Algebra, Functions, and Data Analysis
Vocabulary Cards
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Expressions and Operations

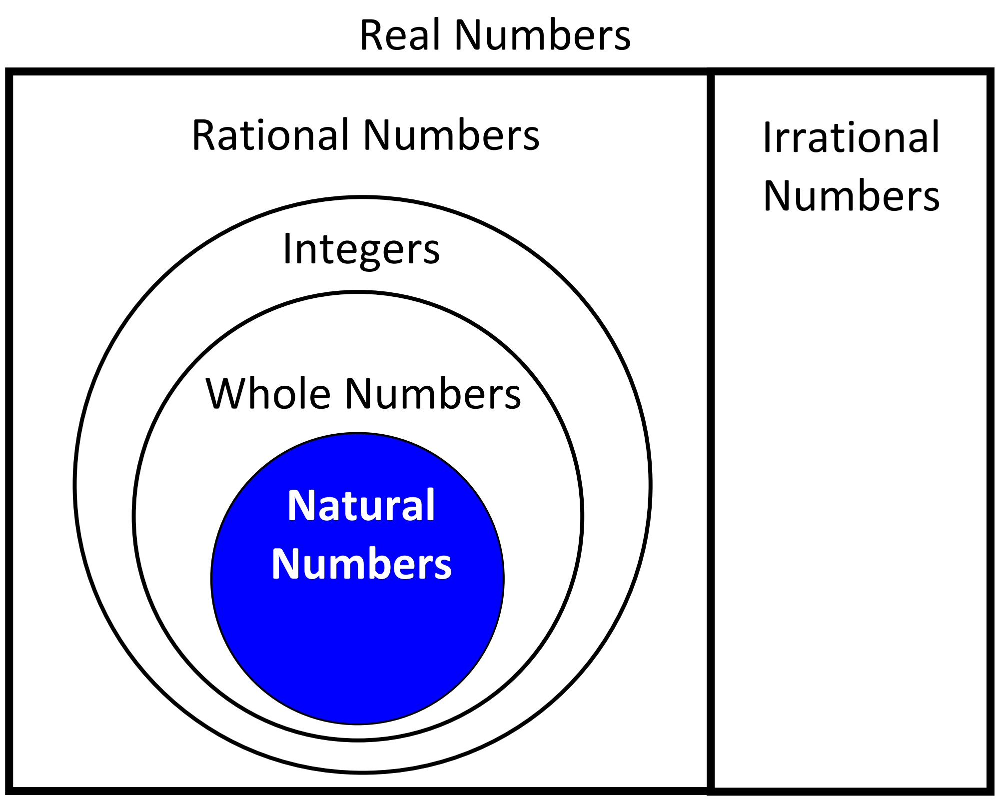
Equations and Inequalities

Relations and Functions

Probability and Statistics

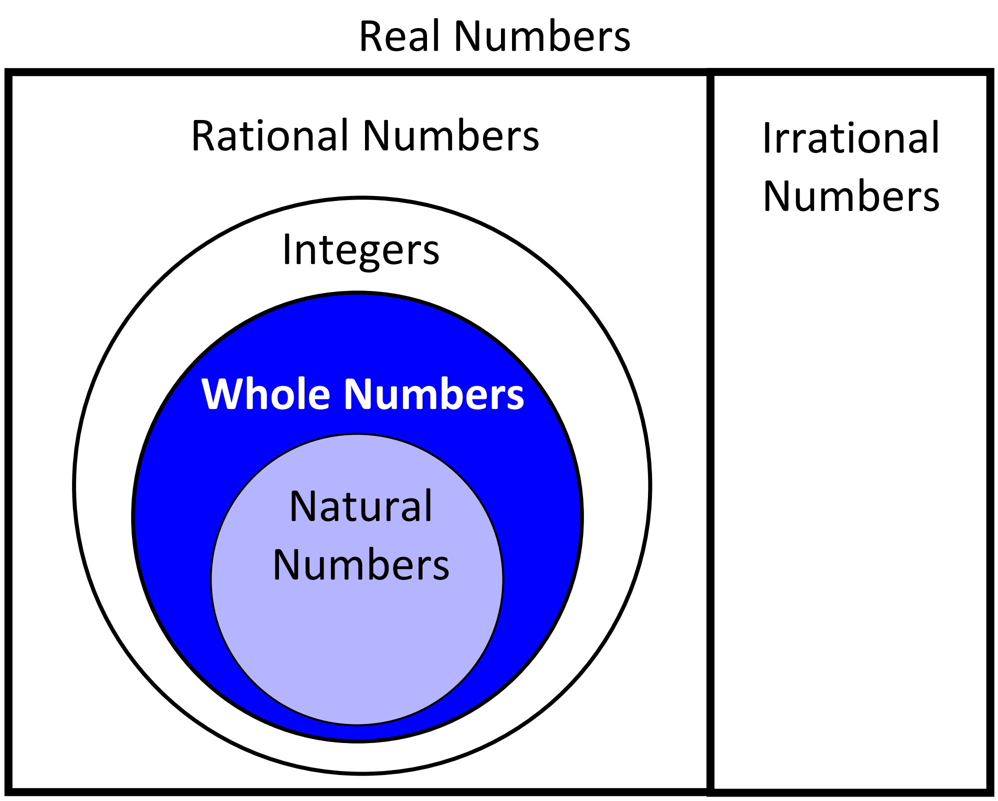
Natural Numbers

The set of numbers
1, 2, 3, 4…



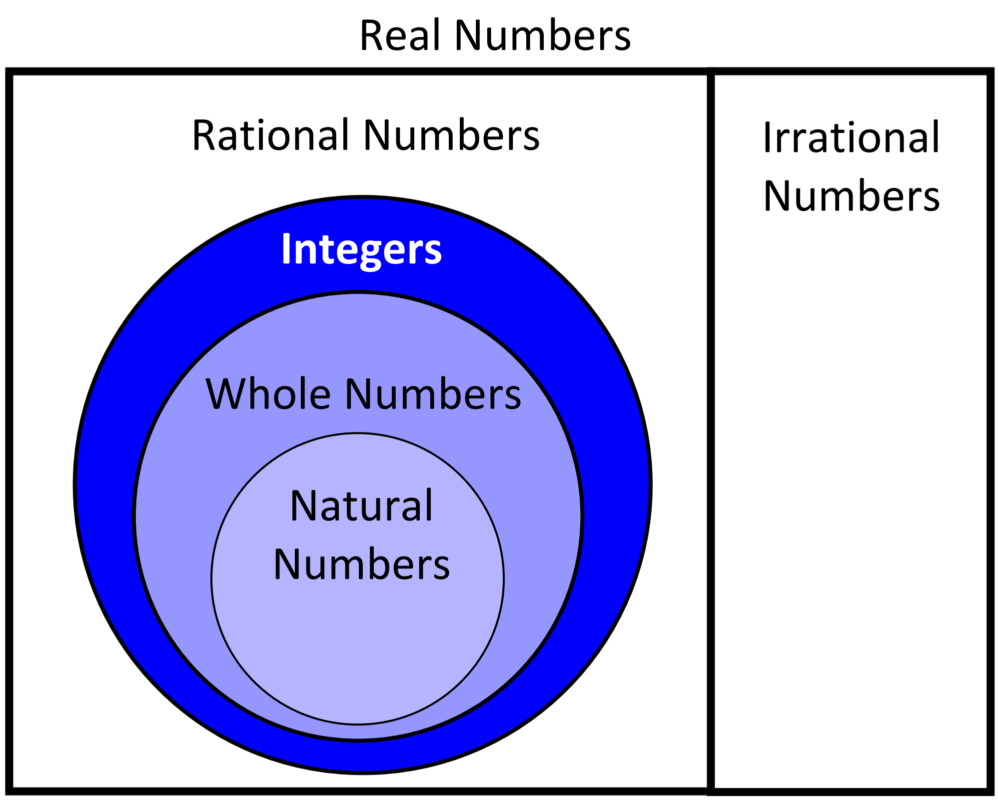
Whole Numbers

The set of numbers
0, 1, 2, 3, 4…

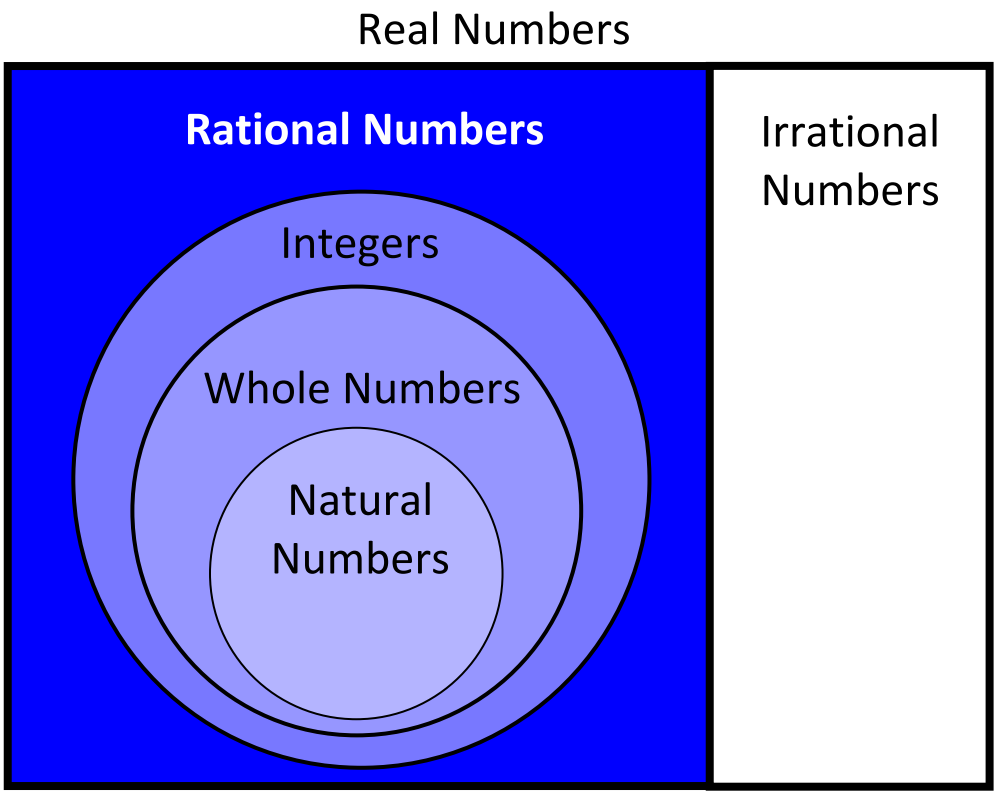


Integers

The set of numbers …-3, -2, -1, 0, 1, 2, 3…



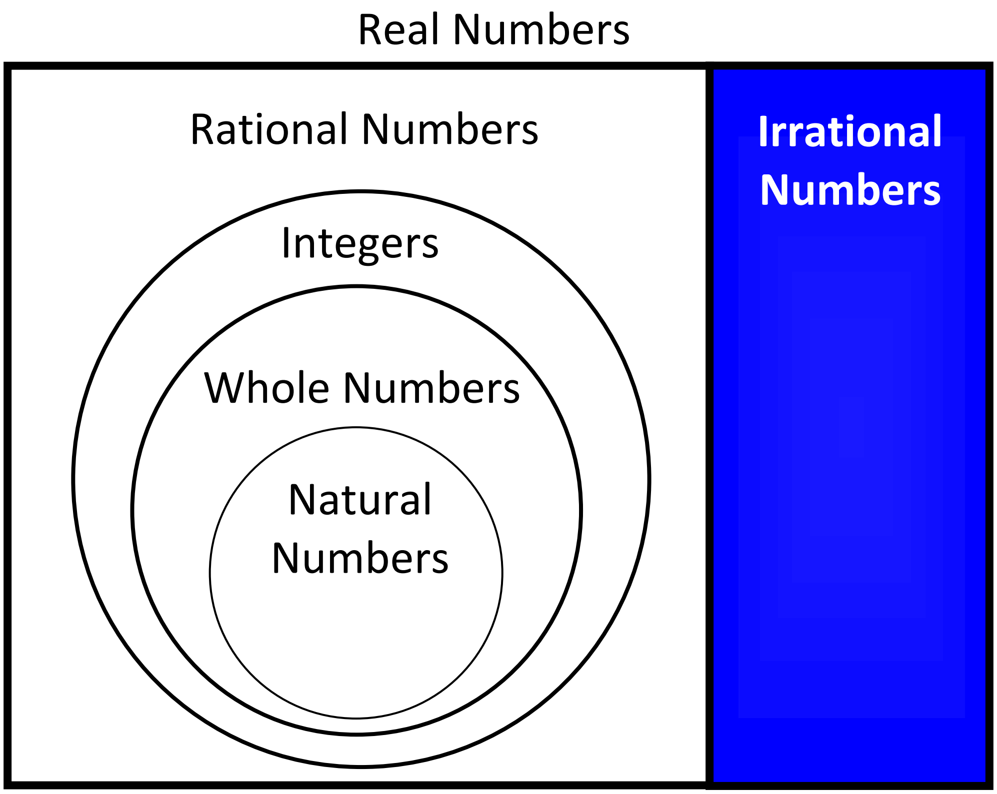
Rational Numbers



The set of all numbers that can be written as the ratio of two integers with a non-zero denominator

