

3. Complete the table to determine what happens when you add a negative constant to x.

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

x	$h(x) = f(x-2)$	$h(x)$
0	$h(0) = f(0-2) = f(-2)$	4
1	$h(1) = f(1-2) = f(-1)$	1

4. Sketch the graph of $h(x)$ on the same coordinate plane with the graph of $f(x)$.



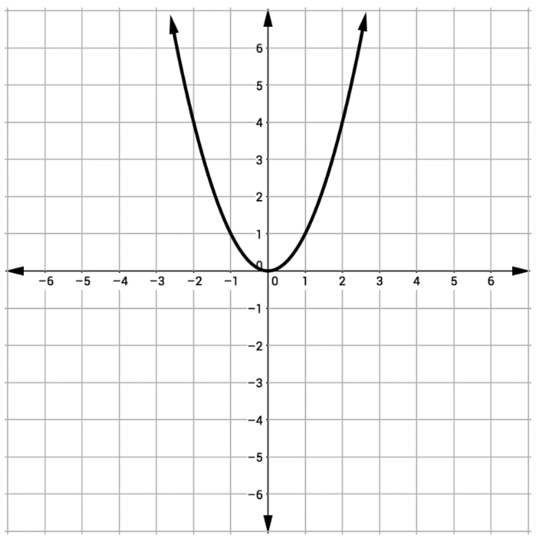
Let's Practice!

5. Complete the table to determine what happens when you multiply x by a number greater than 1.

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

x	$m(x) = f(2x)$	$m(x)$
-1	$m(-1) = f(2 \cdot -1) = f(-2)$	4
$-\frac{1}{2}$	$m\left(-\frac{1}{2}\right) = f\left(2 \cdot -\frac{1}{2}\right) = f(-1)$	1

6. Sketch the graph of $m(x)$ on the same coordinate plane with the graph of $f(x)$.



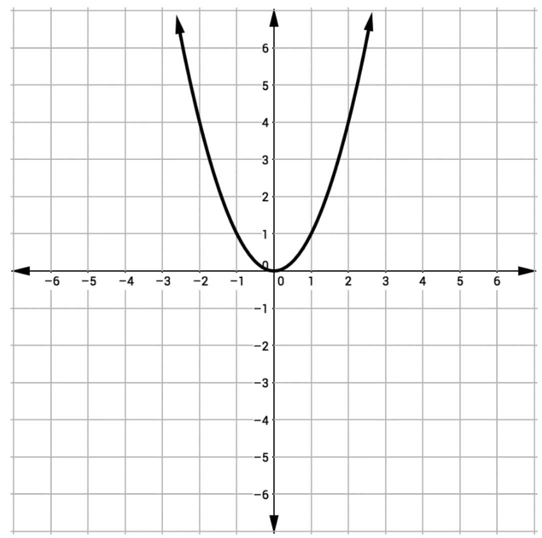
Try It!

7. Complete the table to determine what happens when you multiply x by a constant in between 0 and 1.

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

x	$n(x) = f\left(\frac{1}{2}x\right)$	$n(x)$
-4	$n(-4) = f(\frac{1}{2} \cdot -4) = f(-2)$	4
-2	$n(-2) = f\left(\frac{1}{2} \cdot -2\right) = f(-1)$	1

8. Sketch the graph of $n(x)$ on the same coordinate plane with the graph of $f(x)$.



BEAT THE TEST!

1. The table that represents the quadratic function $g(x)$ is shown below.

x	$g(x)$
-6	12
-4	2
1	12
7	90
11	182

The function $f(x) = g\left(\frac{1}{3}x\right)$. Complete the following table for $f(x)$.

x	$f(x)$

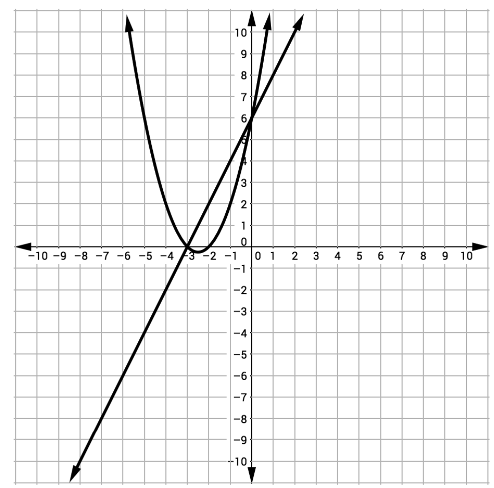
Section 6 – Topic 9
Finding Solution Sets to Systems of Equations Using Tables of Values and Successive Approximations

We can find solutions to systems of linear and quadratic equations by looking at a graph or table.

Consider the following system of equations:

$$f(x) = x^2 + 5x + 6$$
$$g(x) = 2x + 6$$

The graph of the system is shown below.



For which values of x does $f(x) = g(x)$?

We call these the ***solutions*** of $f(x) = g(x)$.

We can also identify the solutions by looking at tables. We can easily find the solutions by looking for the x-coordinate where $f(x) = g(x)$.

The table that represents the system is shown below.

x	$f(x)$	$g(x)$
-3	0	0
-2	0	2
-1	2	4
0	6	6
1	12	8
2	20	10
3	30	12

Use the table to identify the solutions of $f(x) = g(x)$.

We can also use a process called successive approximations.

Consider the following system:

$$f(x) = x^2 + 2x + 1$$

$$g(x) = 2x + 3$$

The table that represents the systems is shown below.

x	$f(x)$	$g(x)$
0	1	3
0.5	2.25	4
1	4	5
1.5	6.25	6
2	9	7
2.5	12.25	8
3	16	9

Since there are no x-coordinates where $f(x) = g(x)$, we must look for the x-coordinates that have the smallest absolute differences in $f(x)$ and $g(x)$.

- Find the absolute differences in $f(x)$ and $g(x)$ on the table above.
- In between which two x values must the positive solution lie?
- Which of the values does the solution lie closest to?

Let's Practice!

1. Using the same system, complete the table below.

$$f(x) = x^2 + 2x + 1$$

$$g(x) = 2x + 3$$

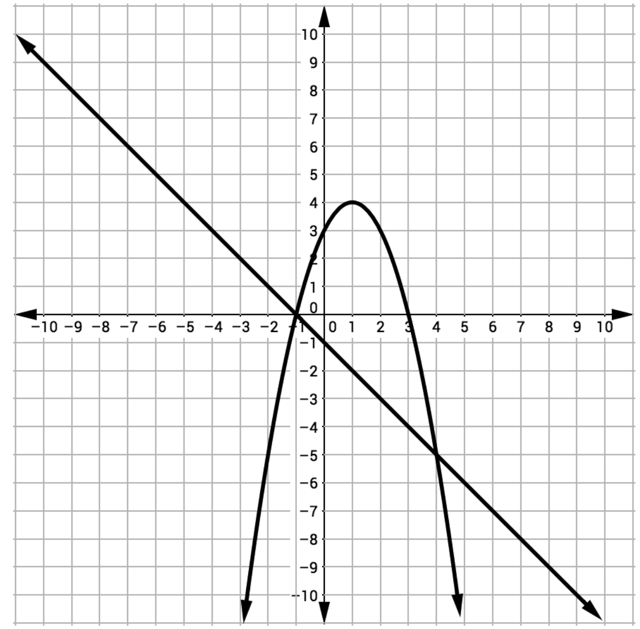
x	$f(x)$	$g(x)$
1	4	5
1.1	4.41	5.2
1.2		5.4
1.3	5.29	
1.4		
1.5	6.25	6

2. Find the absolute differences in $f(x)$ and $g(x)$ on the table above.

3. Use the table to find the positive solution (to the nearest tenth) for $f(x) = g(x)$.

Try It!

4. The graphs of $f(x)$ and $g(x)$ are shown below.



Use the graph to find the negative and positive solution of $f(x) = g(x)$.

BEAT THE TEST!

1. Consider the following system of equations.

$$g(x) = x^2 - 10$$

$$h(x) = x + 8$$

The table below represents the system.

x	$g(x)$	$h(x)$
-4	6	4
-3.5	2.25	4.5
-3	-1	5
-2.5	-3.75	5.5
-2	-6	6
-1.5	-7.75	6.5
-1	-9	7

Use successive approximations to find the negative solution for $g(x) = h(x)$.

Section 7: Exponential Functions

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.A-REI.4.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
MAFS.912.F-BF.2.3	Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them*.

MAFS.912.F-LE.1.1.a.b.c	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MAFS.912.F-LE.1.2	Construct linear and exponential functions, *including arithmetic and geometric sequences*, given a graph, a description of a relationship, or two input-output pairs (*include reading these from a table*).
MAFS.912.F-LE.1.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

MAFS.912.F-LE.2.5	Interpret the parameters in a linear or exponential function in terms of a context.
MAFS.912.F-IF.1.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MAFS.912.F-IF.2.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
MAFS.912.F-IF.2.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MAFS.912.F-IF.3.7.e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude and using phase shift.
MAFS.912.F-IF.3.8.b	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions.
MAFS.912.F-IF.3.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MAFS.912.A-SSE.2.3.c	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. c. Use the properties of exponents to transform expressions for exponential functions.

Topics in this Section

Section 7 – Topic 1
Geometric Sequences

Consider the sequence $3, 6, 12, 24, \ldots$. What pattern do you notice in the sequence?

This is an example of a ***geometric sequence***.

- Each term in the sequence is the _______________ of the previous term and some real number r.

Just like arithmetic sequences, we can represent this sequence in a table:

Term Number	Sequence Term	Term
1	a_1	3
2	a_2	6
3	a_3	12
4	a_4	24
5	a_5	48
$\vdots$	$\vdots$	$\vdots$
n	a_n	

Function Notation	
$f(1)$	a formula to find the 1st term
	a formula to find the 2nd term
$f(3)$	a formula to find the ___ term
$f(4)$	a formula to find the ___ term
	a formula to find the 5th term
$\vdots$	$\vdots$
$f(n)$	a formula to find the ___ term

How can we find the 10th term of this sequence?

- We can use the recursive process, where we use the previous term.

Term Number	Sequence Term	Term
1	a_1	3
2	a_2	$6 = 3 \cdot 2$
3	a_3	$12 = 6 \cdot 2$
4	a_4	$24 = 12 \cdot 2$
5	a_5	$48 = 24 \cdot 2$
6	a_6	$96 = 48 \cdot 2$
7	a_7	$192 = 96 \cdot 2$
8	a_8	$384 = 192 \cdot 2$
9	a_9	
10	a_{10}	

Function Notation	
$A(1)$	a_1
$A(2)$	$2a_1$
$A(3)$	$2a_2$
$A(4)$	$2a_3$
$A(5)$	$2a_4$
$A(6)$	$2a_5$
$A(7)$	$2a_6$
$A(8)$	$2a_7$
$A(9)$	$2a_8$
$A(10)$	$2a_9$

Write a recursive formula that we could use to find any term in the sequence.

- We can use the explicit process, where we relate back to the first term.

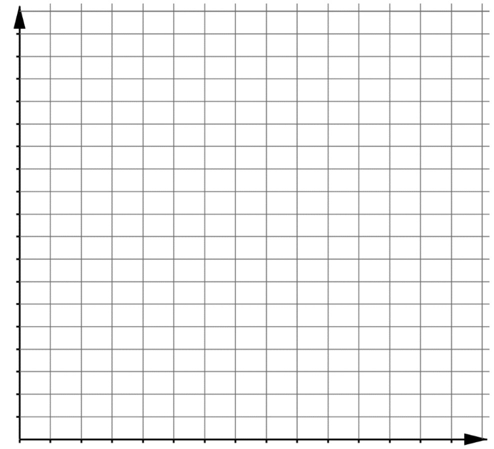
Term Number	Sequence Term	Term
1	a_1	3
2	a_2	$6 = 3 \cdot 2$
3	a_3	$12 = 3 \cdot 2 \cdot 2 = 3 \cdot 2^2$
4	a_4	$24 = 3 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^3$
5	a_5	$48 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^4$
6	a_6	$96 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^5$
7	a_7	$192 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^6$
8	a_8	$384 = 3 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^7$
9	a_9	
10	a_{10}	

Function Notation	
$A(1)$	a_1
$A(2)$	$2a_1$
$A(3)$	2^2a_1
$A(4)$	2^3a_1
$A(5)$	2^4a_1
$A(6)$	2^5a_1
$A(7)$	2^6a_1
$A(8)$	2^7a_1
$A(9)$	
$A(10)$	

Write an explicit formula that we could use to find any term in the sequence.

Sketch the graph of the geometric sequence found in the table.

Term Number	Term
1	3
2	6
3	12
4	24
5	48
6	96



STUDY EDGE TIP

The recursive process uses the previous term while the explicit process uses the first term.

Let's Practice!

1. Consider the sequence $-2, 4, -8, 16, \dots$.

   a. Write a recursive formula for the sequence.

   b. Write an explicit formula for the sequence.

   c. Find the 12^{th} term of the sequence.

Try It!

2. The first four terms of a geometric sequence are $7, 14, 28,$ and 56.

   a. Write a recursive formula for the sequence.

   b. Write an explicit formula for the sequence.

   c. Find the 20^{th} term of the sequence.

BEAT THE TEST!

1. An art gallery was showcasing a $6\ in$ long photo of a geometric landscape. The picture was enlarged ten times, each time by 125% of the previous picture.

   Enter formulas that will give the length of each enlarged print.

   $a_1 =$ [ ]

   Recursive formula:

   $a_n =$ [ ]

   Explicit formula:

   $a_n =$ [ ]

Section 7 – Topic 2
Real-World Examples of Arithmetic and Geometric Sequences

The founder of a popular social media website is trying to inspire gifted algebra students to study computer programming. He is offering two different incentive programs for students:

Option 1: Students will earn one penny for completing their first math, science, or computer-related college course. The amount earned will double for each additional course they complete.

Option 2: Students will earn one penny for completing their first math, science, or computer-related college course. For each subsequent course they complete, they will earn $\$100.00$ more than the previous course.

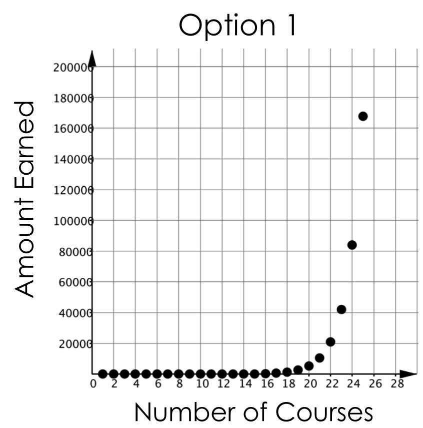
Write an explicit formula for each option.

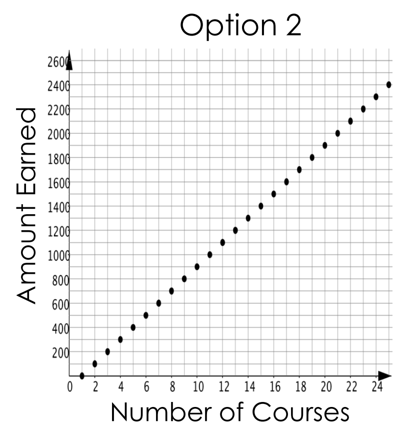
Compare the two scholarship options in the tables below:

Option 1	
Course	Amount
1	$0.01
2	$0.02
3	$0.04
4	$0.08
5	$0.16
6	$0.32
7	$0.64
8	$1.28
9	$2.56
10	$5.12
11	$10.24
12	$20.48
13	$40.96
14	$81.92
15	$163.84
16	$327.68
17	$655.36
18	$1,310.72
19	$2,621.44
20	$5,242.88
21	$10,485.76
22	$20,971.52
23	$41,943.04
24	$83,886.08
25	$167,772.16

Option 2	
Course	Amount
1	$0.01
2	$100.01
3	$200.01
4	$300.01
5	$400.01
6	$500.01
7	$600.01
8	$700.01
9	$800.01
10	$900.01
11	$1,000.01
12	$1,100.01
13	$1,200.01
14	$1,300.01
15	$1,400.01
16	$1,500.01
17	$1,600.01
18	$1,700.01
19	$1,800.01
20	$1,900.01
21	$2,000.01
22	$2,100.01
23	$2,200.01
24	$2,300.01
25	$2,400.01

Compare the two scholarship options in the graphs below:





Option 1 is a geometric sequence.

- Each term is the product of the previous term and two.
- This geometric sequence follows an ___________________ pattern.

Option 2 is an arithmetic sequence.

- Each term is the sum of the previous term and 100.
- Arithmetic sequences follow a _____________________ pattern.

Let's Practice!