$$2\frac{3}{5},\quad -5,\quad 0.3,\quad \sqrt{16},\quad \frac{13}{7}$$

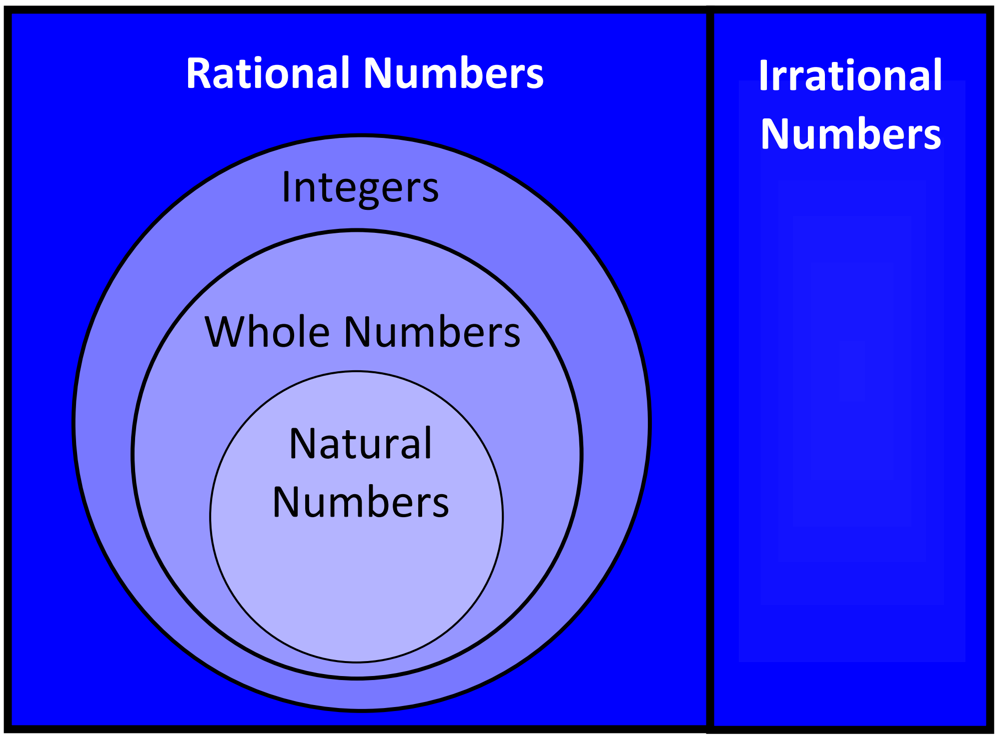
Irrational Numbers



The set of all numbers that cannot be expressed as the ratio of integers

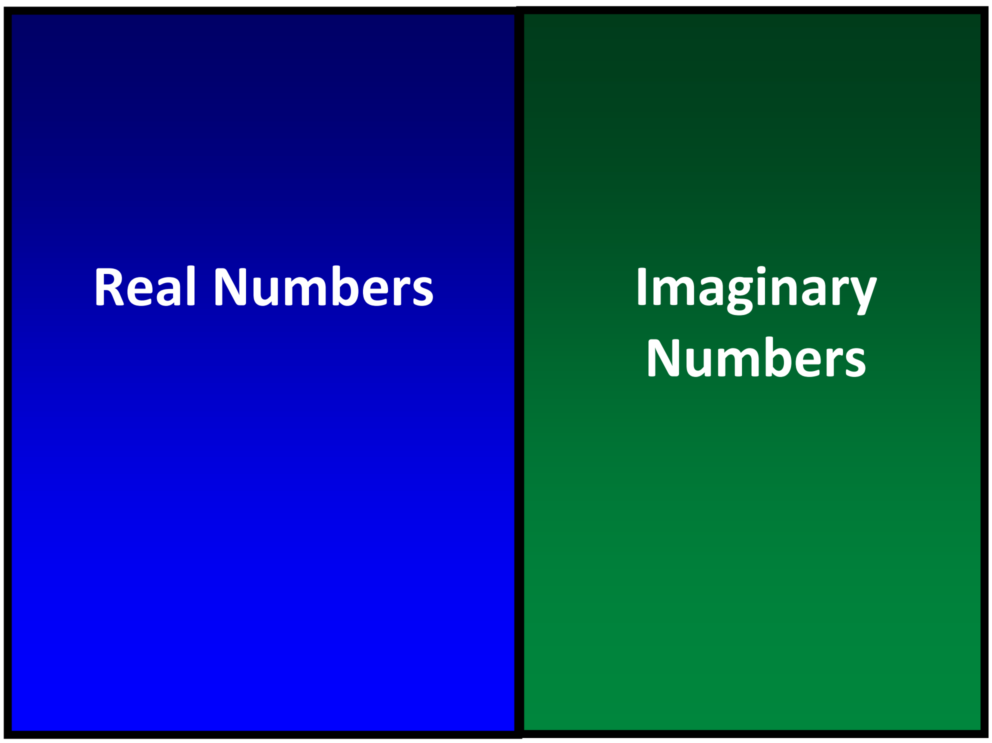
$\sqrt{7}$, π , -0.23223222322223…

Real Numbers



The set of all rational and irrational numbers

Complex Numbers



The set of all real and imaginary numbers

Complex Number

$$a \pm bi$$

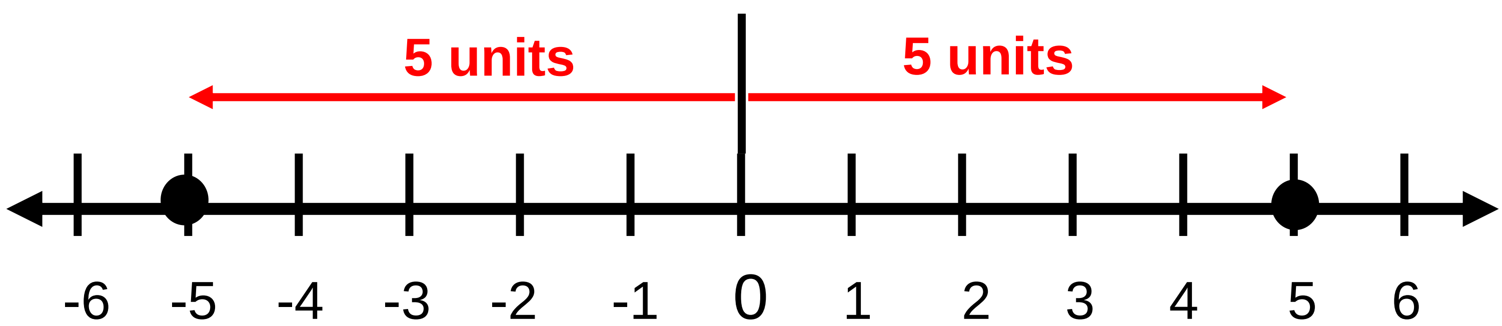
a and *b* are real numbers and $i = \sqrt{-1}$

A complex number consists of both real (*a*) and imaginary (*bi*) but either part can be 0

Case	**Example**
a = 0	$0.01i$, $-i$, $\frac{2i}{5}$
b = 0	$\sqrt{5}$, 4, -12.8
$a \neq 0$, $b \neq 0$	$39 - 6i$, $-2 + \pi i$

Absolute Value

$|5| = 5$ $|-5| = 5$



The distance between a number and zero

Order of Operations

Grouping Symbols	( ) { } [ ] |absolute value| fraction bar
Exponents	a^n
Multiplication Division	→ Left to Right
Addition Subtraction	→ Left to Right

Expression

$$x$$

$$-\sqrt{26}$$

$$3^4 + 2m$$

$$3(y + 3.9)^2 - \frac{8}{9}$$

Variable

$$2(y + \sqrt{3})$$

$$9 + x = 2.08$$

$$d = 7c - 5$$

$$A = \pi r^2$$

Coefficient

$$(-4) + 2x$$

$$-7y^{2}$$

$$\frac{2}{3}ab - \frac{1}{2}$$

$$\pi r^{2}$$

Term

$$\underbrace{3x}+\underbrace{2y}-\underbrace{8}$$

3 terms

$$\underbrace{-5x^2}-\underbrace{x}$$

2 terms

$$\underbrace{\frac{2}{3}ab}$$

1 term

Scientific Notation

$$a \times 10^n$$

$1 \le |a| < 10$ and n is an integer

Examples:

Standard Notation	Scientific Notation
17,500,000	1.75×10^7
-84,623	-8.4623×10^4
0.0000026	2.6×10^{-6}
-0.080029	-8.0029×10^{-2}

Exponential Form

exponent

$$a^n = \underbrace{a \cdot a \cdot a \cdot a}_{\text{factors}} \ldots, \; a \neq 0$$

base

factors

Examples:

$$2 \cdot 2 \cdot 2 = 2^3 = 8$$

$$n \cdot n \cdot n \cdot n = n^4$$

$$3 \cdot 3 \cdot 3 \cdot x \cdot x = 3^3 x^2 = 27x^2$$

Negative Exponent

$$a^{-n} = \frac{1}{a^n}, \; a \neq 0$$

Examples:

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$\frac{x^4}{y^{-2}} = \frac{x^4}{\frac{1}{y^2}} = \frac{x^4}{\frac{1}{y^2}} \cdot \frac{y^2}{y^2} = x^4y^2$$

$$(2-a)^{-2} = \frac{1}{(2-a)^2}, \; a \neq 2$$

Zero Exponent

$$a^0 = 1, \; a \neq 0$$

Examples:

$$(-5)^0 = 1$$

$$(3x + 2)^0 = 1$$

$$(x^2y^{-5}z^8)^0 = 1$$

$$4m^0 = 4 \cdot 1 = 4$$

Product of Powers Property

$$a^m \cdot a^{\mathrm{n}} = a^{m+n}$$

Examples:

$$x^4 \cdot x^2 = x^{4+2} = x^6$$

$$a^3 \cdot a = a^{3+1} = a^4$$

$$w^7 \cdot w^{-4} = w^{7+(-4)} = w^3$$

Power of a Power Property

$$(a^m)^n = a^{m \cdot n}$$

Examples:

$$(y^4)^2 = y^{4 \cdot 2} = y^8$$

$$(g^2)^{-3} = g^{2 \cdot (-3)} = g^{-6} = \frac{1}{g^6}$$

Power of a Product Property

$$(ab)^m = a^m \cdot b^m$$

Examples:

$$(-3ab)^2 = (-3)^2 \cdot a^2 \cdot b^2 = 9a^2b^2$$

$$\frac{-1}{(2x)^3} = \frac{-1}{2^3 \cdot x^3} = \frac{-1}{8x^3}$$

Quotient of Powers Property

$$\frac{a^m}{a^n} = a^{m-n}, \; a \neq 0$$

Examples:

$$\frac{x^6}{x^5} = x^{6-5} = x^1 = x$$

$$\frac{y^{-3}}{y^{-5}} = y^{-3-(-5)} = y^2$$

$$\frac{a^4}{a^4} = a^{4-4} = a^0 = 1$$

Power of Quotient Property

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b \neq 0$$

Examples:

$$\left(\frac{y}{3}\right)^4 = \frac{y^4}{3^4}$$

$$\left(\frac{5}{t}\right)^{-3} = \frac{5^{-3}}{t^{-3}} = \frac{\frac{1}{5^3}}{\frac{1}{t^3}} = \frac{t^3}{5^3} = \frac{t^3}{125}$$

Polynomial

Example	Name	Terms
7 $6x$	monomial	1 term
$3t - 1$ $12xy^3 + 5x^4y$	binomial	2 terms
$2x^2 + 3x - 7$	trinomial	3 terms

Nonexample	Reason
$5m^n - 8$	variable exponent
$n^{-3} + 9$	negative exponent

Degree of a Polynomial

The largest exponent or the largest sum of exponents of a term within a polynomial

Example:

$$6a^3 + 3a^2b^3 - 21$$

Term	Degree
$6a^3$	3
$3a^2b^3$	5
-21	0

Degree of polynomial: 5

Leading Coefficient

The coefficient of the first term of a polynomial written in descending order of exponents

Examples:

$$7a^3 - 2a^2 + 8a - 1$$

$$-3n^3 + 7n^2 - 4n + 10$$

$$16t - 1$$

Add Polynomials

Combine like terms.

Example:

$$(2g^2 + 6g - 4) + (g^2 - g)$$

$$= 2g^2 + 6g - 4 + g^2 - g$$

(Group like terms and add.)

$$= (2g^2 + g^2) + (6g - g) - 4$$

$$= 3g^2 + 5g^2 - 4$$

Add Polynomials

Combine like terms.

Example:

$$(2g^3 + 6g^2 - 4) + (g^3 - g - 3)$$

(Align like terms and add.)

$$\begin{array}{rrrr} 2g^3 & + 6g^2 & & - 4 \\ + \quad g^3 & & - g & - 3 \\ \hline 3g^3 & + 6g^2 & - g & - 7 \end{array}$$

Subtract Polynomials

Add the inverse.

Example:

$$(4x^2 + 5) - (-2x^2 + 4x - 7)$$

(Add the inverse.)

$$= (4x^2 + 5) + (2x^2 - 4x + 7)$$

$$= 4x^2 + 5 + 2x^2 - 4x + 7$$

(Group like terms and add.)

$$= (4x^2 + 2x^2) - 4x + (5 + 7)$$

$$= 6x^2 - 4x + 12$$

Subtract Polynomials

Add the inverse.