1. Consider the two scholarship options for studying computer science.

   a. Which scholarship option is better if your college degree requires 10 math, engineering, or programming courses?

   b. What if your degree requires 25 math, engineering, or programming courses?

   c. Do you think that these graphs represent discrete or continuous functions? Justify your answer.

   d. Do you think Option 1 would ever be offered as a scholarship? Why or why not?

Try It!

2. Pablo and Lily are saving money for their senior trip next month. Pablo's goal is to save one penny on the first day of the month and triple the amount he saves each day for one month. Lily's goal is to save $\$10.00$ on the first day of the month and increase the amount she saves by $\$5.00$ each day.

   a. Pablo's savings plan is an example of a(n)
      - o arithmetic sequence
      - o geometric sequence
      .

   b. Lily's savings plan is an example of a(n)
      - o arithmetic sequence
      - o geometric sequence
      .

   c. Which person do you think will be able to meet their goal? Explain.

BEAT THE TEST!

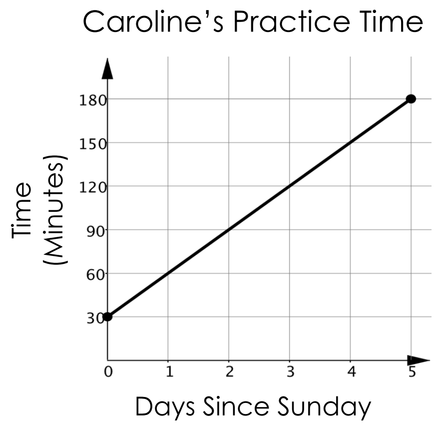
1. On Sunday, Chris and Caroline will begin their final preparations for a piano recital the following Saturday. Caroline plans to practice 30 minutes on the Sunday prior to the recital and increase her practice time by 30 minutes every day leading up to the recital. Chris plans to practice half Caroline's time on Sunday, but will double his practice time every day leading up to the recital.

   Part A: List Caroline and Chris' practice times on the tables below.

Caroline's Practice Time	

Chris' Practice Time	

   Part B: Compare the graphs of Caroline and Chris' practice times. Identify each graph as linear or exponential.



______________________ ______________________

Section 7 – Topic 3
Exponential Functions

Functions can be represented by:

- Verbal descriptions
- Algebraic equations
- Numeric tables
- Graphs

Let's review linear and quadratic functions.

Linear Functions

- Verbal description:

  You are driving to visit your best friend in Gulfport. Since you have a long drive ahead, you turn on your cruise control. The cruise control keeps your car traveling at a constant rate of 60 mph.

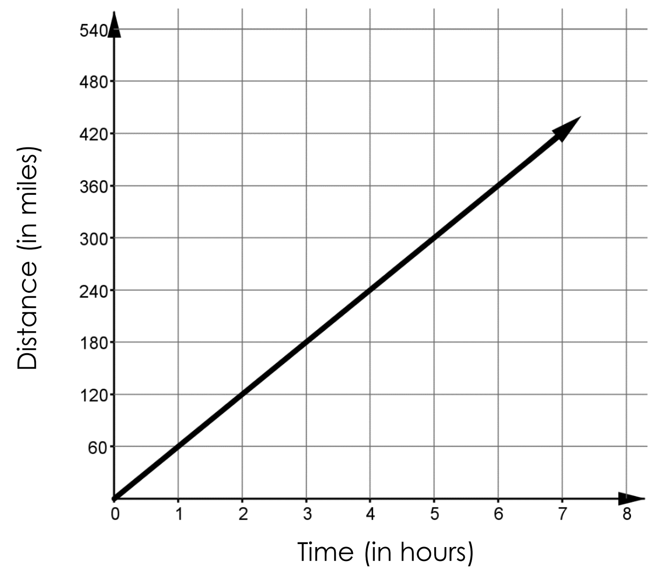
- Algebraic equation:

  The situation is represented by the function $f(h) = 60h$. Your distance, $f(h)$, depends on your time, h, in hours.

- Numeric table:

h	$f(h)$
1	60
2	120
3	180
4	240
5	300
6	360
7	420

- Graph:



Quadratic Functions

- Verbal description:

  You are observing the height of a ball as it's dropped from a $150\ ft$ tall building. Because of the force of gravity, the more time that passes, the faster the ball travels. The ball does not travel at a constant speed, like your car on cruise control.

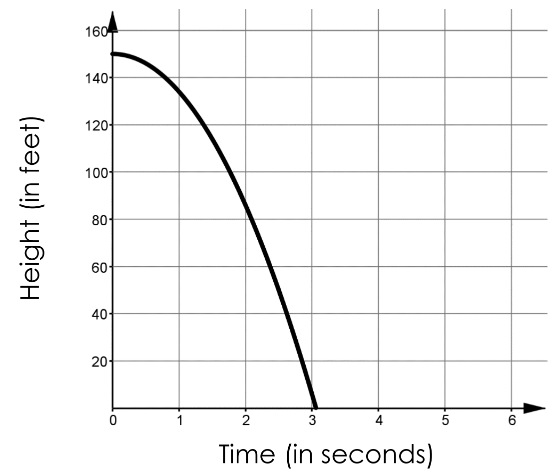
- Algebraic equation:

  The height of the ball (h) is a function of, or depends on, the time (t), in seconds. The quadratic function can be represented by the equation $h(t) = -16t^2 + 150$.

- Numeric table:

t	$h(t)$
1	134
2	86
3	6

- Graph:



Exponential Functions

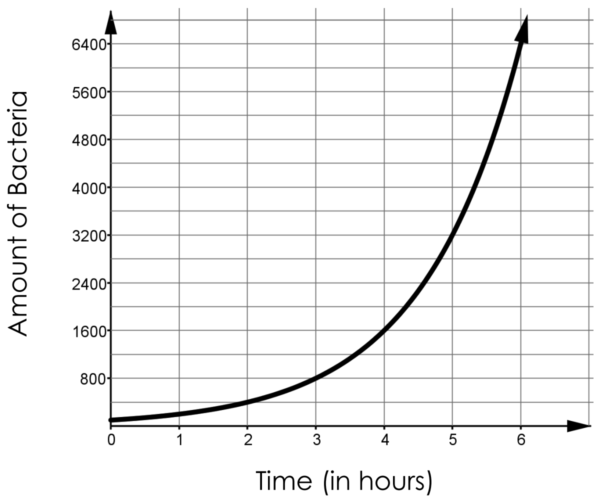
- Verbal description:

  You are performing an experiment in science class in which you start with 100 bacteria and the amount of bacteria doubles every hour.

➢ Numeric table:

t	$b(t)$
0	100
1	200
2	400
3	800
4	1,600
5	3,200
6	6,400

➢ Graph:



➢ Algebraic equation:

Use the following steps to write the equation for the exponential function.

- Pick two points. It's helpful to use the y-intercept and the coordinate where $x = 1$.

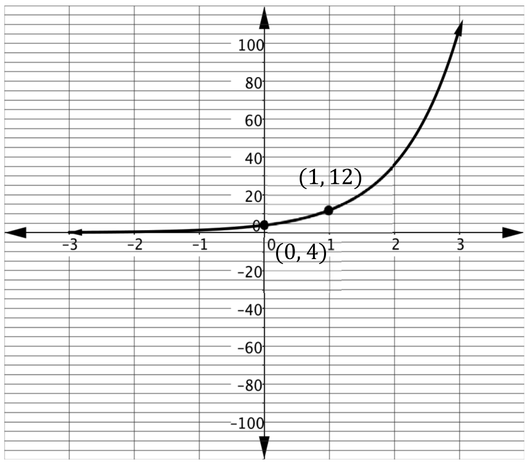
- Substitute the coordinates into the exponential equation $y = ab^x$. Solve for a and b.

- Substitute a and b into the equation $y = ab^x$.

Let's Practice!

1. The table and graph below represent an exponential function:

x	y
-1	$\frac{4}{3}$
0	4
1	12
2	36
3	108



a. Write an equation for the exponential function.

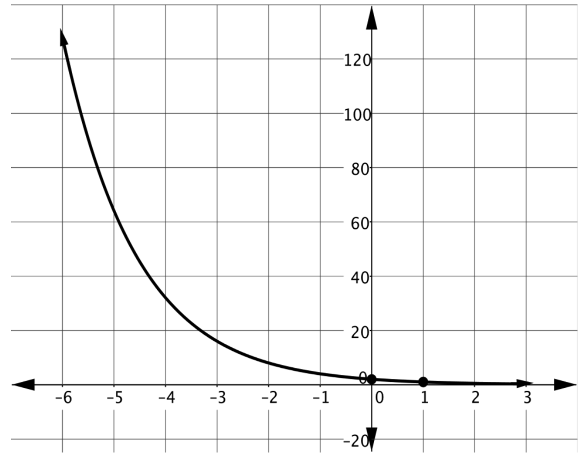
b. Form a hypothesis relating the a term to one of the key features of the graph.

c. Form a hypothesis relating the b term to one of the key features of the graph.

Try It!

2. The table and graph below represent an exponential function:

x	y
-1	4
0	2
1	1
2	$\frac{1}{2}$
3	$\frac{1}{4}$



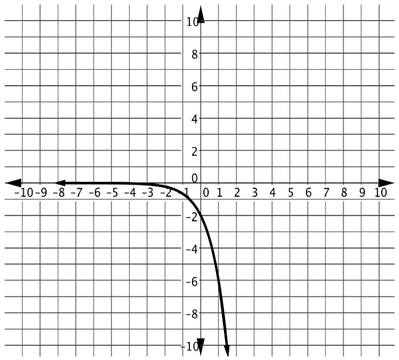
a. Write an equation to represent the exponential function.

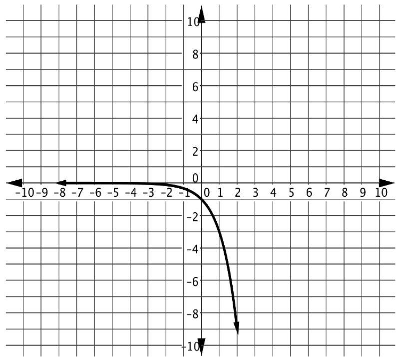
b. Did your earlier hypothesis hold true for this equation?

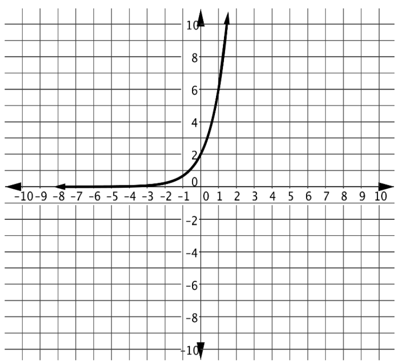
BEAT THE TEST!

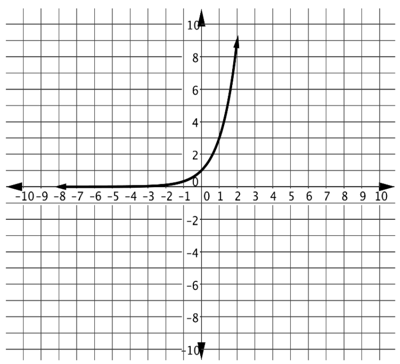
1. Match the graphs below with the following functions.

$f(x) = 3^x$	$f(x) = 2 \cdot 3^x$	$f(x) = -3^x$	$f(x) = -2 \cdot 3^x$





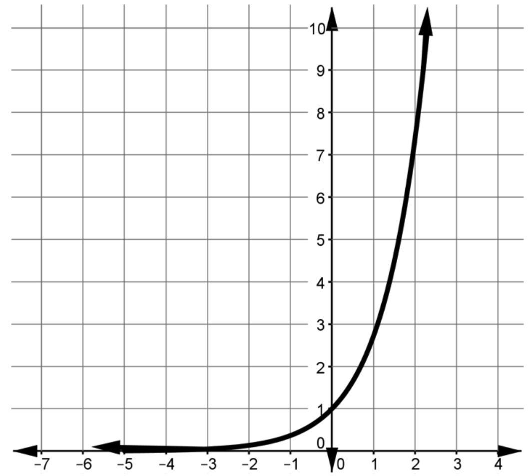




Section 7 – Topic 4
Graphs of Exponential Functions – Part 1

Let's review what we learned in the previous video about exponential functions.

Consider an exponential function written in the form $f(x) = a \cdot b^x$.



Which key feature of the exponential function does the a term represent?

- o x-intercept
- o y-intercept
- o common ratio

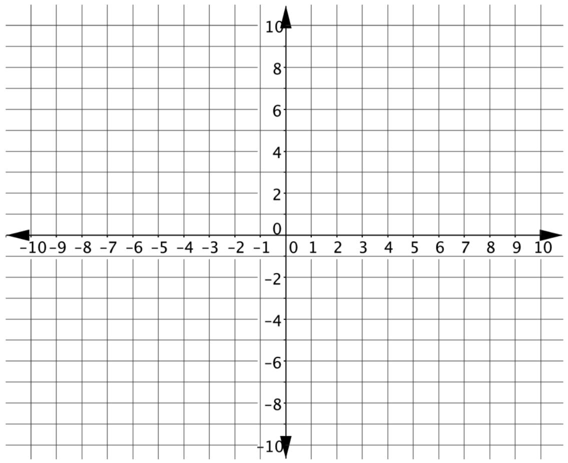
Which key feature of the exponential function does the b term represent?

- o x-intercept
- o y-intercept
- o common ratio

Let's Practice!

1. Consider the exponential equation $y = 2^x$.

   a. Sketch the graph of the exponential equation.



   b. Is the graph increasing or decreasing?

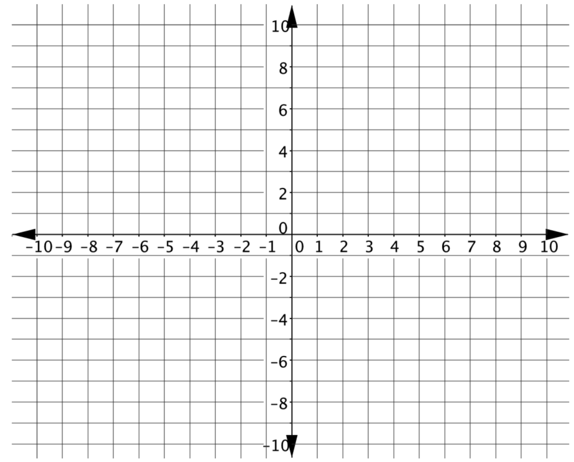
   c. Describe the end behavior of the graph.

   As x increases, y ______________________.

   As x decreases, y ______________________.

2. Consider the exponential equation $y = \left(\frac{1}{2}\right)^x$.

   a. Sketch the graph of the exponential equation.



   b. Is the graph increasing or decreasing?

   c. Describe the end behavior of the graph.

   As x increases, y ______________________.

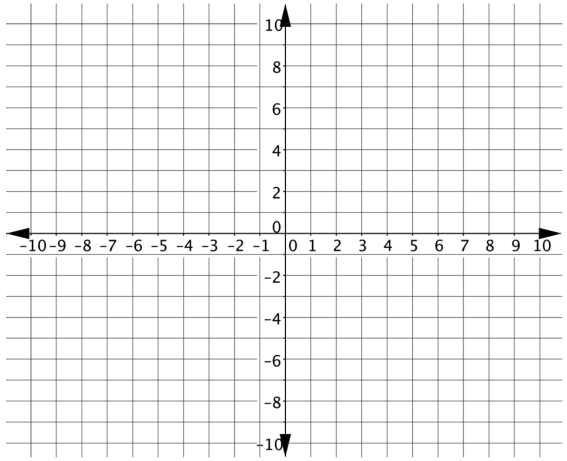
   As x decreases, y ______________________.

STUDY EDGE TIP

Remember, you can always write an exponential function such as $g(x) = 3^x$ in the form $g(x) = a \cdot b^x$ by writing the understood 1 in the front.

3. Consider the exponential equation $y = -2^x$.

a. Sketch the graph of the exponential equation.



b. Is the graph increasing or decreasing?

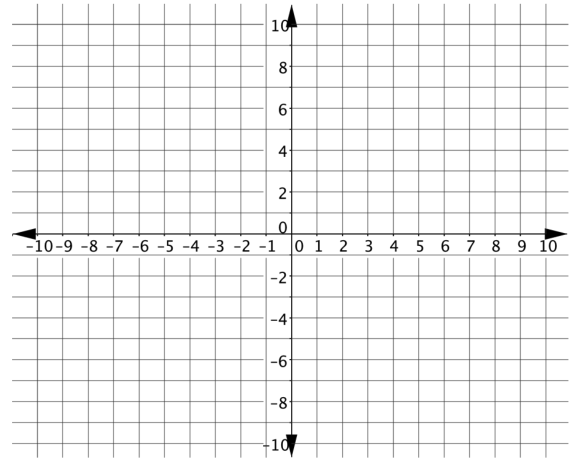
c. Describe the end behavior of the graph.