Example:

$$(4x^2 + 5) - (-2x^2 + 4x - 7)$$

(Align like terms then add the inverse and add the like terms.)

$$\begin{array}{rcl} \begin{array}{r} 4x^2 \qquad\qquad + 5 \\ \underline{-(2x^2 + 4x - 7)} \end{array} & \longrightarrow & \begin{array}{r} 4x^2 \qquad\qquad + 5 \\ \underline{+\ 2x^2 - 4x + 7} \\ 6x^2 - 4x + 12 \end{array} \end{array}$$

Multiply Polynomials

Apply the distributive property.

$$(a + b)(d + e + f)$$

$$(a + b)(d + e + f)$$

$$= a(d + e + f) + b(d + e + f)$$

$$= ad + ae + af + bd + be + bf$$

Multiply Binomials

Apply the distributive property.

$$(a + b)(c + d) =$$
$$a(c + d) + b(c + d) =$$
$$ac + ad + bc + bd$$

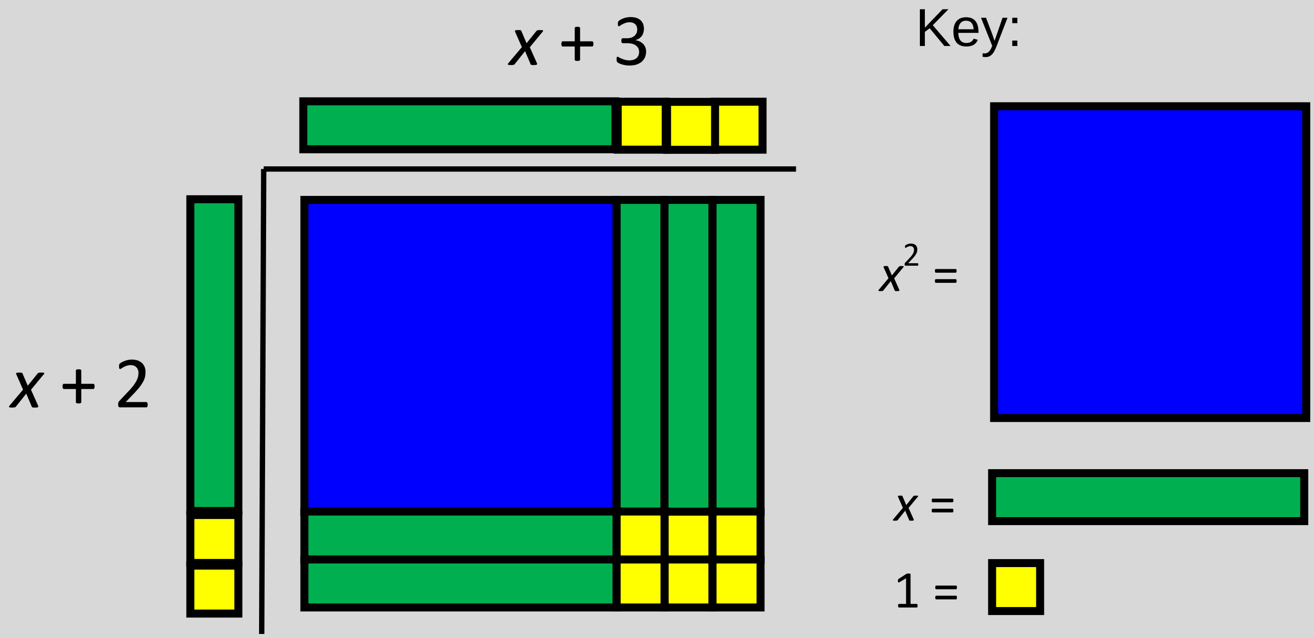
Example: $(x + 3)(x + 2)$

$$= x(x + 2) + 3(x + 2)$$
$$= x^2 + 2x + 3x + 6$$
$$= x^2 + 5x + 6$$

Multiply Binomials

Apply the distributive property.

Example: $(x + 3)(x + 2)$



$$x^2 + 2x + 3x + 6 = x^2 + 5x + 6$$

Multiply Binomials

Apply the distributive property.

Example: $(x + 8)(2x - 3)$

$= (x + 8)(2x + -3)$

	$2x$	$+$	-3
x	$2x^2$		$-3x$
$+$			
8	$8x$		-24

$$2x^2 + 8x + -3x + -24 = 2x^2 + 5x - 24$$

Multiply Binomials: Squaring a Binomial

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Examples:

$$(3m + n)^2 = 9m^2 + 2(3m)(n) + n^2$$
$$= 9m^2 + 6mn + n^2$$

$$(y - 5)^2 = y^2 - 2(5)(y) + 25$$
$$= y^2 - 10y + 25$$

Multiply Binomials: Sum and Difference

$$(a + b)(a - b) = a^2 - b^2$$

Examples:

$$(2b + 5)(2b - 5) = 4b^2 - 25$$

$$\begin{aligned}(7 - w)(7 + w) &= 49 + 7w - 7w - w^2 \\ &= 49 - w^2\end{aligned}$$

Factors of a Monomial

The number(s) and/or variable(s) that are multiplied together to form a monomial

Examples:	Factors	Expanded Form
$5b^2$	$5 \cdot b^2$	$5 \cdot b \cdot b$
$6x^2y$	$6 \cdot x^2 \cdot y$	$2 \cdot 3 \cdot x \cdot x \cdot y$
$\frac{-5p^2q^3}{2}$	$\frac{-5}{2} \cdot p^2 \cdot q^3$	$\frac{1}{2} \cdot (-5) \cdot p \cdot p \cdot q \cdot q \cdot q$

Factoring: Greatest Common Factor

Find the greatest common factor (GCF) of all terms of the polynomial and then apply the distributive property.

Example: $20a^4 + 8a$

$$2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot a \cdot a + 2 \cdot 2 \cdot 2 \cdot a$$

common factors

$$\text{GCF} = \underbrace{2 \cdot 2 \cdot a}_{\text{common factors}} = 4a$$

$$20a^4 + 8a = 4a(5a^3 + 2)$$

Factoring: Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Examples:

$$x^2 + 6x + 9 = x^2 + 2 \cdot 3 \cdot x + 3^2$$
$$= (x + 3)^2$$

$$4x^2 - 20x + 25 = (2x)^2 - 2 \cdot 2x \cdot 5 + 5^2$$
$$= (2x - 5)^2$$

Factoring: Difference of Two Squares

$$a^2 - b^2 = (a + \mathrm{b})(a - b)$$

Examples:

$$x^2 - 49 = x^2 - 7^2 = (x + 7)(x - 7)$$

$$4 - n^2 = 2^2 - n^2 = (2 - n)\ (2 + n)$$

$$9x^2 - 25y^2 = (3x)^2 - (5y)^2$$

$$= (3x + 5y)(3x - 5y)$$

Factoring: Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

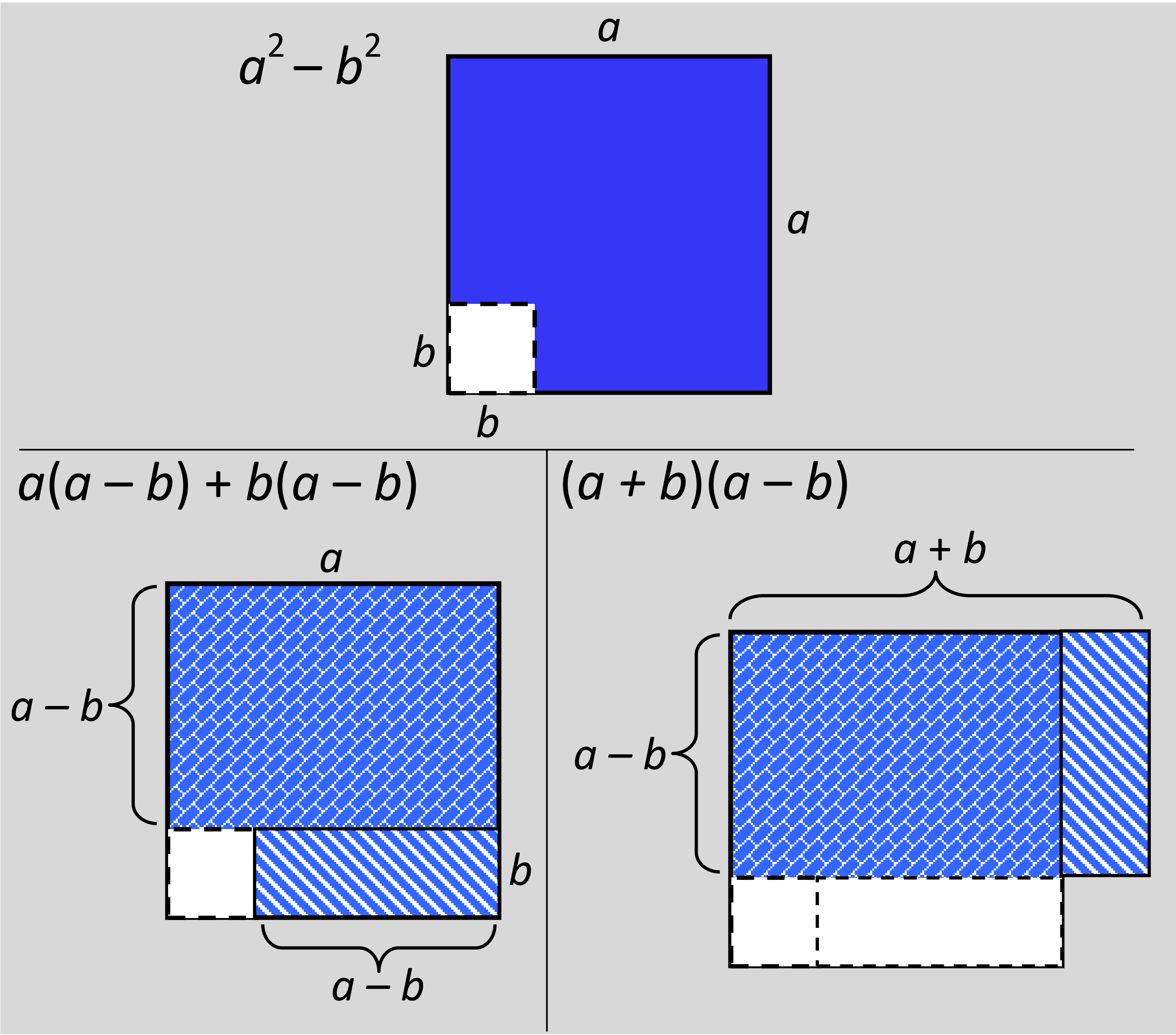
Examples:

$$27y^3 + 1 = (3y)^3 + (1)^3$$
$$= (3y + 1)(9y^2 - 3y + 1)$$

$$x^3 - 64 = x^3 - 4^3 = (x - 4)(x^2 + 4x + 16)$$

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$



Divide Polynomials

Divide each term of the dividend by the monomial divisor

Example:

$$(12x^3 - 36x^2 + 16x) \div 4x$$

$$= \frac{12x^3 - 36x^2 + 16x}{4x}$$

$$= \frac{12x^3}{4x} - \frac{36x^2}{4x} + \frac{16x}{4x}$$

$$= 3x^2 - 9x + 4$$

Divide Polynomials by Binomials

Factor and simplify

Example:

$$(7w^2 + 3w - 4) \div (w + 1)$$

$$= \frac{7w^2 + 3w - 4}{w + 1}$$

$$= \frac{(7w - 4)(w + 1)}{w + 1}$$

$$= 7w - 4$$

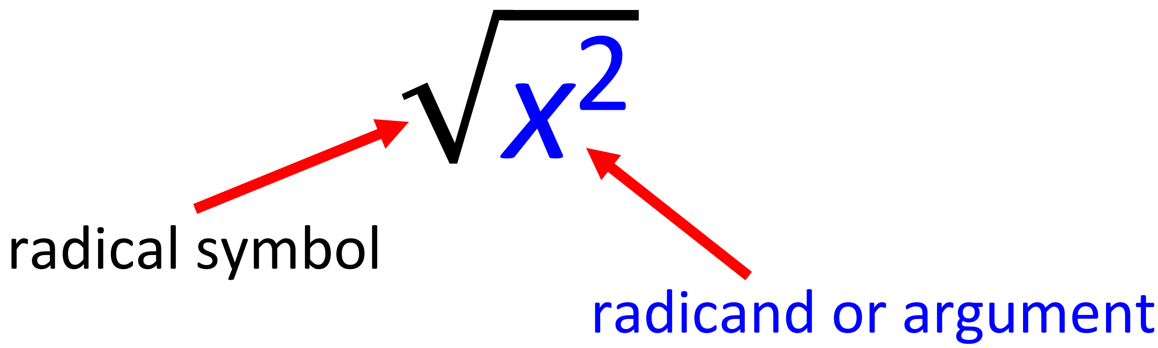
Prime Polynomial

Cannot be factored into a product of lesser degree polynomial factors

Example
r
$3t + 9$
$x^2 + 1$
$5y^2 - 4y + 3$

Nonexample	Factors
$x^2 - 4$	$(x + 2)(x - 2)$
$3x^2 - 3x + 6$	$3(x + 1)(x - 2)$
x^3	$x \cdot x^2$

Square Root



Simply square root expressions.

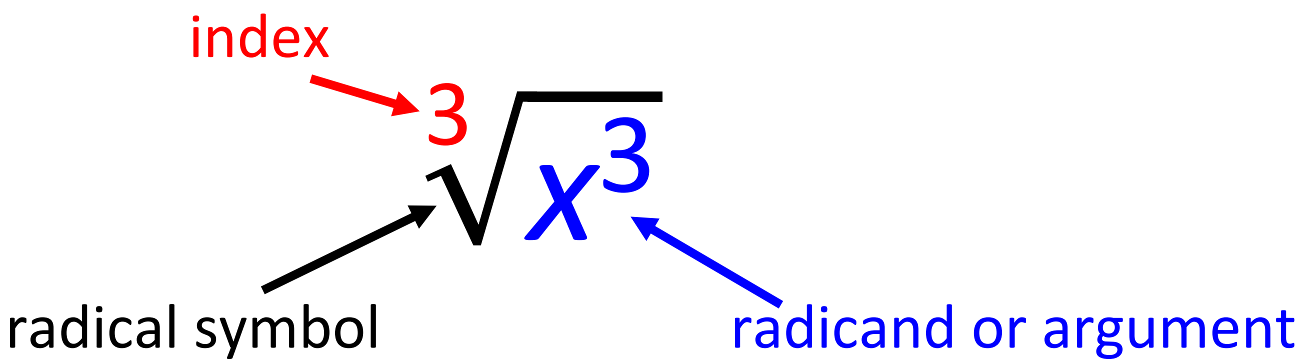
Examples:

$$\sqrt{9x^2} = \sqrt{3^2 \cdot x^2} = \sqrt{(3x)^2} = 3x$$

$$-\sqrt{(x-3)^2} = -(x-3) = -x + 3$$

Squaring a number and taking a square root are inverse operations.

Cube Root



Simplify cube root expressions.

Examples:

$$\sqrt[3]{64} = \sqrt[3]{4^3} = 4$$

$$\sqrt[3]{-27} = \sqrt[3]{(-3)^3} = -3$$

$$\sqrt[3]{x^3} = x$$

Cubing a number and taking a cube root are inverse operations.

n^{th} Root

index

$$\sqrt[n]{x^m} = x^{\frac{m}{n}}$$

radical symbol

radicand or argument

Examples:

$$\sqrt[5]{64} = \sqrt[5]{4^3} = 4^{\frac{3}{5}}$$

$$\sqrt[6]{729x^9y^6} = 3x^{\frac{3}{2}}y$$

Product Property of Radicals

The square root of a product equals the product of the square roots of the factors.

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

$a \geq 0$ and $b \geq 0$

Examples:

$$\sqrt{4x} = \sqrt{4} \cdot \sqrt{x} = 2\sqrt{x}$$

$$\sqrt{5a^3} = \sqrt{5} \cdot \sqrt{a^3} = a\sqrt{5a}$$

$$\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$$

Quotient Property of Radicals

The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$a \geq 0$ and $b > 0$

Example:

$$\sqrt{\frac{5}{y^2}} = \frac{\sqrt{5}}{\sqrt{y^2}} = \frac{\sqrt{5}}{y},\ y \neq 0$$

Zero Product Property

If $ab = 0$,
then $a = 0$ or $b = 0$.

Example:

$$(x + 3)(x - 4) = 0$$

$$(x + 3) = 0 \text{ or } (x - 4) = 0$$

$$x = -3 \text{ or } x = 4$$

The solutions are -3 and 4, also called roots of the equation.

Solutions or Roots

$$x^2 + 2x = 3$$

Solve using the zero product property.

$$x^2 + 2x - 3 = 0$$

$$(x + 3)(x - 1) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = -3 \text{ or } x = 1$$

The solutions or roots of the polynomial equation are -3 and 1.

Zeros

The zeros of a function *f*(x) are the values of x where the function is equal to zero.

$f(x) = x^2 + 2x - 3$

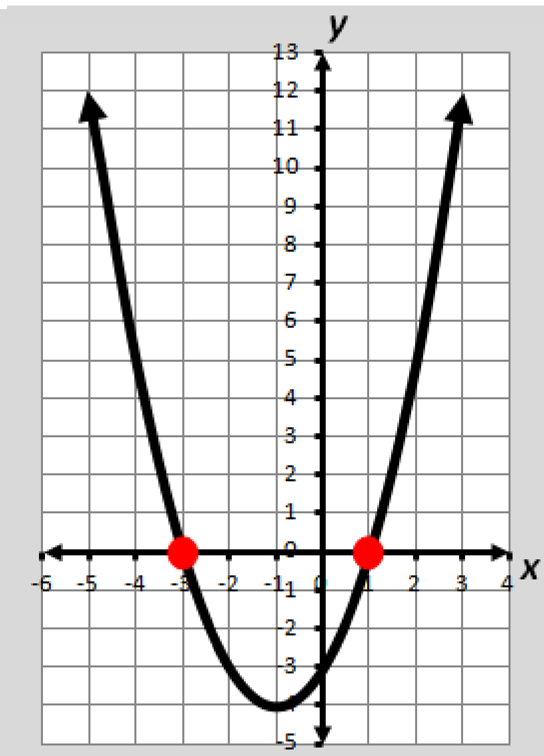
Find $f(x) = 0$.

$0 = x^2 + 2x - 3$

$0 = (x + 3)(x - 1)$

$x = -3$ or $x = 1$

The zeros are -3 and 1 located at (-3,0) and (1,0).



The zeros of a function are also the solutions or roots of the related equation.

x-Intercepts

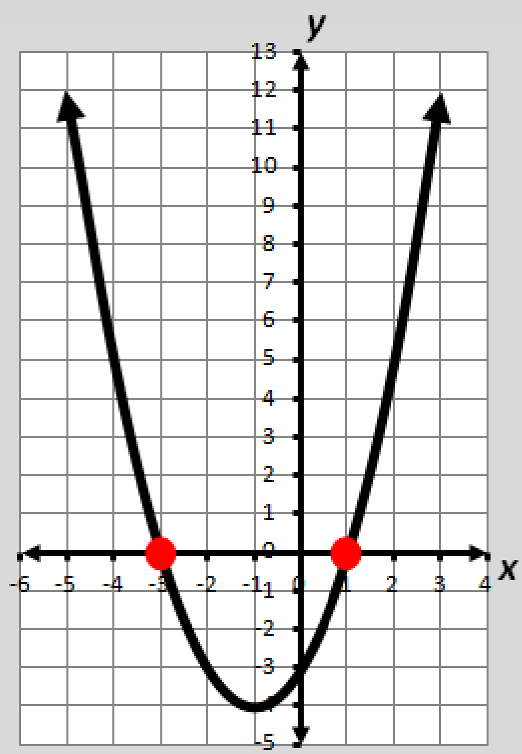
The *x*-intercepts of a graph are located where the graph crosses the x-axis and where $f(x) = 0$.

$$f(x) = x^2 + 2x - 3$$

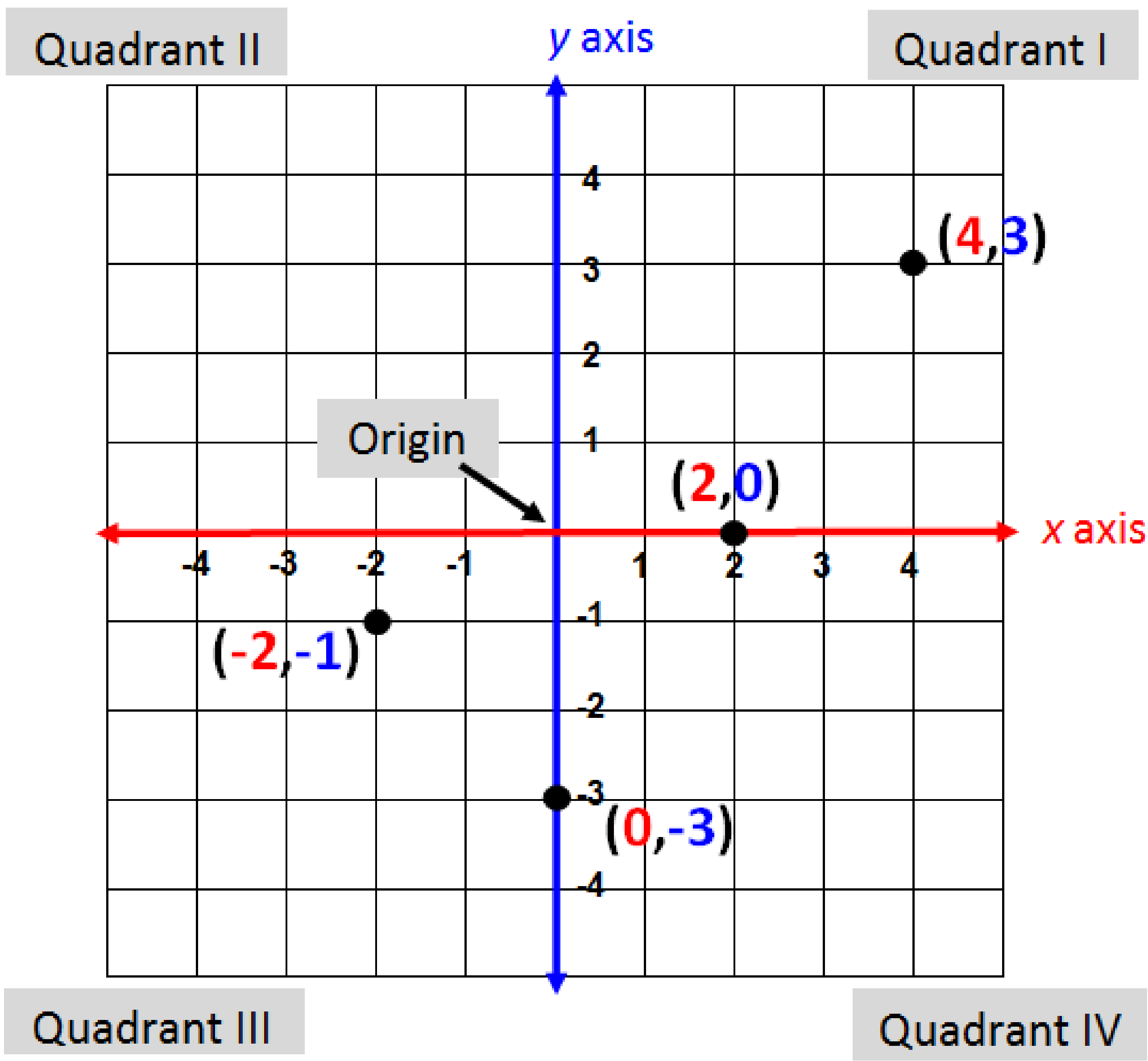
$$0 = (x + 3)(x - 1)$$
$$0 = x + 3 \text{ or } 0 = x - 1$$
$$x = -3 \text{ or } x = 1$$

The zeros are -3 and 1.
The *x*-intercepts are:

- -3 or (-3,0)
- 1 or (1,0)



Coordinate Plane



ordered pair (x,y)

(abscissa, ordinate)

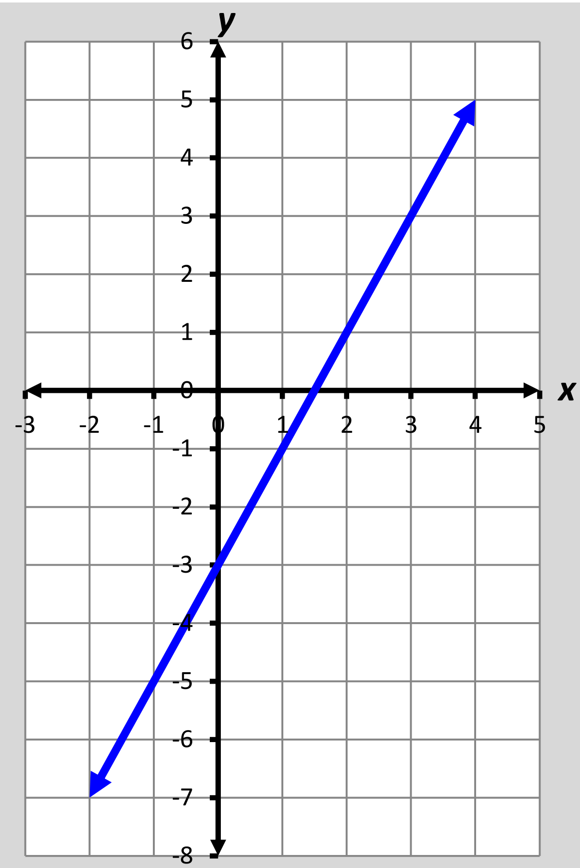
Linear Equation

$Ax + By = C$

(A, B and C are integers; A and B cannot both equal zero.)

Example:

$-2x + y = -3$



The graph of the linear equation is a straight line and represents all solutions (x, y) of the equation.

Linear Equation: Standard Form

$$Ax + By = C$$

(A, B, and C are integers;
A and B cannot both equal zero.)