As x increases, y ________________________.

As x decreases, y ________________________.

4. Consider the exponential equation $y = -\left(\frac{1}{2}\right)^x$

a. Sketch the graph of the exponential equation.



b. Is the graph increasing or decreasing?

c. Describe the end behavior of the graph.

As x increases, y ______________________.

As x decreases, y ______________________.

5. Make a hypothesis about the relationship between the y-intercept, common ratio, and end behavior of a graph. Use your hypothesis to complete the table below.

y –intercept	Common Ratio, r	Increasing or Decreasing	End Behavior: As x increases	End Behavior: As x decreases
positive	$r > 1$			
positive	$0 < r < 1$			
negative	$0 < r < 1$			
negative	$r > 1$			

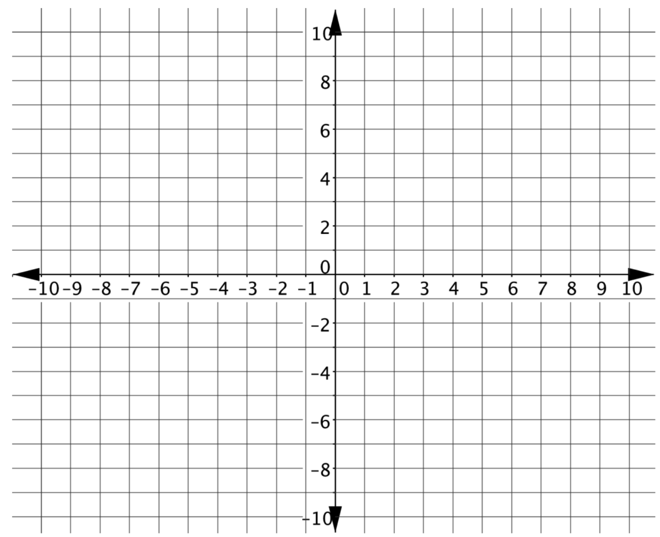
STUDY EDGE TIP

If you get confused about end behavior, you can sketch the graph of $y = a \cdot b^x$ and the key features to see the end behavior.

Section 7 – Topic 5
Graphs of Exponential Functions – Part 2

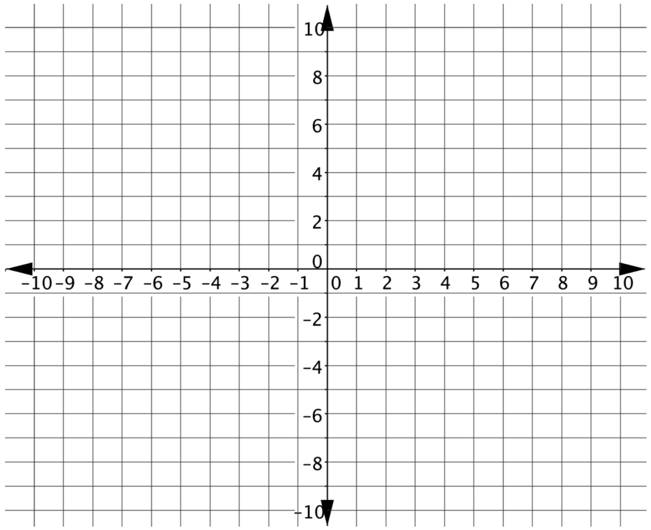
Sometimes we can use the properties of exponents to easily sketch exponential functions.

How can we use the properties of exponents to sketch the graph of $y = 2^{x+2}$?



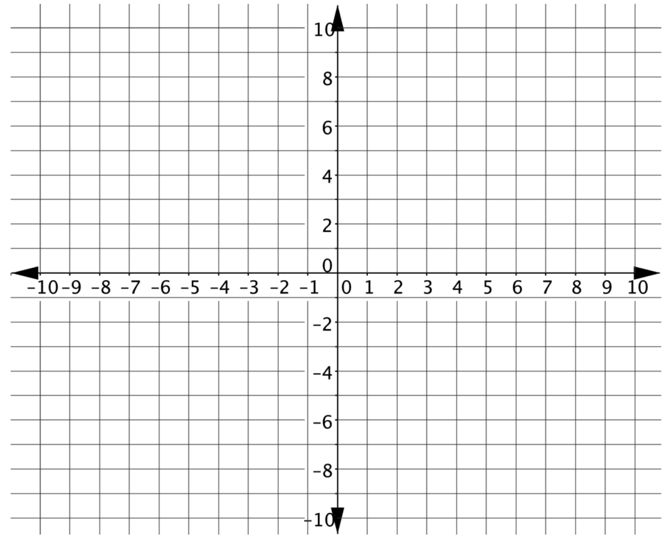
Let's Practice!

1. Use the properties of exponents to sketch the graph of $y = 3^{-x}$.



Try It!

2. Use the properties of exponents to sketch the graph of $y = 2^{x-3}$.



BEAT THE TEST!

1. The graph that represents the function $f(x) = -3 \cdot 2^x$ has

a y-intercept of [○ -3 / ○ 2]. The graph is increasing by a

common ratio of [○ -3 / ○ 2], is decreasing as

[○ x increases / ○ x decreases], and approaches 0 as [○ x increases. / ○ x decreases.]

2. Which of the following have the same graphic representation as the function $f(x) = 8 \cdot 2^x$? Select all that apply.

- ☐ $y = (2^x)^3$
- ☐ $y = 2^{4x}$
- ☐ $y = 2^{x+3}$
- ☐ $y = 2 \cdot 2^{2x}$
- ☐ $y = 4 \cdot 2^{x+1}$

Section 7 – Topic 6
Growth and Decay Rates of Exponential Functions

Consider an exponential function in the form $f(x) = a \cdot b^x$.
Assume that a (the ______________________) is positive.

➢ If b (the ________________________) is greater than 1, the function is ______________________.

➢ If b is between 0 and 1, the function is ____________.

What are some examples of exponential growth?

What are some examples of exponential decay?

Let's Practice!

1. Consider the exponential function $f(x) = 500 \cdot 1.05^x$, which models the amount of money in Tyler's savings account, where x represents the number of years since Tyler invested the money.

   a. Is the money in the account growing or decaying?

   b. What is the rate of growth or decay?

   c. What does 500 represent?

STUDY EDGE TIP

You will see the rate of growth/decay expressed as a decimal or a percent.

2. Consider the exponential function $f(x) = 21{,}000 \cdot 0.91^x$, which models the value of Robert's car, where x represents the number of years since he purchased the car.

   a. Is the value of Robert's car growing or decaying?

   b. What is the rate of growth or decay?

   c. What does $21{,}000$ represent?

STUDY EDGE TIP

To find the decay rate, you must subtract b from 1. To find the growth rate, you subtract 1 from b.

Try It!

3. Consider the exponential function $f(x) = 1{,}250 \cdot 1.08^x$, which models the amount of money invested in a bond fund, where x represents the number of years since the money was invested.

   a. What is the rate of growth or decay?

   b. What does $1{,}250$ represent?

4. Consider the exponential function $f(x) = 25{,}000 \cdot 0.88^x$, which models the amount of money remaining in Lola's retirement fund, where x represents the number of years since Lola began withdrawing the money.

   a. What is the rate of growth or decay?

   b. What does $25{,}000$ represent?

BEAT THE TEST!

1. The equation $y = 250 \cdot 1.04^x$ models
   - o exponential growth
   - o exponential decay

   .

   The rate of growth/decay is
   - o 4%
   - o 96%
   - o 104%

   .

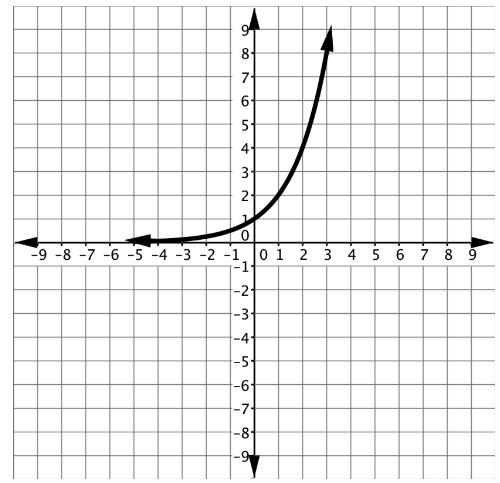
2. The function $f(x) = 350 \cdot 0.75^x$ models the amount of money remaining in Alicia's summer budget, where x represents the number of weeks since summer began. Which of the following are true statements? Select all that apply.

   - ☐ The function models exponential decay.
   - ☐ 350 represents the amount of money Alicia had in the budget at the beginning of summer.
   - ☐ The rate of decay is 25%.
   - ☐ Alicia spent $\$262.50$ during the first week of summer.
   - ☐ At the end of the second week, Alicia will have less than $\$200.00$ in the budget.

Section 7 – Topic 7
Transformations of Exponential Functions

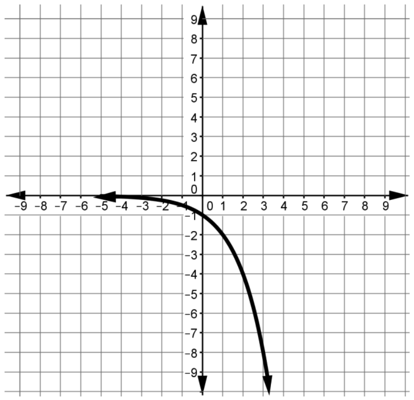
Consider the following exponential function.

$f(x) = 2^x$



Consider the following transformations of $f(x)$. Write a function to represent each transformed function and describe the transformation.

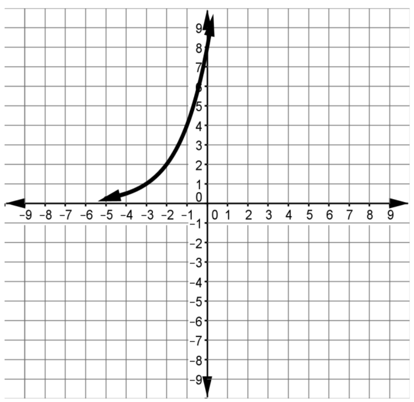
$-f(x)$



Transformed function:

Description:

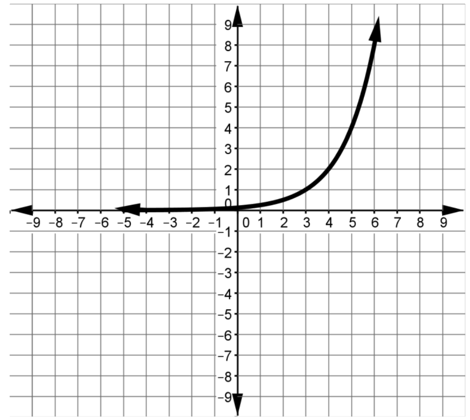
$f(x+3)$



Transformed function:

Description:

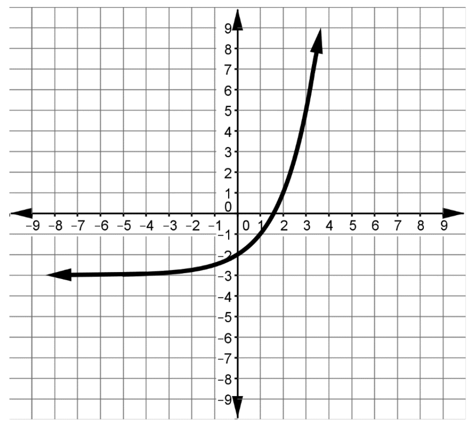
$f(x-3)$



Transformed function:

Description:

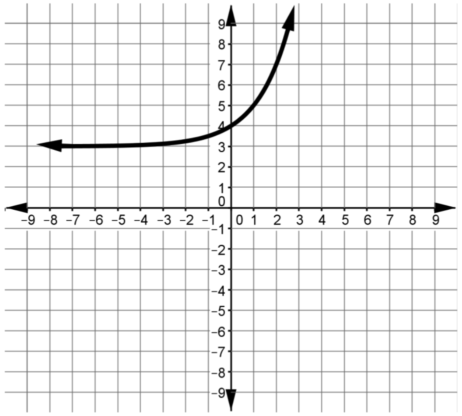
$f(x)-3$



Transformed function:

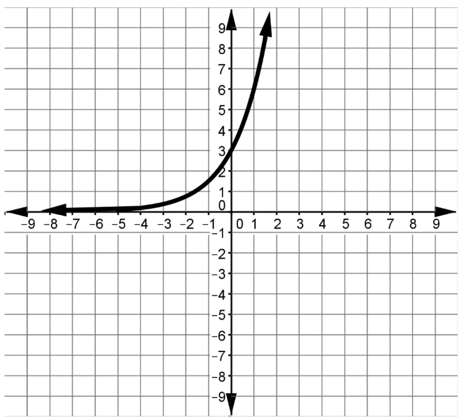
Description:

$f(x) + 3$



Transformed function:
Description:

$3f(x)$



Transformed function:
Description:

Let's Practice!

1. Describe how k affects the graph of the function $f(x) = 2^x$ in each of the following situations. Assume $k > 1$.

   a. $f(x) - k$

   b. $f(x + k)$

   c. $kf(x)$

2. The function $g(x)$ represents an exponential function. The ordered pair $(6, -3)$ lies on the graph of $g(x)$.

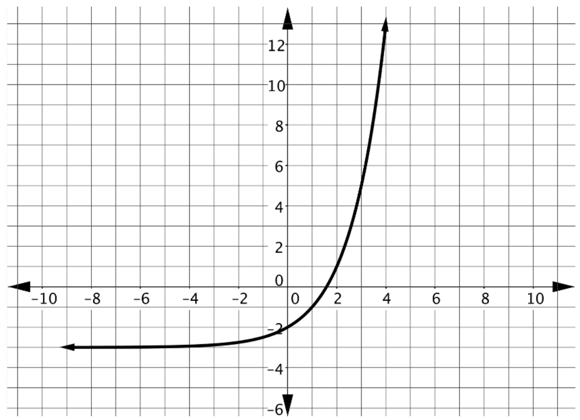
   a. The function $f(x) = g(x) + 5$. Name a point on the graph of $f(x)$.

   b. The function $h(x) = g(2x)$. Name a point on the graph of $h(x)$.

Try It!

3. Recall the graph of $f(x) = 2^x$. Describe the graph of $f(x-3)+2$.

4. The following graph represents the function $f(x)$.



$f(x)$ is a transformation of the exponential function $g(x) = 2^x + 1$. Write the exponential function for the graph.

BEAT THE TEST!

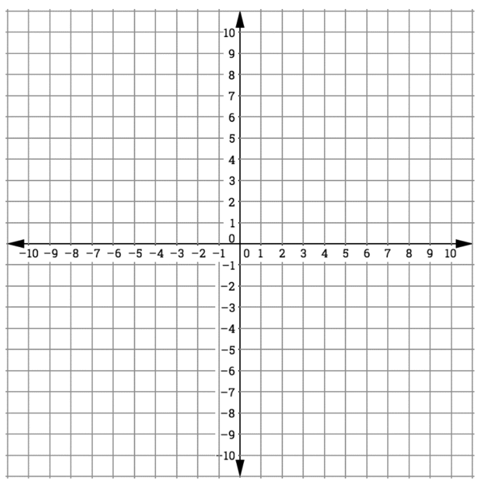
1. Consider the function $f(x) = \left(\frac{1}{2}\right)^x$. Describe the graph of each transformation.

$g(x) = f(x+2)$	
$h(x) = f(x) - 2$	
$m(x) = -2f(x)$	
$n(x) = f(x-4)$	
$r(x) = f(x) + 3$	

Section 7 – Topic 8
Comparing Linear, Quadratic, and Exponential Functions – Part 1

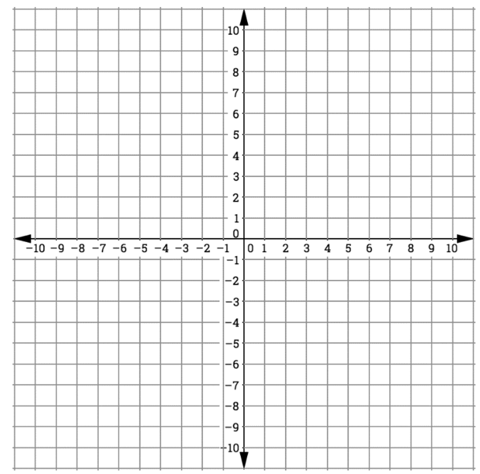
Linear Functions	
Equation:	
Shape:	
Rate of Change:	
Number of x-intercepts:	
Number of y-intercepts:	
Number of vertices:	
Domain:	
Range:	

Sketch the graphs of three linear functions that show all of the possible combinations above.