Examples:

$$4x + 5y = -24$$

$$x - 6y = 9$$

Literal Equation

A formula or equation which consists primarily of variables

Examples:

$$ax + \mathrm{b} = \mathrm{c}$$

$$A = \frac{1}{2}bh$$

$$V = lwh$$

$$F = \frac{9}{5}C + 32$$

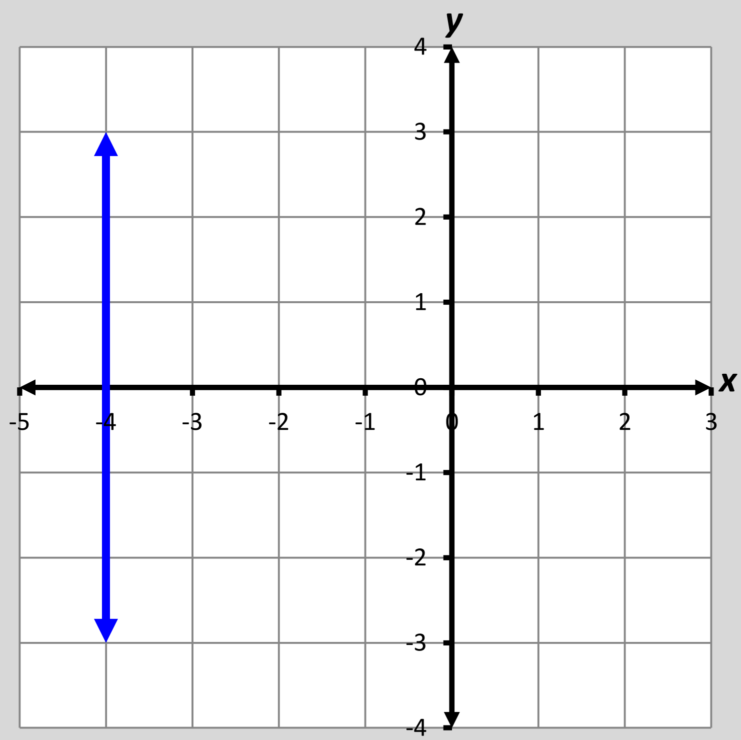
$$A = \pi r^2$$

Vertical Line

$$x = a$$

(where a can be any real number)

Example: $x = -4$



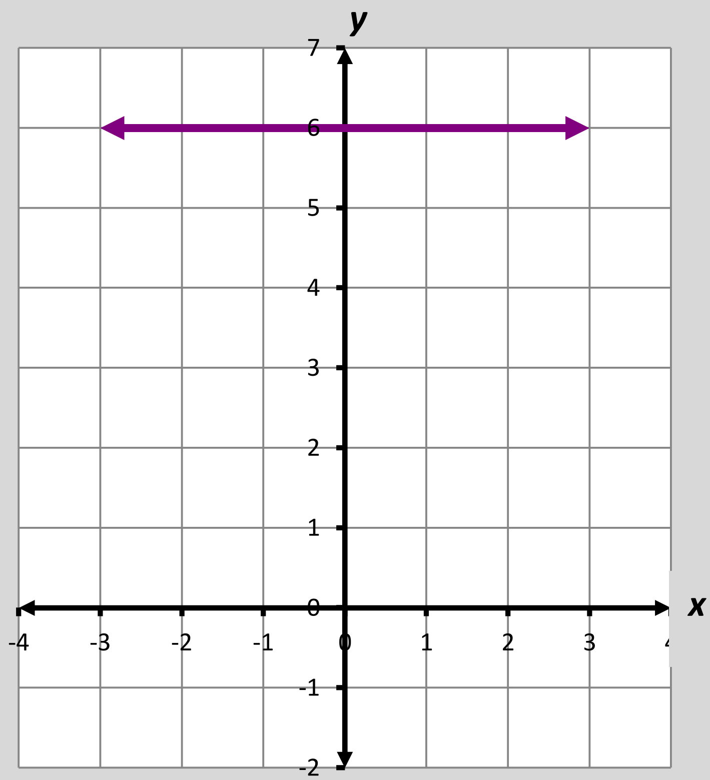
Vertical lines have an undefined slope.

Horizontal Line

$y = c$

(where c can be any real number)

Example: $y = 6$



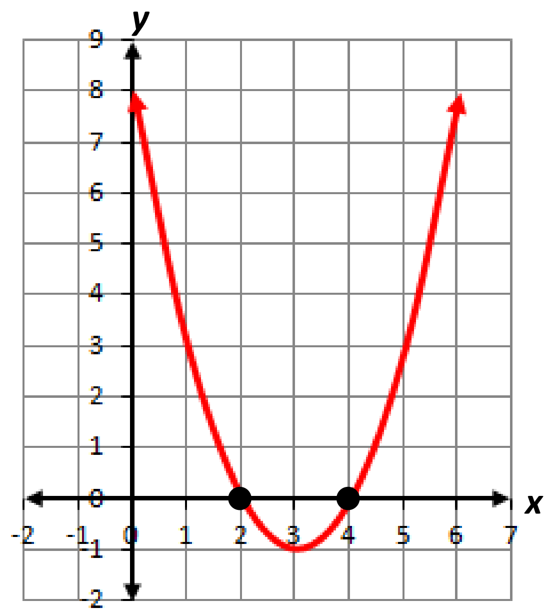
Horizontal lines have a slope of 0.

Quadratic Equation

$$ax^2 + bx + c = 0$$

$$a \neq 0$$

Example: $x^2 - 6x + 8 = 0$

Solve by factoring	**Solve by graphing**
$x^2 - 6x + 8 = 0$ $(x - 2)(x - 4) = 0$ $(x - 2) = 0$ or $(x - 4) = 0$ $x = 2$ or $x = 4$	Graph the related function $f(x) = x^2 - 6x + 8$. 

Solutions to the equation are 2 and 4;
the *x*-coordinates where the curve crosses the x-axis.

Quadratic Equation

$$ax^2 + bx + c = 0$$

$a \neq 0$

Example solved by factoring:

$x^2 - 6x + 8 = 0$	Quadratic equation
$(x - 2)(x - 4) = 0$	Factor
$(x - 2) = 0$ or $(x - 4) = 0$	Set factors equal to 0
$x = 2$ or $x = 4$	Solve for x

Solutions to the equation are 2 and 4.

Quadratic Equation

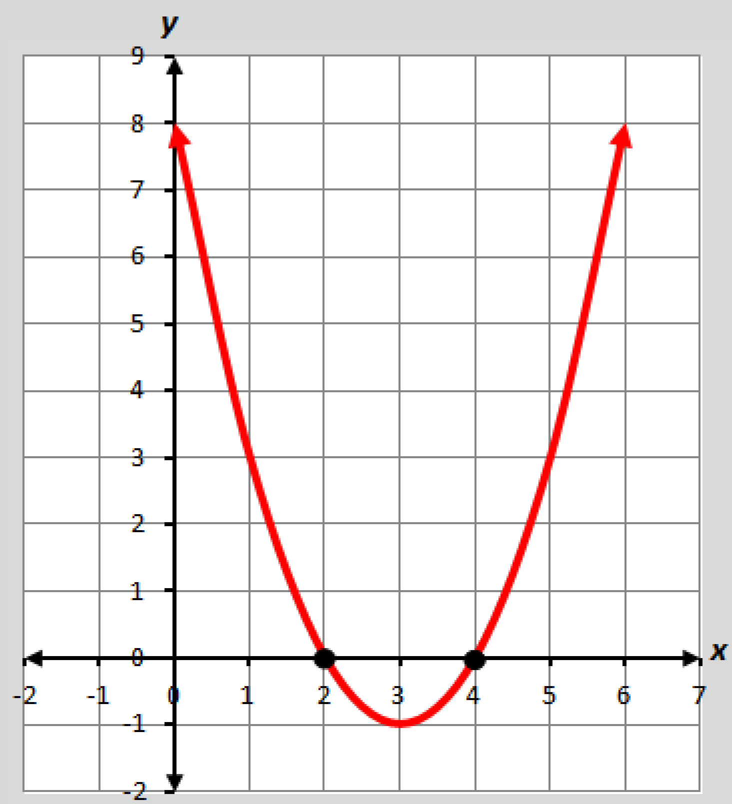
$$ax^2 + bx + c = 0$$

$$a \neq 0$$

Example solved by graphing:

$$x^2 - 6x + 8 = 0$$

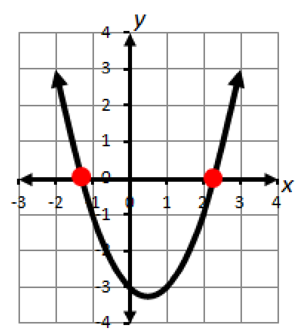
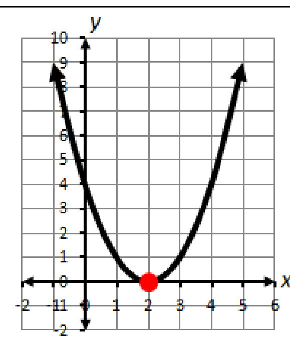
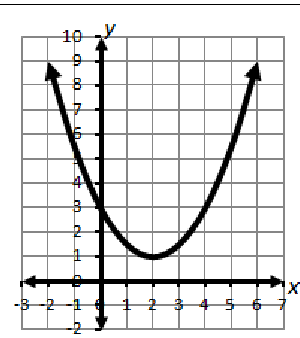
Graph the related function $f(x) = x^2 - 6x + 8$.



Solutions to the equation are the *x*-coordinates (2 and 4) of the points where the curve crosses the x-axis.

Quadratic Equation: Number of Real Solutions

$$ax^2 + bx + c = 0, \quad a \neq 0$$

Examples	**Graphs**	**Number of Real Solutions/Roots**
$x^2 - x = 3$		2
$x^2 + 16 = 8x$		1 distinct root with a multiplicity of two
$2x^2 - 2x + 3 = 0$		0

Identity Property of Addition

$$a + 0 = 0 + a = a$$

Examples:

$$3.8 + 0 = 3.8$$

$$6x + 0 = 6x$$

$$0 + (-7 + r) = -7 + r$$

Zero is the additive identity.