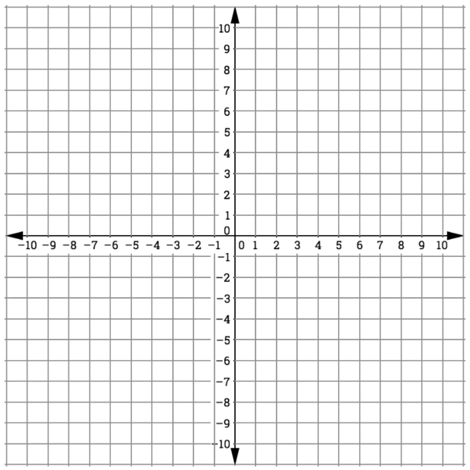
Quadratic Functions	
Equation:	
Shape:	
Rate of Change:	
Number of x-intercepts:	
Number of y-intercepts:	
Number of vertices:	
Domain:	
Range:	

Sketch the graphs of three quadratic functions that show all of the possible combinations above.



Exponential Functions	
Equation:	
Shape:	
Rate of Change:	
Number of x-intercepts:	
Number of y-intercepts:	
Number of vertices:	
Domain:	
Range:	

Sketch the graphs of two exponential functions that show all of the possible combinations above.



Consider the following tables that represent a linear and a quadratic function and find the differences.

Linear Function

x	$f(x)$
0	5
1	7
2	9
3	11
4	13

Quadratic Function

x	$f(x)$
0	3
1	4
2	7
3	12
4	19

How can you distinguish a linear function from a quadratic function?

Consider the following table that represents an exponential function.

Exponential Function		
x	$f(x)$	
0	1	
1	3	____
2	9	____
3	27	____
4	81	____
5	243	____

How can you determine if a function is exponential by looking at a table?

Section 7 – Topic 9
Comparing Linear, Quadratic, and Exponential Functions – Part 2

Let's Practice!

1. Identify whether the following key features indicate a model could be linear, quadratic, or exponential.

Key Feature	Linear	Quadratic	Exponential
Rate of change is constant	○	○	○
2nd differences, but not 1st are constant	○	○	○
Graph has a vertex	○	○	○
Graph has no x-intercept	○	○	○
Graph has 2 x-intercepts	○	○	○
Graph has 1 y-intercept	○	○	○
Domain is all real numbers	○	○	○
Range is $\{y \mid y > 0\}$	○	○	○
Range is $\{y \mid y \leq 0\}$	○	○	○
Range is all real numbers	○	○	○

Try It!

2. Determine whether each table represents a linear, quadratic, or exponential function.

x	y
0	1
1	2
2	5
3	10
4	17

o Linear
o Quadratic
o Exponential

x	y
0	7
3	13
6	19
9	25
15	37

o Linear
o Quadratic
o Exponential

x	y
0	2
1	6
2	18
3	54
4	162

o Linear
o Quadratic
o Exponential

BEAT THE TEST!

1. Identify whether the following real-world examples should be modeled by a linear, quadratic, or exponential function.

Real-World Example	Linear	Quadratic	Exponential
Growing a culture of bacteria	○	○	○
Selling fruit and vegetables at the same price all day	○	○	○
Kicking a ball into the air	○	○	○
Running a race at a constant speed	○	○	○
A dead body decaying	○	○	○
Appreciating value of property	○	○	○
Jumping from a high dive	○	○	○

2. Complete the following table so that $f(x)$ represents a linear function and $g(x)$ represents an exponential function.

x	$f(x)$	$g(x)$
-5		
-4		
-3		
-2		
-1		

Section 8: Polynomial Functions

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.F-IF.1.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MAFS.912.F-IF.3.7.c	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable functions are available, and showing end behavior.
MAFS.912.A-APR.2.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Topics in this Section

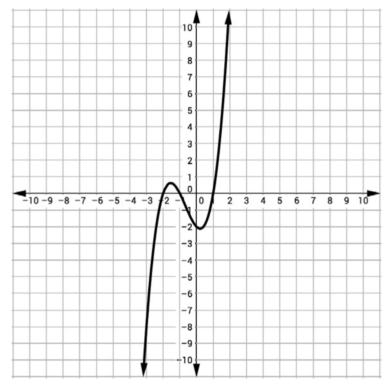
Section 8 – Topic 1
Finding Zeros of Polynomial Functions of Higher Degrees

How can you find zeros when given the graph of a polynomial function?

How can you find zeros when given the equation of a polynomial function in factored form?

How do you determine if x is a solution or zero for $f(x)$?

Consider the following graph of $f(x)$.



What are the zeros of $f(x)$?

Consider the following fourth degree polynomial function.

$$g(x) = x^4 - 4x^2$$

Find the range of $g(x)$ for the given domain $\{-2, -1, 0, 1, 2\}$.

Does the above domain contain zeros of $g(x)$? Justify your answer.

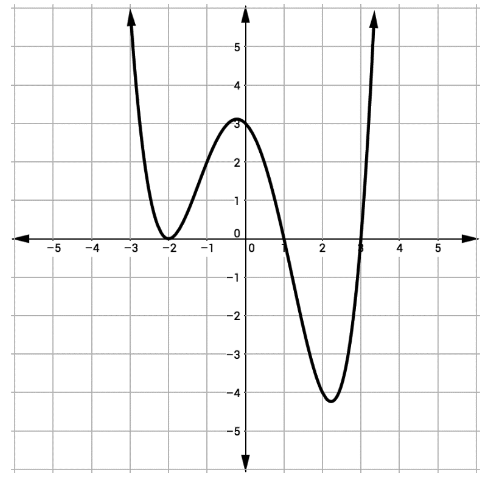
Consider the following third degree polynomial function.

$$h(x) = -x^3 - 5x^2$$

Find the zeros of the function $h(x)$.

Let's Practice!

1. Consider the following graph of $f(x)$.



What are the zeros of $f(x)$?

2. What are the zeros of $g(x) = x(x+1)(x-2)^2$?

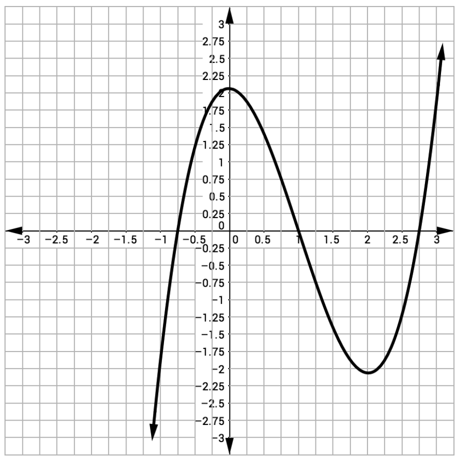
Try It!

3. Consider the function $h(x) = x^3 - 3x^2 + 2$.

   a. Find the range of $h(x)$ given the domain $\{-1, 1, 3\}$.

   b. Are any zeros of $h(x)$ found in the above domain? Justify your answer.

   c. Consider the graph of $h(x)$.

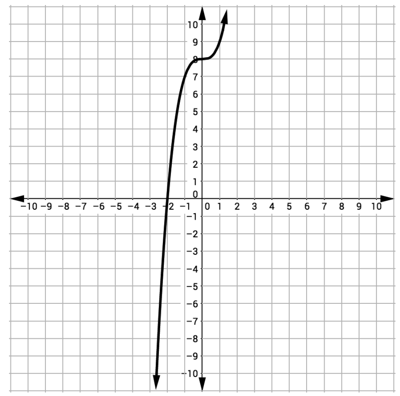


What are the other zeros of $h(x)$?

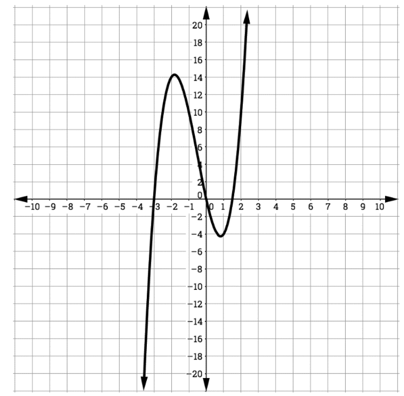
BEAT THE TEST!

1. Which of the graphs has the same zeros as the function $f(x) = 2x^3 + 3x^2 - 9x$?

Ⓐ



Ⓑ



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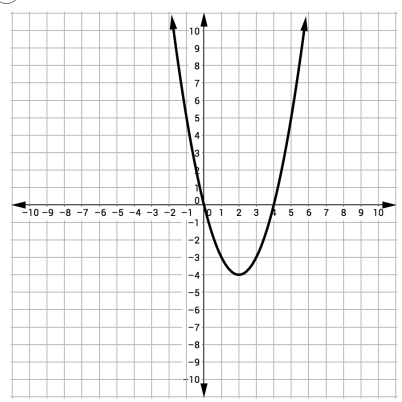
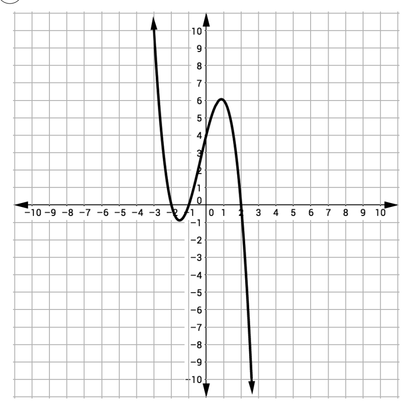


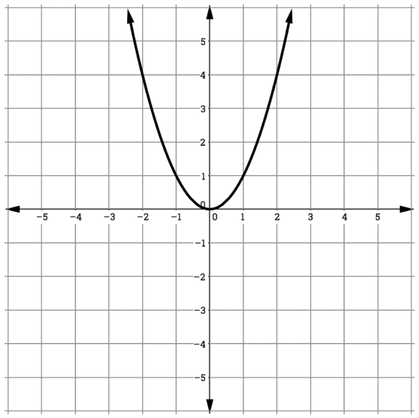
Ⓓ



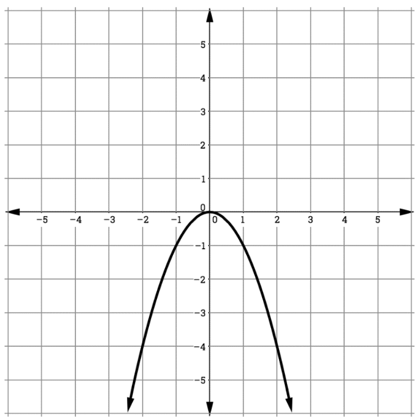
Section 8 – Topic 2
End Behavior of Graphs of Polynomials

Make observations about the end behavior of the following graphs.

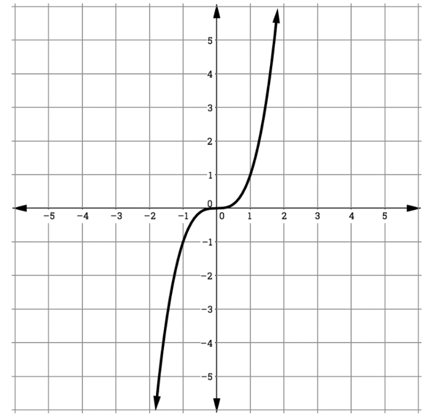
$y = x^2$



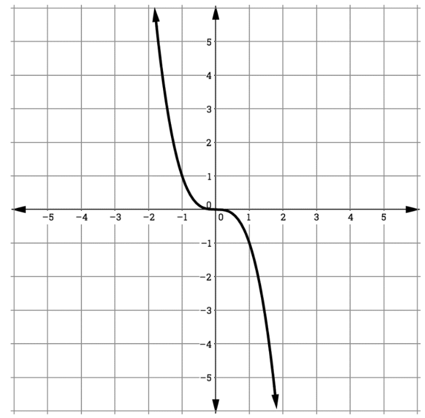
$y = -x^2$



$y = x^3$



$y = -x^3$



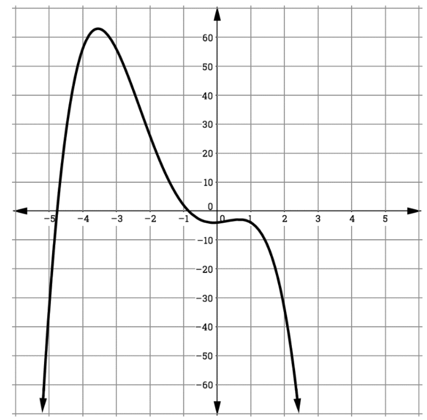
Use your observations to sketch the graphs and make conjectures to complete the table.

End Behavior of Polynomials

	Leading Coefficient is Positive	Leading Coefficient is Negative
Degree of Polynomial is Even	$f(x) = x^2$	$f(x) = -x^2$
	As $x \to \infty, f(x)$______ As $x \to -\infty, f(x)$______	As $x \to \infty, f(x)$______ As $x \to -\infty, f(x)$______
Degree of Polynomial is Odd	$f(x) = x^3$	$f(x) = -x^3$
	As $x \to \infty, f(x)$______ As $x \to -\infty, f(x)$______	As $x \to \infty, f(x)$______ As $x \to -\infty, f(x)$______

Let's Practice!

1. Consider the following graph of $f(x)$.



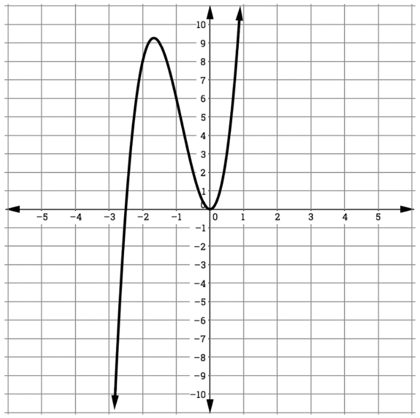
   a. Does the function $f(x)$ have an even or odd degree? Justify your answer.

   b. Is the leading coefficient of $f(x)$ positive or negative? Justify your answer.

2. Describe the end behavior of the function $g(x) = -5x^3 + 8x^2 - 9x$.

Try It!

3. Consider the following graph of $f(x)$.



   a. Does the function $f(x)$ have an even or odd degree? Justify your answer.

   b. Is the leading coefficient of $f(x)$ positive or negative? Justify your answer.

4. Describe the end behavior of the function below.

$$p(x) = \frac{1}{2}x^6 - x^5 - x^4 + 2x^3 - 2x + 2$$

BEAT THE TEST!

1. Determine which of the following statements is true for the function $f(x) = 3x^5 + 7x - 4247$?

   Ⓐ As $x \to \infty, f(x) \to \infty$ and as $x \to -\infty, f(x) \to \infty$
   Ⓑ As $x \to \infty, f(x) \to -\infty$ and as $x \to -\infty, f(x) \to -\infty$
   Ⓒ As $x \to \infty, f(x) \to -\infty$ and as $x \to -\infty, f(x) \to \infty$
   Ⓓ As $x \to \infty, f(x) \to \infty$ and as $x \to -\infty, f(x) \to -\infty$

Section 8 – Topic 3
Graphing Polynomial Functions of Higher Degrees

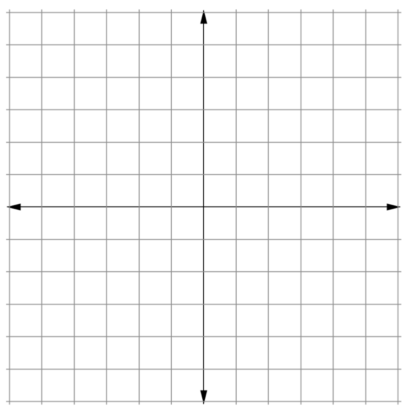
Consider the following function.

$$g(x) = -(x+3)(x-1)(x-2)$$

Describe the end behavior of the graph of $g(x)$.

Find the zeros of $g(x)$.

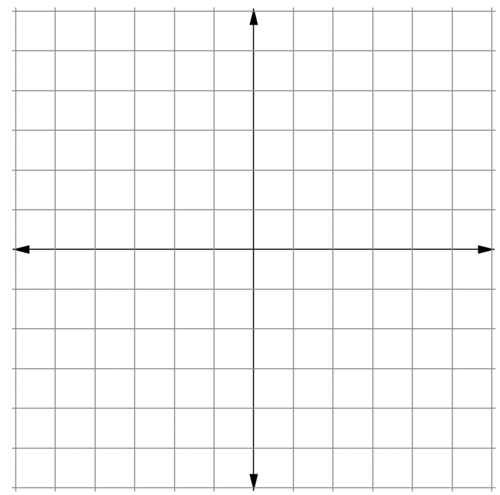
Use the end behavior and zeros to sketch the graph of $g(x)$.



Let's Practice!

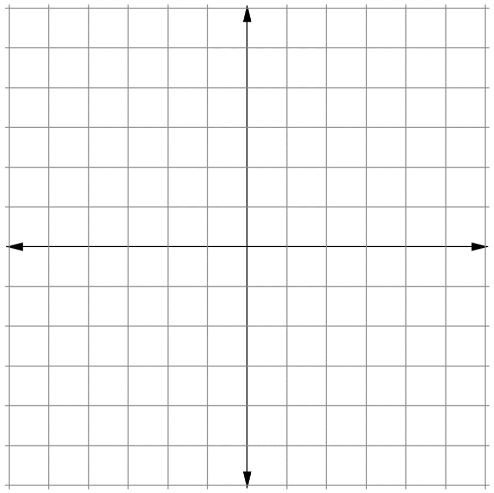
1. Sketch the graph of the following polynomial.

$$f(x) = (x-2)(x+3)(x+5)$$



2. Sketch the graph of the following polynomial.

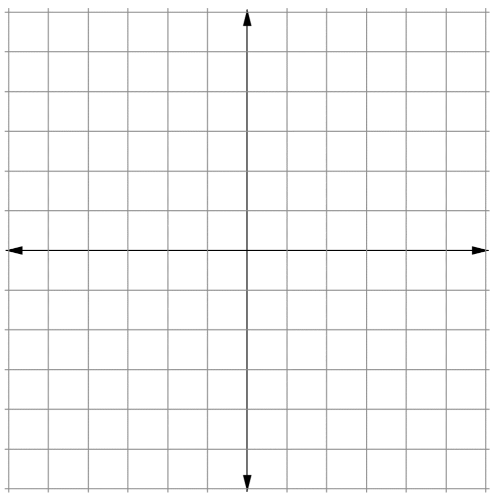
$$f(x) = -(x-5)(x+4)(3x-1)(x+2)$$



Try It!

3. Sketch a graph of the following polynomial.

$$f(x) = (x-1)(x+2)(x-3)(x+1)$$



BEAT THE TEST!

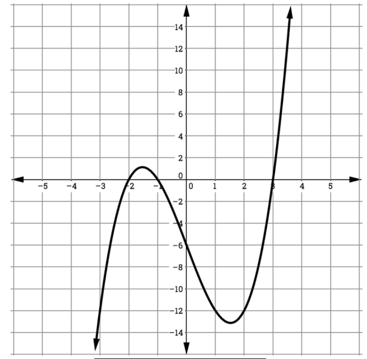
1. Match each equation with its corresponding graph.

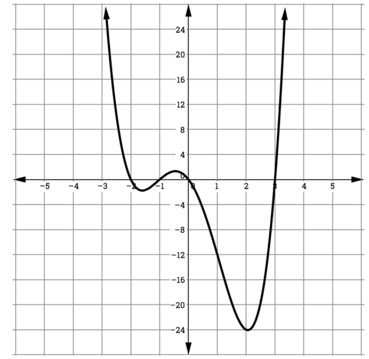
A. $y = (x+1)(x-3)(x+2)$

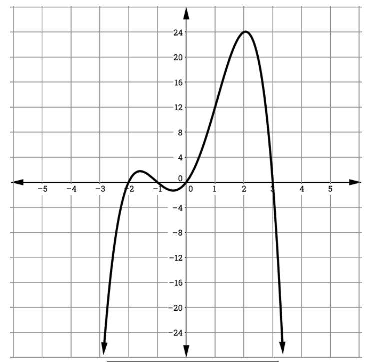
B. $y = -(x+1)(x-3)(x+2)$

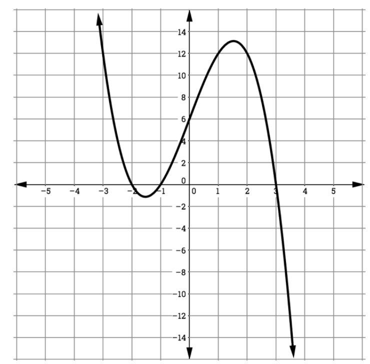
C. $y = -x(x+1)(x-3)(x+2)$

D. $y = x(x+1)(x-3)(x+2)$









Section 9: One Variable Statistics

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.S-ID.1.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).
MAFS.912.S-ID.1.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
MAFS.912.S-ID.1.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Topics in this Section

Section 9 – Topic 1
Dot Plots

Statistics is the science of collecting, organizing, and analyzing data.

Two major classifications of data:

- ***Categorical*** (________________________)
  - o based on "qualities" such as color, taste, or texture, rather than measurements
- ***Quantitative*** (________________________)
  - o based on measurements

There are two types of quantitative data:

- ***Discrete***
  - o There is a finite number of possible data values.
- ***Continuous***
  - o There are too many possible data values so data needs to be measured over intervals.

Classify the following variables.

Height
- o Categorical
- o Discrete quantitative
- o Continuous quantitative

Favorite subject
- o Categorical
- o Discrete quantitative
- o Continuous quantitative

Number of televisions in a household
- o Categorical
- o Discrete quantitative
- o Continuous quantitative

Area code
- o Categorical
- o Discrete quantitative
- o Continuous quantitative

Distance a football is thrown
- o Categorical
- o Discrete quantitative
- o Continuous quantitative

Number of siblings
- o Categorical
- o Discrete quantitative
- o Continuous quantitative

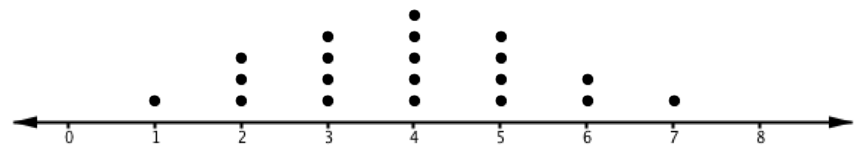
STUDY EDGE TIP

To differentiate between quantitative and categorical data ask yourself: Can I take the average of this data and is it meaningful? If the average is meaningful, then the data is quantitative.

Consider the following sample.

1, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 7

➢ Let's display the above data in a ***dot plot***.



➢ Each data value is represented with a _______ above the number line.

➢ Shows the ________________________ of data values.

➢ Always include the title and an appropriate scale on the number line for the dot plot.

➢ Dot plots are often used for:
- o smaller sets of data
- o discrete data

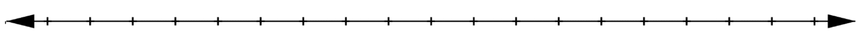
*What is **frequency**?*

Let's Practice!

1. The amount of time 26 students spent on their phone on a given day (rounded to the nearest hour) is recorded as follows.

   0, 3, 4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 9, 10, 10, 10, 11, 11, 12

   Create a dot plot of the data above.

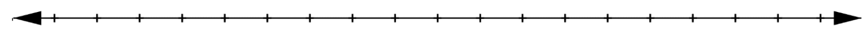


Try It!

2. The responses below are from Mrs. Ferrante's class survey about the number of siblings each student has:

   0, 4, 2, 2, 3, 4, 8, 1, 0, 1, 2, 2, 3, 0, 3, 1, 1, 2

   a. Construct a dot plot of the data.



   b. What observations can you make about the shape of the distribution?

   c. Are there any values that seem not to fit? Justify your answer.

BEAT THE TEST!

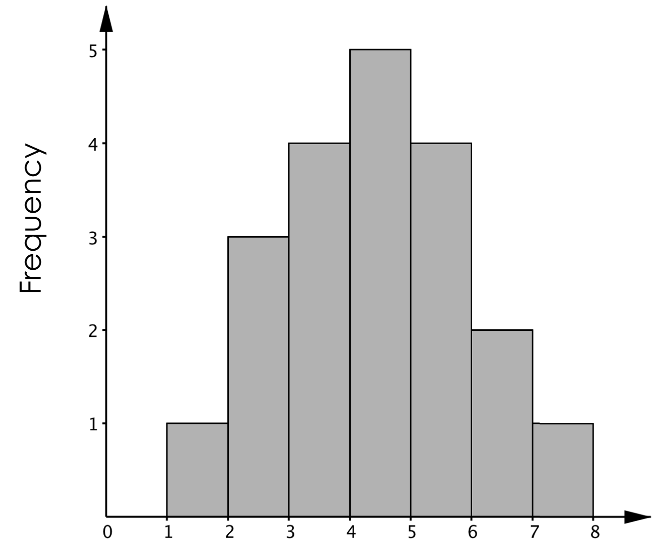
1. The cafeteria at *Just Dance Academy* offers items at seven different prices. The manager recorded the price each time an item was sold in a 2-hour period and created a dot plot to display the data.

Describe the data from the dot plot.

Section 9 – Topic 2
Histograms

Consider the following sample displayed using a histogram.

$1, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 7$



- A ***histogram*** is a bar-style data display showing frequency of data measured over ________________, rather than displaying each individual data value.
- Each interval length must be the __________.
- Always _________ the graph and _________ both axes.
- Choose the appropriate scale on the y-axis and the appropriate intervals on the x-axis.
- Histograms are often used for:
  - larger sets of data
  - continuous data

Describe an interval.

Let's Practice!

1. The amount of time, rounded to the nearest hour, that 26 students spent playing video games on a given day were recorded as follows:

   $0, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 8$

   Construct a histogram to represent the data.

Try It!

2. Determine the sets of data where it would be better to use a histogram than a dot plot.

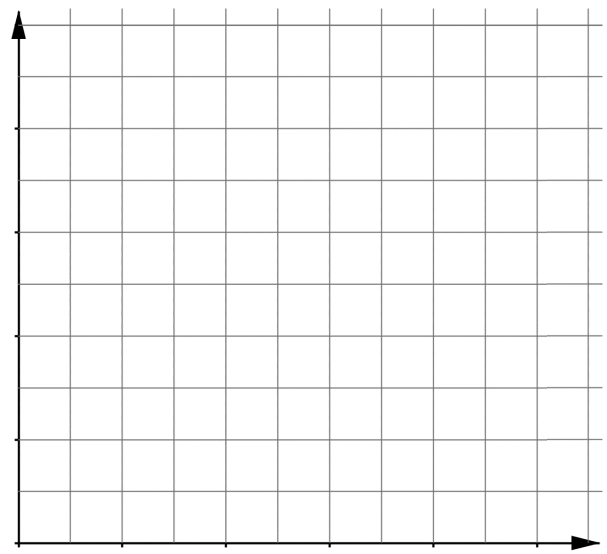
   - ☐ Average daily weather temperatures for Orlando over a year
   - ☐ Daily weather temperatures for Orlando over a month
   - ☐ The results of rolling two dice over and over
   - ☐ Height of high school football players statewide
   - ☐ Time, rounded to the nearest second, needed to run a 100-meter race for 125 randomly selected athletes

BEAT THE TEST!

1. Last year, the local men's basketball team had a great season. The total points scored by the team for each of the 20 games are listed below:

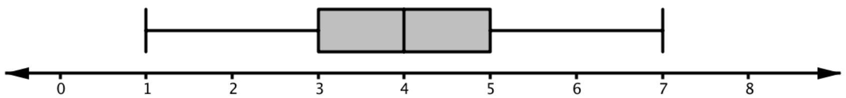
   $45, 46, 46, 52, 53, 53, 55, 56, 57, 58, 62, 62, 64, 64, 65, 67, 67, 76, 76, 89$

   Create a frequency table and construct a histogram of the data.



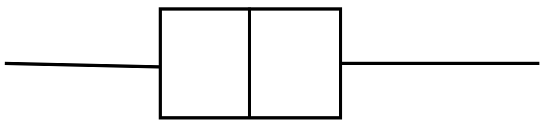
Section 9 – Topic 3
Box Plots – Part 1

The following ***box plot*** graphically displays a summary of the data set $\{1, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 7\}$.



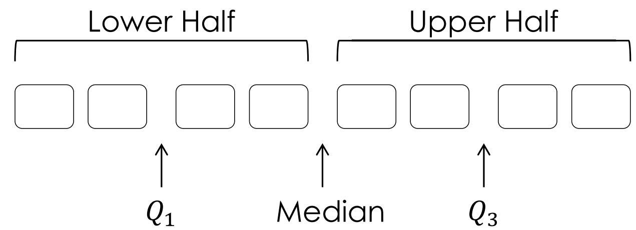
A box plot displays the ***five-number summary*** for a data set.

- The five-number summary of a data set consists of the minimum, first quartile, median, third quartile, and maximum values.

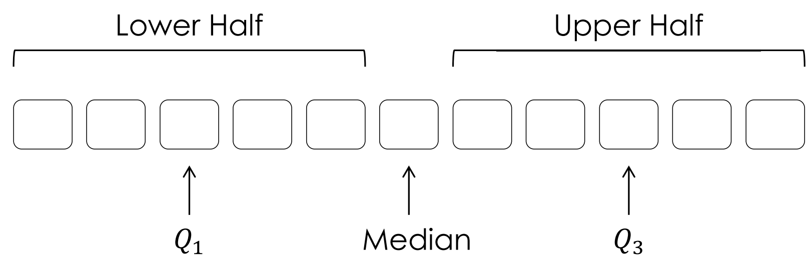


What is a quartile?

Even data set:



Odd data set:



Consider the following data set with an even number of data values:

$$6, 2, 1, 4, 7, 3, 8, 5$$

The minimum value of the data set is ______.

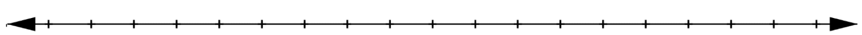
The maximum value of the data set is ______.

The median is the number in the middle when the data is ordered from least to greatest. The median of the data set is ______.

The first quartile of the data set is ______.

The third quartile of the data set is ______.

Use the five-number summary to represent the data with a box plot.



Some observations from our boxplot:

- The lowest 50% of data values are from ____ to ____.
- The highest 50% of data values are from ____ to ____.
- The middle 50% (the box area) represents the values from ______ to ______.
  - The middle 50% is also known as the ***IQR*** (interquartile range).
- The first quartile represents the lower 25% of the data (______ percentile).
- The third quartile represents the first 75% of the data (______ percentile).
- 75% of the values are above ______________________.
- 25% of the values are above _______________________.
- The median of the lower half of the data is ________.
- The median of the upper half of the data is ________.

Section 9 – Topic 4
Box Plots – Part 2

Consider the following data sets:

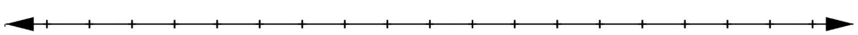
Data set #1: $1, 3, 5, 7, 9, 11, 13, 23$

Data set #2: $1, 3, 5, 7, 9, 11, 13, 15$

Complete the following table.

	Minimum	Maximum	Median	First Quartile	Third Quartile
Data Set #1					
Data Set #2					

Construct the box plots for both data sets, one above the other.



Compare and contrast both box plots.

Explain which boxplot is not symmetrical. Justify your answer.

Let's Practice!

1. Consider the following data set with an odd number of data values:

$$3, 7, 10, 11, 15, 18, 21$$

   a. The minimum value of the data set is ______.

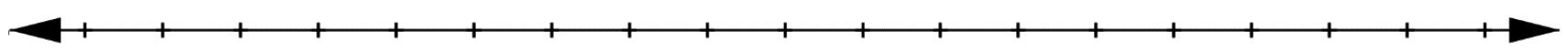
   b. The maximum value of the data set is ______.

   c. The median of the data set is ______.

   d. The first quartile of the data set is ______.

   e. The third quartile of the data set is ______.

f. Use the five-number summary to construct a box plot.

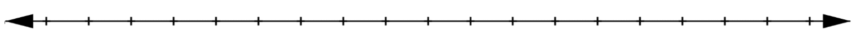


Try It!

2. The time, rounded to the nearest hour, that 26 tourists spent on excursions in Cat Island, Mississippi on a given day was recorded as follows (Cat Island is not actually an island for cats):

$0, 3, 4, 4, 5, 5, 6, 6, 6, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 9, 10, 10, 10, 11, 11, 12$

a. Construct a box plot to represent the data. Label the minimum, maximum, first quartile, third quartile, and median.



b. The bottom 25% of tourists spent, at most, _____ hours on excursions.

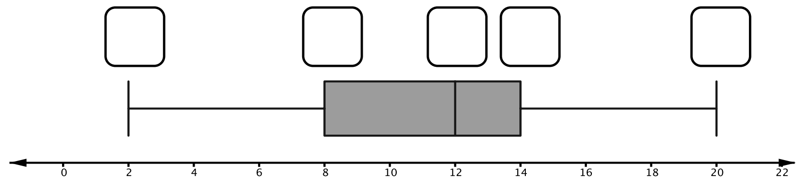
BEAT THE TEST!

1. Mrs. Bridgewater recorded the number of snapchats 10 different students sent in one day and constructed the box plot below for the data.

Part A: Use the following vocabulary to label the box plot. Hint: You will not use all of the words on the list.

A. Average	E. Median
B. First Quartile	F. Minimum
C. Maximum	G. Third Quartile
D. Mean	

Snapchats Sent In One Day



Part B: The 50^{th} percentile of the data set is ______.

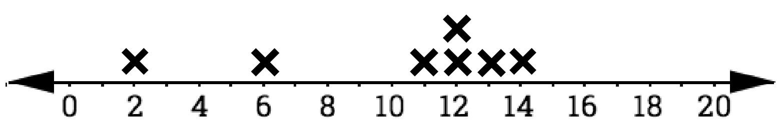
Part C: Half of the data values are between

2 and 20.
8 and 12.
8 and 14.
10 and 12.

Part D: 75% of students send [12 / 13 / 14 / 15] or fewer snapchats per day.

12
13
14
15

Part E: Add dots to the number line below to complete the dot plot so that it could also represent the data.



Section 9 – Topic 5
Measures of Center and Shapes of Distributions

Data displays can be used to describe the following elements of a data set's distribution:

- Center
- Shape
- Spread

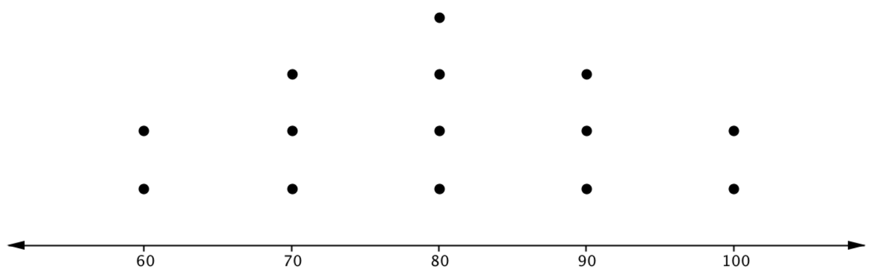
There are three common ***measures of center***:

- ***Mean*** – The ______________ of the data values.
- ***Median*** – The ______________ value of the ordered data set.
- ***Mode*** – The __________ ___________________ occurring value(s).

Mr. Gray gave a test on a regular school day with no special activities. The scores are listed below:

$$60, 60, 70, 70, 70, 80, 80, 80, 80, 90, 90, 90, 100, 100$$

The dot plot for the data is as follows:



Looking at the dot plot, what do you think is the value of the median?

What is the value of the mean?

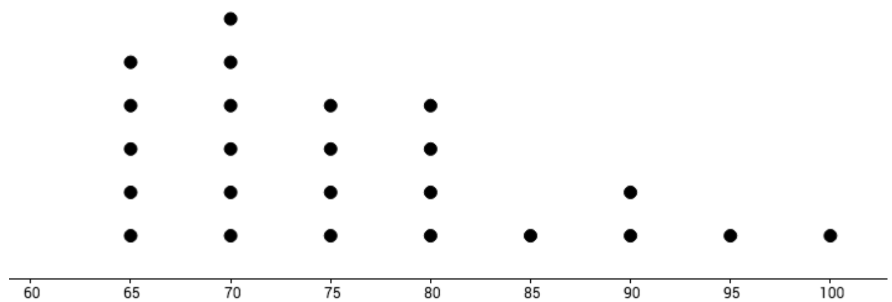
Why is it important to know where the center is?

The shape of a dot plot also gives important information about a data set's distribution. The data in the previous dot plot is symmetrical and follows a ***normal distribution***. What do you notice about the shape of a normal distribution?

Let's Practice!

1. Mr. Gray then gave a test the day after a basketball game against the school's rival. The scores were as follows:

$$65, 65, 65, 65, 65, 70, 70, 70, 70, 70, 70\ 75, 75, 75, 75,$$
$$80, 80, 80, 80, 85, 90, 90, 95, 100$$



   a. What are the mean and the median of this data set?

   b. Which measure is a more appropriate measure of center, the mean or the median?

   c. Does this data set have a normal distribution? Why or why not?

   d. The shape of this distribution is __________ _________.

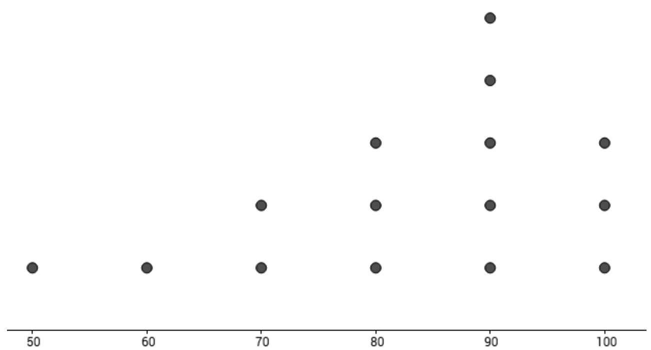
Try It!

2. Mr. Gray then gave a test the day after a mid-week early release day. The scores were as follows:

   $$50, 60, 70, 70, 80, 80, 80, 90, 90, 90, 90, 90, 100, 100, 100$$

   a. Which value do you think will be smaller, the mean or the median?

   b. Consider the dot plot for the data.

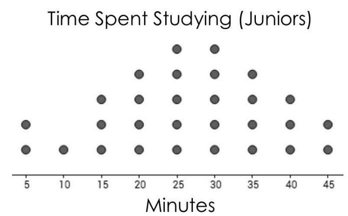
   Which measure is a more appropriate measure of center, the mean or the median?

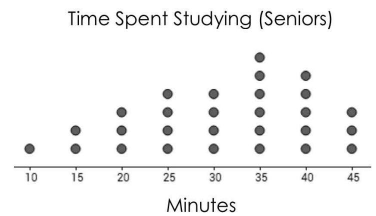
   c. The shape of this distribution is ___________ ________.

   d. For a normal-shaped data set the best measure of center is the ____________, whereas for a skewed-shaped data set, the ____________ is better.

BEAT THE TEST!

1. Mr. Logan surveyed his junior and senior students about the time they spent studying math in one day. He then tabulated the results and created a dot plot displaying the data for both groups.





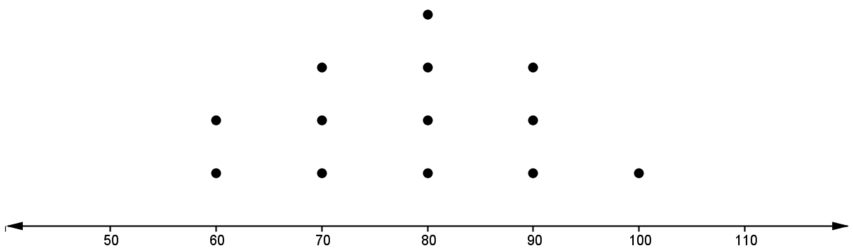
Part A: The value of the larger median for the two groups is ______.

Part B: The value of the larger mean for the two groups is ______.

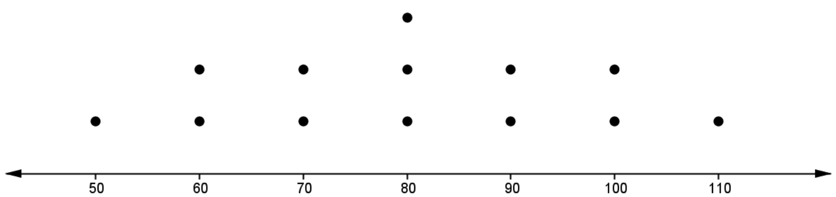
Part C: Using one to two sentences, describe the difference between the number of minutes the juniors and seniors studied by comparing the center and shapes for the groups.

Section 9 – Topic 6
Measures of Spread – Part 1

A meteorologist recorded the average weekly weather temperatures over a 13-week period and displayed the data below.



A meteorologist in a different state also recorded the average weekly weather temperatures over a 13-week period and displayed the data below.



Measures of spread tell us how much a data sample is spread out or scattered.

What are the differences between the spreads of the two data sets?

There are two primary ways to measure the spread of data:

- ***Interquartile Range (IQR)*** =
  - The *IQR* is typically used to describe the spread of skewed data.

Consider the following data set:

$$5, 5, 6, 7, 8, 8, 8, 9, 10, 12, 12$$

What are the first and third quartiles of the data?

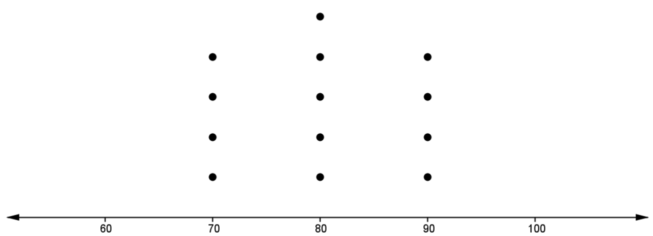
Calculate the interquartile range (IQR) of the data.

Why do you think IQR is used to measure spread in skewed data?

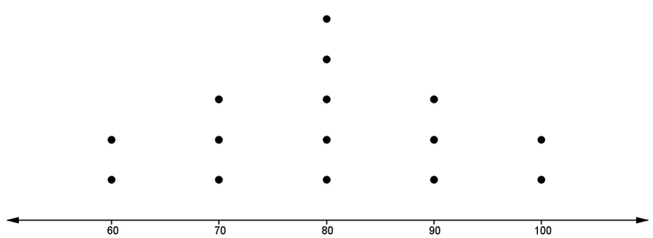
- ➢ ***Standard deviation*** is the typical distance of the data values from the mean. The larger the standard deviation, the ___________ the individual values are from the mean.
  - o Typically used for __________ ________________.

Consider the dot plots below:

A.



B.



Which has a larger standard deviation? Explain your answer.

Section 9 – Topic 7
Measures of Spread – Part 2

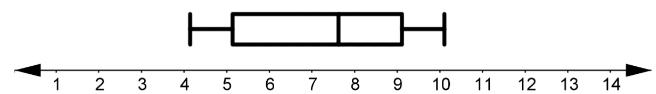
Let's Practice!

1. The Bozeman Bucks and Tate Aggies cross-country teams ran an obstacle course. The times for each team are summarized below:

Bozeman Obstacle Course Times

4:25	4:43	4:49	5:02	5:12
5:21	5:31	5:32	5:37	5:52
5:54	6:08	6:20	6:26	6:33
6:48	6:53	7:16	7:23	8:05

Tate Aggies Obstacle Course Times

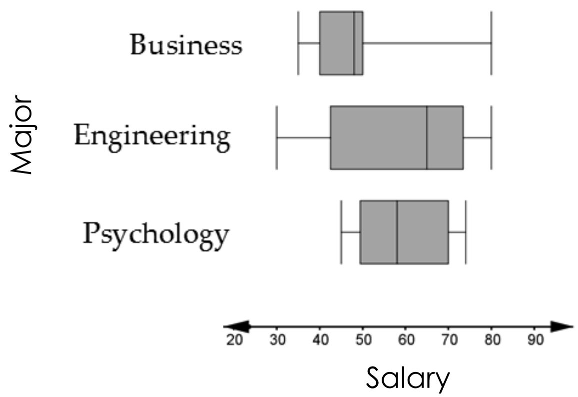


Which statements are true about the data for the Bozeman Bucks and the Tate Aggies? Select all that apply.

- ☐ The median time of the Bozeman Bucks is less than the median time of the Tate Aggies.
- ☐ The fastest 25% of athletes on both teams complete the obstacle course in about the same amount of time.
- ☐ The interquartile range of the Bozeman Bucks is less than the interquartile range of the Tate Aggies.
- ☐ Approximately 50% of Tate Aggies have times between 5 and 6 minutes.
- ☐ The data for the Bozeman Bucks is skewed to the left.

Try It!

2. The following box plots represent the starting salaries (in thousands of dollars) of 12 recent business graduates, 12 recent engineering graduates, and 12 recent psychology graduates:



   a. Describe the shape of each major's data distribution.

      Business:

      Engineering:

      Psychology:

   b. Which major has the largest median salary? The largest IQR?

BEAT THE TEST!

1. Data on the time that Mrs. Lanister's students spend studying math and science on a given night is summarized below:

<u>Math</u>

Mean: 75 minutes
Minimum: 0 minutes
First Quartile: 65 minutes
Median: 78 minutes
Third Quartile: 100 minutes
Maximum: 145 minutes
Standard deviation: 8 minutes

<u>Science</u>

Mean: 25 minutes
Minimum: 0 minutes
First Quartile: 15 minutes
Median: 30 minutes
Third Quartile: 35 minutes
Maximum: 50 minutes
Standard deviation: 12 minutes

Tyrion spent 10 minutes studying math and 50 minutes studying science. If Tyrion spent all 60 minutes studying math, which of the following would be affected? Check all that apply.

	Increase	Decrease	Stays the Same
Interquartile Range for Math Time	☐	☐	☐
Standard Deviation of Math Time	☐	☐	☐

2. The data from a survey of the ages of people in a CrossFit class was skewed to the right.

Part A: The appropriate measure of center to describe the data distribution is the

- ○ mean
- ○ median

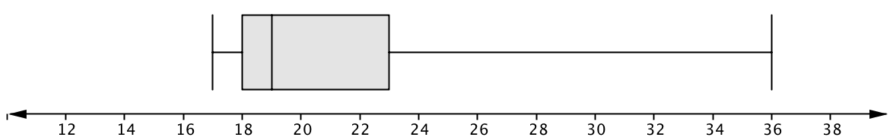
.

The

- ○ interquartile range
- ○ standard deviation

is the appropriate measure to describe the spread.

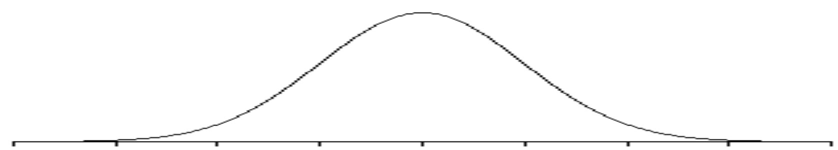
Part B: The boxplot below represents the data. Calculate the appropriate measure of spread.



Section 9 – Topic 8
The Empirical Rule

Assume that we have a data set so large that we are not given a list of all the values. We are told the data follows a normal distribution with a mean of 16 and standard deviation of 4.

Label the distribution below with the values using the mean and standard deviation.



Suppose one of the data values is 20. An observation of 20 is _____ standard deviation(s) __________ the mean.

Suppose one of the data values is 8. An observation of 8 is ______ standard deviation(s) _____________ the mean.

Suppose an observation is 1.5 standard deviations above the mean. The value of that observation is ____________.

We can use the empirical rule to understand the data distribution.

Empirical Rule

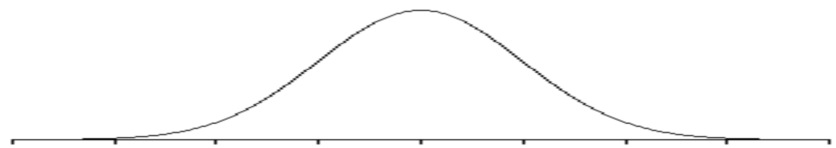
- Approximately 68% of values are within one standard deviation of the mean.
- Approximately 95% of values are within two standard deviations of the mean.
- Approximately 99.7% of values are within three standard deviations of the mean.

Label the percentages on the previous distribution.

Let's Practice!

1. Suppose the amount of water a machine dispenses into plastic bottles has a normal distribution with a mean of 16.2 ounces and a standard deviation of 0.1 ounces.

   a. Label the distribution below with the values using the mean and standard deviation.



   b. The middle 95% of bottles contain between ______ and ______ ounces of water.

   c. Approximately 68% of bottles have between ______ and ______ounces of water.

   d. What percentage of bottles contain more than 16.4 ounces of water?

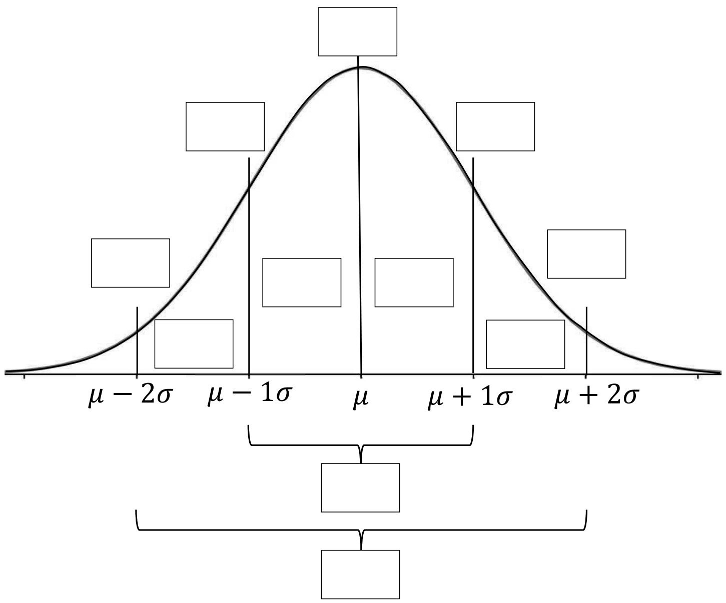
   e. What is the probability that a randomly selected bottle contains less than 16.3 ounces of water?

   f. What percentage of bottles contain between 16.1 and 16.4 ounces of water?

Try It!

2. Drag and drop the correct numbers to build a normal distribution graph based on a mean of 45.5 and standard deviation of 3.92 (All numbers will not be used, and some may be used more than once).

13.5%	13.5%	37.66	34%
57.26	33.74	53.34	49.42
2.45%	2.45%	68%	45.5
34%	41.58	95%	99.7%



BEAT THE TEST!

1. SAT mathematics scores for a particular year are approximately normally distributed with a mean of 510 and a standard deviation of 80.

Part A: What is the probability that a randomly selected score is greater than 590?

Part B: What is the probability that a randomly selected score is greater than 670?

Part C: What percentage of students score between 350 and 670?

Part D: A student who scores a 750 is in the __________ percentile.

Section 9 – Topic 9
Outliers in Data Sets

A survey about the average number of text messages sent per day was conducted at a retirement home:

$$5, 5, 5, 5, 5, 5, 5, 10, 10, 10, 10, 10, 15, 15, 15$$

The mean for this data set is 8.7 and the median is 10.

Grandma Gadget is up-to-date on the latest technology and loves to text her 25 grandchildren. Her data is substituted for one of the original values of 15. She sends an average of 85 texts per day.

The new data set is:

$$5, 5, 5, 5, 5, 5, 5, 10, 10, 10, 10, 10, 15, 15, 85$$

Which measure of center will be most affected by substituting Grandma Gadget – the mean or the median? Justify your answer.

Does Grandma Gadget's data have a greater effect on standard deviation or interquartile range? Justify your answer.

Grandma Gadget's data point is called an ***outlier***.

An ***outlier*** is an ____________________value in a data set that is very distant from the others.

Let's Practice!

1. The number of customers received by a car dealership during 30 randomly selected days is listed.

26	29	27	33	29	28
31	36	26	31	35	32
34	34	28	11	35	35
33	37	31	26	37	33
29	35	37	29	27	33

Identify the outlier, and describe how it affects the mean and the standard deviation.

The outlier is _______. The outlier in the data set causes the mean to _________________ and the standard deviation to __________________.

Try It!

2. The students in Mrs. Gomez's class were surveyed about the number of text messages they send per day. The data is as follows:

   $0, 24, 26, 28, 28, 30, 33, 35, 35, 36, 38, 39, 42, 42, 45, 50$

   a. What value would you predict to be an outlier?

   b. How does the outlier affect the mean?

   c. How does the outlier affect the median?

   d. Which measure of center would best describe the data, the mean or the median?

   e. How does the outlier affect the standard deviation?

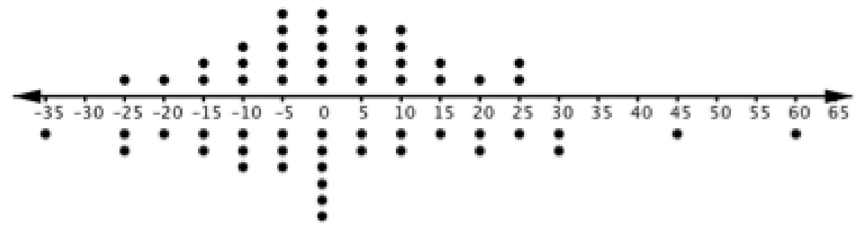
   f. How does the outlier affect the interquartile range?

   g. Which measure of spread would best describe the data, the standard deviation or the interquartile range?

BEAT THE TEST!

1. The dot plot below compares the arrival times of 30 flights for each of two different airlines:

   Airline P

   Airline Q

   Negative numbers represent the number of minutes the flight arrived before its scheduled time.

   Positive numbers represent the number of minutes the flight arrived after its scheduled time.

   Zero indicates the flight arrived at its scheduled time.

   Based on this data, from which airline will you choose to buy your ticket? Use your knowledge of shape, center, outliers, and spread to justify your choice.

2. After a long day at Disney World, a group of students were asked how many times they each rode Space Mountain. The values are as follows:

$$4, 3, 19, 1, 2, 2, 4, 3, 5, 3, 4, 5, 4, 5$$

Part A: Are there any outliers in the data set above? Explain.

Part B: The outlier causes the
- o mean
- o median

to be greater than the
- o mean.
- o median.

Part C: If the outlier was changed to 5, the interquartile range would
- o increase
- o decrease
- o stay the same

and the standard deviation would
- o increase.
- o decrease.
- o stay the same.

Section 10: Two Variable Statistics

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.S-ID.2.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (*including joint, marginal, and conditional relative frequencies*). Recognize possible associations and trends in the data.
MAFS.912.S-ID.2.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit the function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. *Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.
MAFS.912.S-ID.3.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
MAFS.912.S-ID.3.8	Compute (using technology) and interpret the correlation coefficient of a linear lift.
MAFS.912.S-ID.3.9	Distinguish between correlation and causation.

Topics in this Section

Section 10 – Topic 1
Relationship between Two Categorical Variables – Marginal and Joint Probabilities – Part 1

Two categorical variables can be represented with a ***two-way frequency table.***

Consider the following survey:

There were 149 elementary students interviewed and asked to choose whether they prefer math or English class. The data was broken down by gender:

42 males prefer math class

47 males prefer English class

35 females prefer math class

25 females prefer English class

A two-way frequency table is a visual representation of the frequency counts for each categorical variable. The table can also be called a ***contingency table***.

Elementary Students Survey

	Math	English	Total
Males			
Females			
Total			

The total frequency for any row or column is called a ***marginal frequency.***

- Why do you think these total frequencies are called marginal frequencies?

Joint frequencies are the counts in the body of the table.

- Why do you think these frequencies are called joint frequencies?

Draw a box around the marginal frequencies and circle the joint frequencies in the "Elementary Students Survey" contingency table.

The frequency table can be easily changed to show ***relative frequencies***.

- To calculate relative frequency, divide each count in the frequency table by the overall total.

Complete the following relative frequency table.

Elementary Students Survey

	Math	English	Total
Males			
Females			
Total			

Why do you think these ratios are called relative frequencies?

Draw a box around the **marginal relative frequencies** and circle the **joint relative frequencies** in the table.

Interpret the marginal relative frequency for male students.

Interpret the joint relative frequency for females who prefer math.

Section 10 – Topic 2
Relationship between Two Categorical Variables – Marginal and Joint Probabilities – Part 2

Let's Practice!

1. A survey of high school students asked if they play video games. The following frequency table was created based on their responses.

Video Games Survey

	Plays Video Games	Does Not Play Video Games	Total
Males	69	60	
Females	65	85	
Total			

a. Compute the joint and marginal relative frequencies in the table.

b. How many female students do not play video games?

c. What percentage of students interviewed were females who do not play video games?

Try It!

2. Consider the frequency table Video Games Survey.

   a. How many male students were interviewed?

   b. What is the probability that a student interviewed is male?

   c. Which numbers represent joint frequencies?

   d. Which numbers represent joint relative frequencies?

   e. What percentage of the subjects interviewed play video games?

BEAT THE TEST!

1. A survey conducted at Ambidextrous High School asked all 1,700 students to indicate their grade level and whether they were left-handed or right-handed. Only 59 of the 491 freshmen are left-handed. Out of the 382 students in the sophomore class, 289 of them are right-handed. There are 433 students in the junior class and 120 of them are left-handed. There are 307 right-handed seniors.

   Part A: Complete the frequency table to display the results of the survey.

					Total
Right-handed					
Left-handed					
Total					

   Part B: What is the joint relative frequency for right-handed freshmen?

   Part C: What does the relative frequency $\frac{491}{1{,}700}$ represent?

   Part D: Circle the smallest marginal frequency.

Section 10 – Topic 3
Relationship between Two Categorical Variables – Conditional Probabilities

Recall the students' class preference data:

Elementary Students Survey

	Math	English	Total
Males	42	47	89
Females	35	25	60
Total	77	72	149

The principal says that males in the interview have a stronger preference for math than females. Why might the principal say this?

We can determine the answer to questions like this by comparing ***conditional relative frequencies***.

Complete the conditional relative frequency table on the following page to determine whether males or females showed stronger math preference in the survey.

Conditional Relative Frequency Table

	Math	English	Total
Males			
Females			
Total			

What percentage of male students prefer math?

What percentage of female students prefer math?

These percentages are called ***conditional relative frequencies***.

- Make a conjecture as to why they are called conditional relative frequencies.

When trying to predict a person's class preference, does it help to know their gender?

When we evaluate whether there seems to be a relationship between two categorical variables, we look at the conditional relative frequencies.

- If there is a significant difference between the conditional relative frequencies, then there is evidence of an association between two categorical variables.

Is there an association between gender and class preference?

Let's Practice!

Consider the high school students who were asked if they play video games:

Video Games Survey

	Plays Video Games	Does Not Play Video Games	Total
Males	69	60	129
Females	65	85	150
Total	134	145	279

1. What percentage of the students who do not play video games are female?

2. Given that a student is female, what is the probability that the student does not play video games?

Try It!

3. Of the students who are male, what is the probability that the student plays video games?

4. What percentage of the students who play video games are male?

BEAT THE TEST!

1. Freshmen and sophomores were asked about their preferences for an end-of-year field trip for students who pass their final examinations. Students were given the choice to visit an amusement park, a water park, or a mystery destination. A random sample of 100 freshmen and sophomores was selected. The activities coordinator constructed a frequency table to analyze the data:

	Amusement Park	Water Park	Mystery Destination	Total
Freshmen	25	10	20	55
Sophomores	35	5	5	45
Total	60	15	25	100

Part A: What does the relative frequency $\frac{10}{55}$ represent?

Part B: What percentage of students who want to go to an amusement park are sophomores?

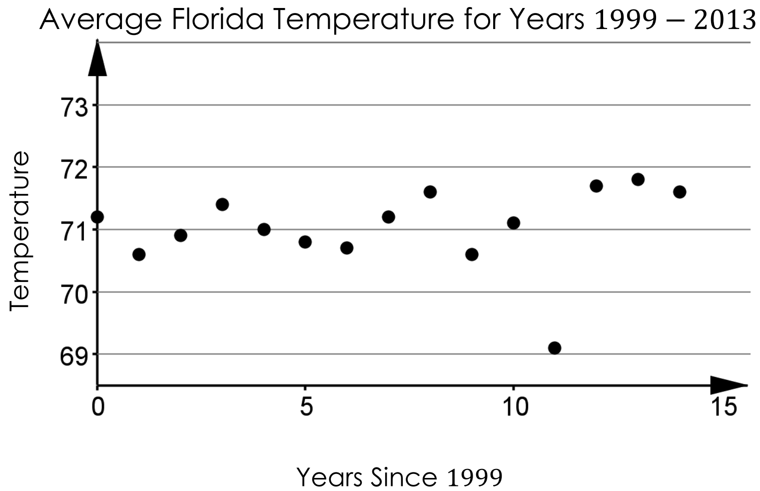
Part C: What activity should the coordinator schedule for sophomores? Justify your answer.

Section 10 – Topic 4
Scatter Plots and Function Models

Let's consider quantitative data involving two variables.

Consider the data below of the statewide average temperature in Florida each year since 1999. A ***scatterplot*** of the data is also shown on the following page.

Year (1999 – 2013)	Average Statewide Temperature (°F)	Year (1999 – 2013) cont.	Average Statewide Temperature (°F) cont.
1999	71.2	2007	71.6
2000	70.6	2008	70.6
2001	70.9	2009	71.1
2002	71.4	2010	69.1
2003	71.0	2011	71.7
2004	70.8	2012	71.8
2005	70.7	2013	71.6
2006	71.2		



What do the values on the x-axis represent?

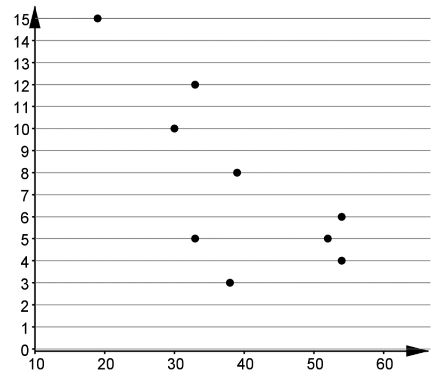
What do the values on the y-axis represent?

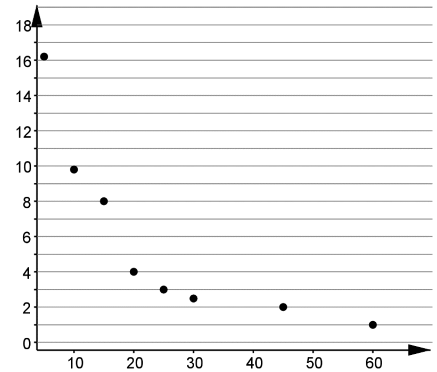
What does the ordered pair $(3, 71.4)$ represent?

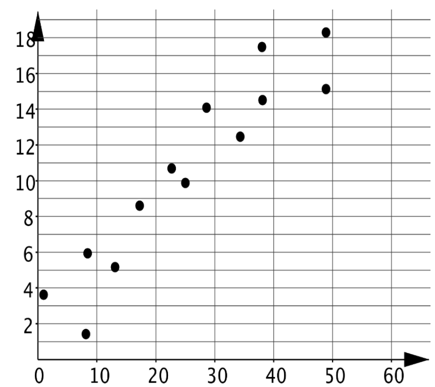
Describe the relationship between year and temperature using the line of best fit.

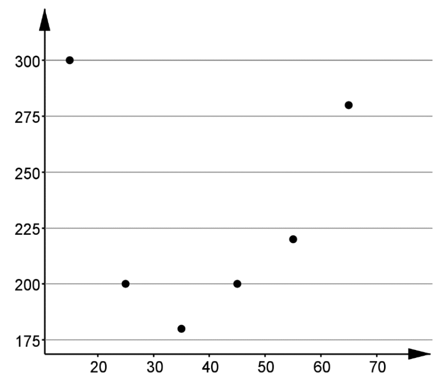
Let's Practice!

1. A scatterplot is a graphical representation of the relationship between two quantitative variables. Classify the relationship represented in each of the scatterplots below as linear, quadratic, or exponential.





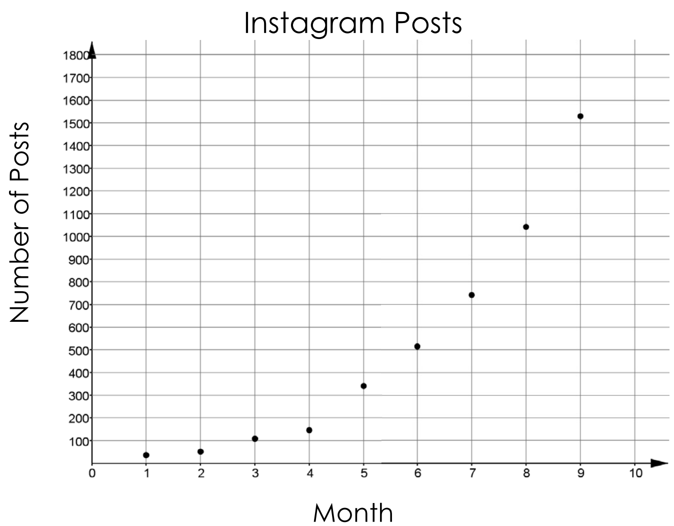




Try It!

2. Over a nine-month period, students in Mrs. Coleman's class at Satellite High School collected data on their total number of Instagram posts each month. The data is summarized below.

Month	1	2	3	4	5	6	7	8	9
# Posts	36	52	108	146	340	515	742	1,042	1,529



The linear regression equation fit to this data is $f(x) = 176.32x - 380.47$, and the exponential regression equation fit to this data is $g(x) = 23.30 \cdot 1.62^x$.

a. What is the predicted number of posts for month 11 using the linear function?

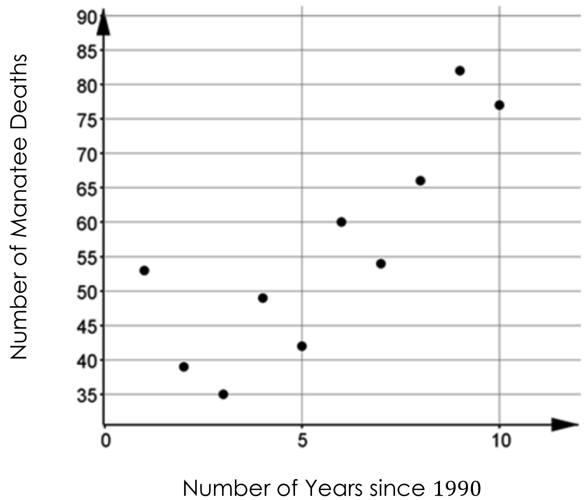
b. What is the predicted number of posts for month 11 using the exponential function?

c.! Is the linear equation or the exponential equation the best model for this data?

BEAT THE TEST!

1. The scatterplot below shows the number of manatees killed by watercraft each year in Florida from $1991 - 2000$.

Manatee Deaths



The linear equation that best models this relationship is $y = 4.3697x + 31.6667$, where x represents the number of years since 1990 and y represents the number of manatee deaths.

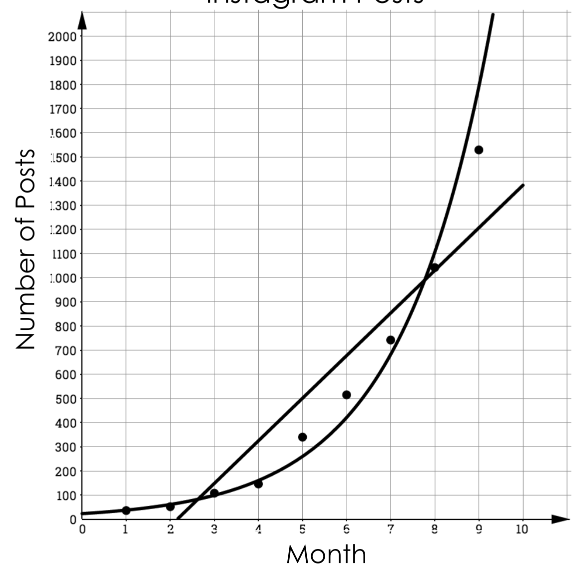
If the trend continues, predict the number of manatees that will be killed by watercraft in 2020.

Section 10 – Topic 5
Residuals and Residual Plots – Part 1

Over a nine-month period, students in Mrs. Coleman's class at Satellite High School collected data on their total number of Instagram posts each month. The data is summarized below:

Month	1	2	3	4	5	6	7	8	9
# Posts	36	52	108	146	340	515	742	1,042	1,529

Instagram Posts



Let's consider which function should be used to fit the data – the linear function $f(x) = 176.32x - 380.47$ or the exponential function $g(x) = 23.30 \cdot 1.62^x$.

A ***residual*** is the vertical distance between an actual data point and the function fitted to the data.

- Residual = actual y – predicted y

Compute the residuals for each function:

Linear Function: $f(x) = 176.32x - 380.47$

Month	# Posts	Predicted Value	Residual
1	36	−204.15	240.15
2	52	−27.83	79.83
3	108	148.49	
4	146	324.81	−178.81
5	340	501.13	−161.13
6	515	677.45	−162.45
7	742	853.77	−111.77
8	1,042	1,030.09	11.91
9	1,529		322.59

Exponential Function: $g(x) = 23.30 \times 1.62^x$

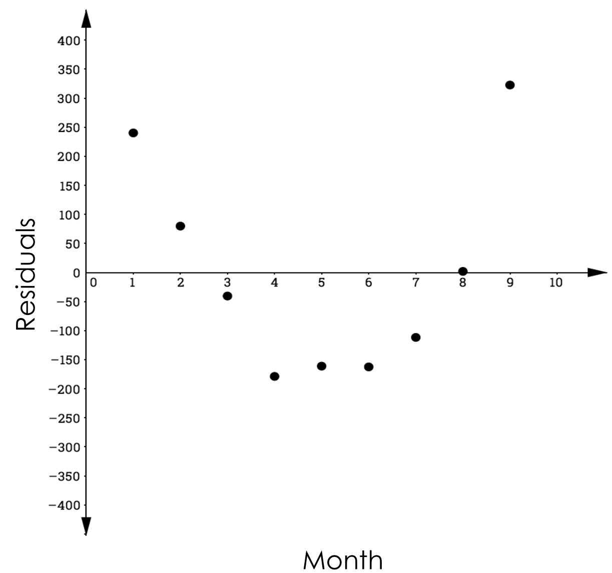
Month	# Posts	Predicted Value	Residual
1	36	37.75	−1.75
2	52	61.15	−9.15
3	108	99.06	8.94
4	146	160.48	−14.48
5	340		
6	515	421.16	93.84
7	742	682.28	59.72
8	1,042		
9	1,529	1,790.57	−261.57

What do you notice about the values of the residuals for the two models?

To determine whether or not a function is a good fit, look at a ***residual plot*** of the data.

- A residual plot is a graph of the residuals (y-axis) versus the x-values (x-axis).

Residual plot for the linear function $f(x) = 176.32x - 380.47$:



STUDY EDGE TIP

The sum of the residuals is always 0. This is because some of the data points are above their predicted value and some are below their predicted value.

Let's Practice!

1. The residuals for the exponential function fitted to model the number of posts on Instagram are shown below. Use the table of residuals to construct a residual plot of the data.

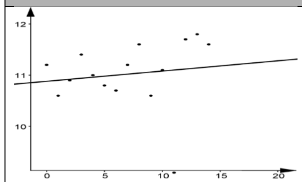
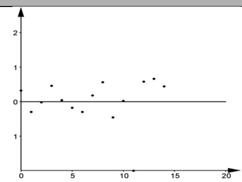
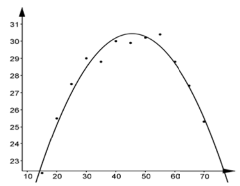
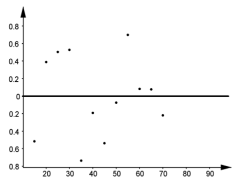
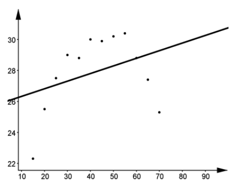
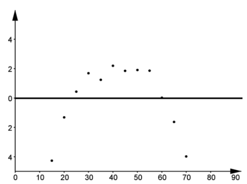
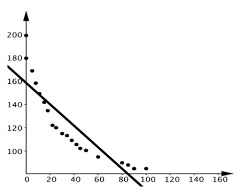
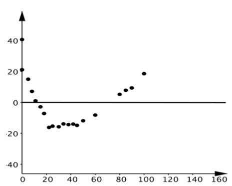
x	1	2	3	4	5	6	7	8	9
Residual	-1.75	-9.15	8.94	-14.48	80.03	93.84	59.72	-63.29	-261.57

Try It!

2. Consider the residual plots for the linear and exponential models.

   Which function fits the data better – the linear or the exponential function? How do you know?

Section 10 – Topic 6
Residuals and Residual Plots – Part 2

Scatter Plot		Residual Plot	What do you notice about the scatter plot and its residual plot?
	→		
	→		
	→		
	→		

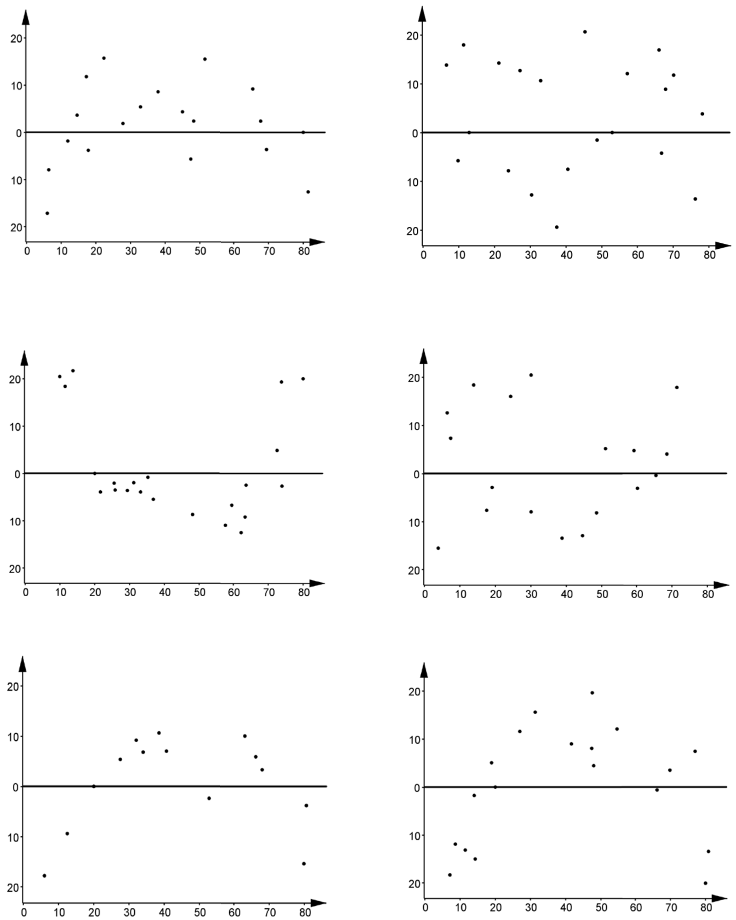
Let's Practice!

1. If a data set has a quadratic trend and a quadratic function is fit to the data, what will the residual plot look like?

2. If a data set has a quadratic trend and a linear function is fit to the data, what will the residual plot look like?

Try It!

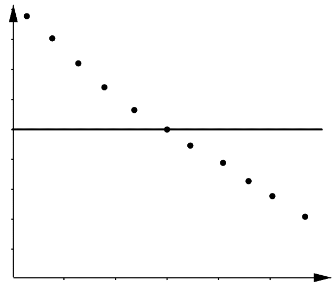
3. Suppose models were fitted for several data sets using linear regression. Residual plots for each data set are shown below. Circle the plot(s) that indicate that the original data set has a linear relationship.



BEAT THE TEST!

1. Suppose a quadratic function is fit to a set of data. Which of the following residual plots indicates that this function was an appropriate fit for the data?

Ⓐ



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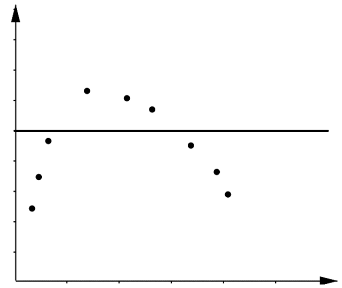

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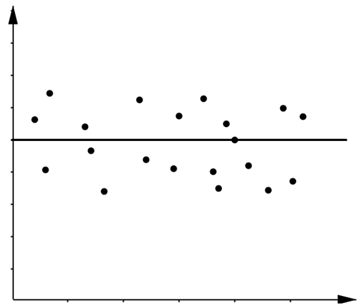

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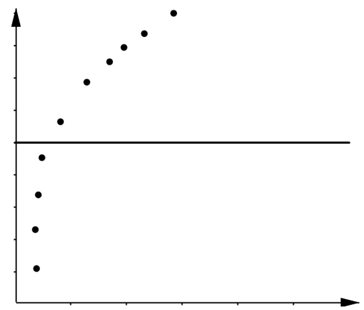
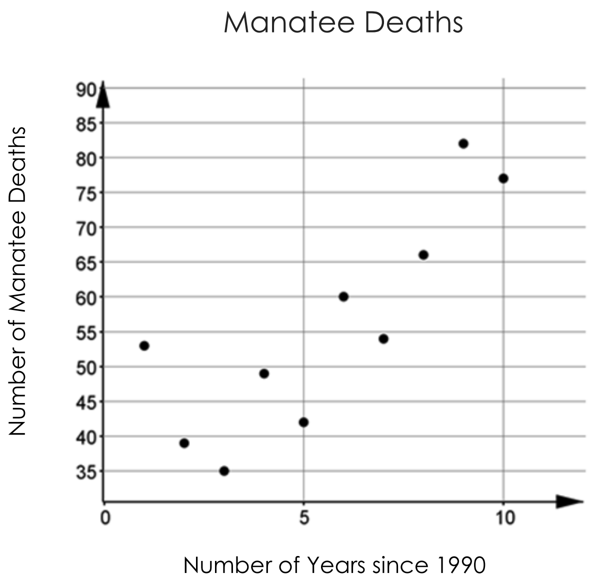


Section 10 – Topic 7
Examining Correlation

The scatter plot below shows the number of manatees killed by watercraft each year in Florida from 1991 – 2000.



Describe the relationship between the years since 1990 and the number of manatees killed by watercraft in Florida.

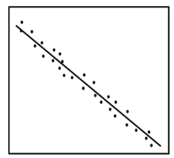
The ***correlation coefficient***, r, measures the strength and direction of the linear association between two quantitative variables.

- $-1 \leq r \leq +1$
- r is unitless

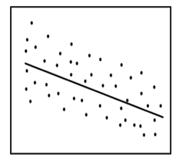
Match each of the following values of r with the appropriate scatter plot:

$r = -0.001$	$r = +0.790$	$r = -0.991$	$r = +0.990$	$r = -0.547$

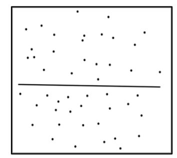
$r =$



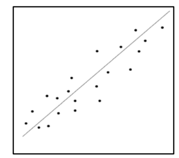
$r =$



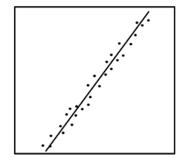
$r =$



$r =$



$r =$



- The closer the points are to the line, the ____________ the absolute value of r will be.
- The closer r is to 0, the ____________ the relationship is between x and y.
- $r = +0.450$ and $r = -0.450$ both indicate the ________ strength of association between the variables.

Strength of a Linear Relationship

$r = -1.00$ $r = 0.00$ $r = +1.00$

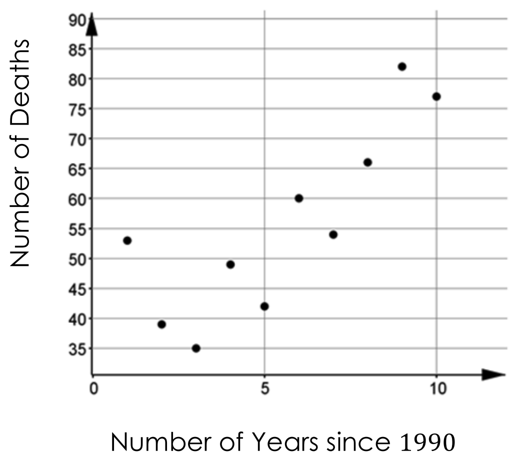
Perfect Negative Linear Relationship	No Linear Relationship	Perfect Positive Linear Relationship

Let's Practice!

1. How do you think outliers affect the value of the correlation coefficient?

2. For the data below, $r = +0.828$. What does the value of the correlation coefficient mean?

Manatee Deaths

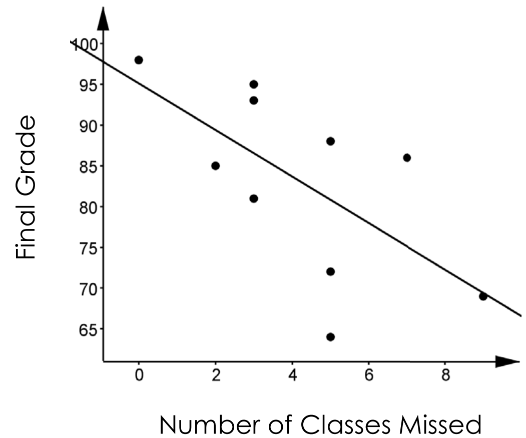


Try It!

3. The table and scatter plot below show the relationship between the number of classes missed and final grade for a sample of 10 students.

Student	1	2	3	4	5	6	7	8	9	10
Missed Classes	0	7	3	2	3	9	5	3	5	5
Final Grade	98	86	95	85	81	69	72	93	64	88

Final Grade and Attendance



For the data above, $r = -0.639$. What does the value of the correlation coefficient mean?

4. There is a strong positive association between the amount of fire damage (y) and the number of firefighters on the scene (x). Does having more firefighters on the scene cause greater fire damage?

STUDY EDGE TIP

Correlation does not imply causation!

- ***Causation*** is when one event causes another to happen.
- Two variables can be correlated without one causing the other.

BEAT THE TEST!

1. Determine whether the correlation in each situation below implies causation. Select all that apply.

   - ☐ There is a positive correlation between smoking cigarettes and lung cancer.
   - ☐ Daily ice cream sales in Florida is positively correlated to the number of shark attacks.
   - ☐ The number of miles driven is negatively correlated to the amount of gas left in the gas tank.
   - ☐ Household income is positively correlated to the number of television sets in a household.
   - ☐ A person's height is positively correlated to his/her weight.