Inverse Property of Addition

$$a + (-a) = (-a) + a = 0$$

Examples:

$$4 + (-4) = 0$$

$$0 = (-9.5) + 9.5$$

$$x + (-x) = 0$$

$$0 = 3y + (-3y)$$

Commutative Property of Addition

$$a + b = b + a$$

Examples:

$$2.76 + 3 = 3 + 2.76$$

$$x + 5 = 5 + \mathrm{x}$$

$$(a + 5) - 7 = (5 + a) - 7$$

$$11 + (\mathrm{b} - 4) = (\mathrm{b} - 4) + 11$$

Associative Property of Addition

$$(a + b) + c = a + (b + c)$$

Examples:

$$\left(5 + \frac{3}{5}\right) + \frac{1}{10} = 5 + \left(\frac{3}{5} + \frac{1}{10}\right)$$

$$3x + (2x + 6\mathrm{y}) = (3x + 2x) + 6\mathrm{y}$$

Identity Property of Multiplication

$$a \cdot 1 = 1 \cdot a = a$$

Examples:

$$3.8\,(1) = 3.8$$

$$6x \cdot 1 = 6x$$

$$1(-7) = -7$$

One is the multiplicative identity.

Inverse Property of Multiplication

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$$

$$a \neq 0$$

Examples:

$$7 \cdot \frac{1}{7} = 1$$

$$\frac{5}{x} \cdot \frac{x}{5} = 1, \; x \neq 0$$

$$\frac{-1}{3} \cdot (-3p) = 1p = p$$

The multiplicative inverse of a is $\frac{1}{a}$.

Commutative Property of Multiplication

$$ab = ba$$

Examples:

$$(-8)\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)(-8)$$

$$y \cdot 9 = 9 \cdot y$$

$$4(2x \cdot 3) = 4(3 \cdot 2x)$$

$$8 + 5x = 8 + x \cdot 5$$

Associative Property of Multiplication

$$(ab)c = a(bc)$$

Examples:

$$(1 \cdot 8) \cdot 3\frac{3}{4} = 1 \cdot \left(8 \cdot 3\frac{3}{4}\right)$$

$$(3x)x = 3(x \cdot x)$$

Distributive Property

$$a(b + c) = ab + ac$$

Examples:

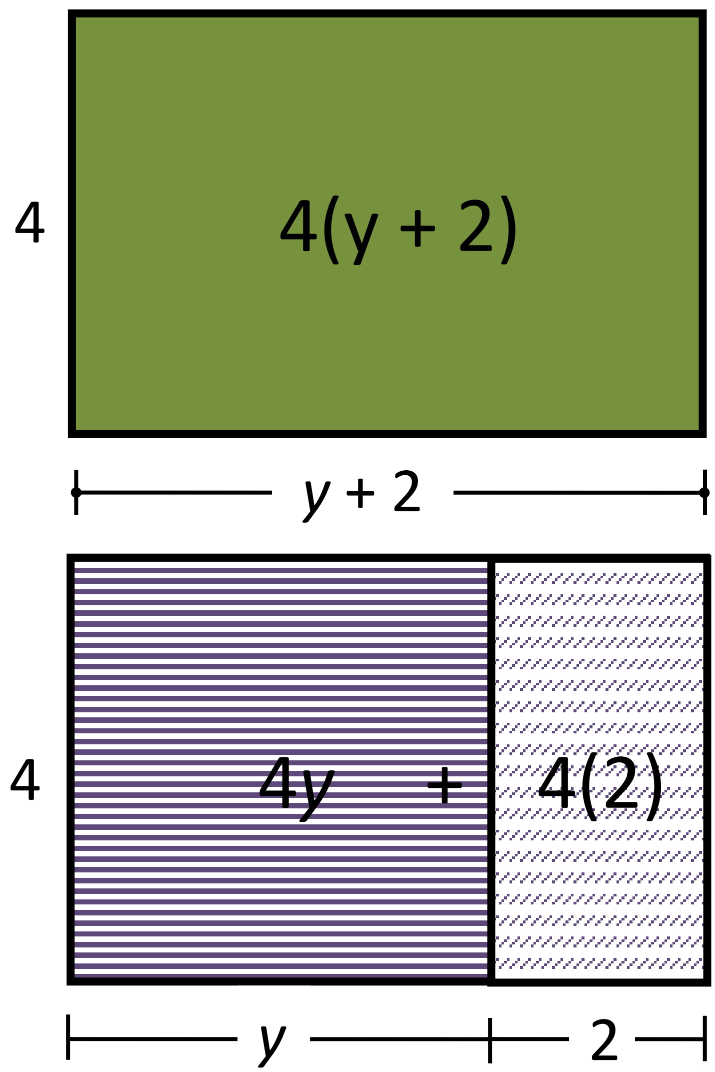
$$5\left(y - \frac{1}{3}\right) = (5 \cdot y) - \left(5 \cdot \frac{1}{3}\right)$$

$$2 \cdot x + 2 \cdot 5 = 2(x + 5)$$

$$3.1a + (1)(a) = (3.1 + 1)a$$

Distributive Property

$$4(y + 2) = 4y + 4(2)$$



Multiplicative Property of Zero

$a \cdot 0 = 0$ or $0 \cdot a = 0$

Examples:

$$8\frac{2}{3} \cdot 0 = 0$$

$$0 \cdot (-13y - 4) = 0$$

Substitution Property

If $a = b$, then b can replace a in a given equation or inequality.

Examples:

Given	Given	Substitution
$r = 9$	$3r = 27$	$3(9) = 27$
$b = 5a$	$24 < b + 8$	$24 < 5a + 8$
$y = 2x + 1$	$2y = 3x - 2$	$2(2x + 1) = 3x - 2$

Reflexive Property of Equality

$$a = a$$

a is any real number

Examples:

$$-4 = -4$$

$$3.4 = 3.4$$

$$9y = 9y$$

Symmetric Property of Equality

If $a = b$, then $b = a$.

Examples:

If $12 = r$, then $r = 12$.

If $-14 = z + 9$, then $z + 9 = -14$.

If $2.7 + y = x$, then $x = 2.7 + y$.

Transitive Property of Equality

If $a = b$ and $b = c$,
then $a = c$.

Examples:

If $4x = 2y$ and $2y = 16$,
then $4x = 16$.

If $x = y - 1$ and $y - 1 = -3$,
then $x = -3$.

Inequality

An algebraic sentence comparing two quantities

Symbol	Meaning
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to
$\neq$	not equal to

Examples:

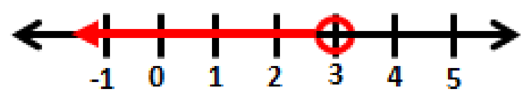
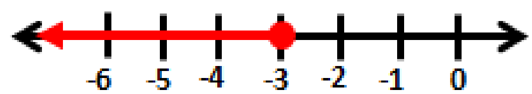
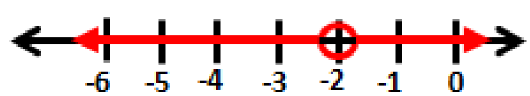
$$-10.5 > -9.9 - 1.2$$

$$8 > 3t + 2$$

$$x - 5y \geq -12$$

$$r \neq 3$$

Graph of an Inequality

Symbol	Examples	Graph
< or >	$x < 3$	
≤ or ≥	$-3 \geq y$	
≠	$t \neq -2$	

Transitive Property of Inequality

If	Then
$a < b$ and $b < c$	$a < c$
$a > b$ and $b > c$	$a > c$

Examples:

If $4x < 2y$ and $2y < 16$,
then $4x < 16$.

If $x > y - 1$ and $y - 1 > 3$,
then $x > 3$.

Addition/Subtraction Property of Inequality

If	Then
$a > b$	$a + c > b + c$
$a \geq b$	$a + c \geq b + c$
$a < b$	$a + c < b + c$
$a \leq b$	$a + c \leq b + c$

Example:

$$d - 1.9 \geq -8.7$$

$$d - 1.9 + 1.9 \geq -8.7 + 1.9$$

$$d \geq -6.8$$

Multiplication Property of Inequality

If	Case	Then
$a < b$	$c > 0$, positive	$ac < bc$
$a > b$	$c > 0$, positive	$ac > bc$
$a < b$	$c < 0$, negative	$ac > bc$
$a > b$	$c < 0$, negative	$ac < bc$

Example: if $c = -2$

$$5 > -3$$

$$5(-2) < -3(-2)$$

$$-10 < 6$$

Division Property of Inequality

If	Case	Then
a < b	c > 0, positive	$\frac{a}{c} < \frac{b}{c}$
a > b	c > 0, positive	$\frac{a}{c} > \frac{b}{c}$
a < b	c < 0, negative	$\frac{a}{c} > \frac{b}{c}$
a > b	c < 0, negative	$\frac{a}{c} < \frac{b}{c}$

Example: if c = -4

$$-90 \geq -4t$$

$$\frac{-90}{-4} \leq \frac{-4t}{-4}$$

$$22.5 \leq t$$

Linear Equation: Slope-Intercept Form

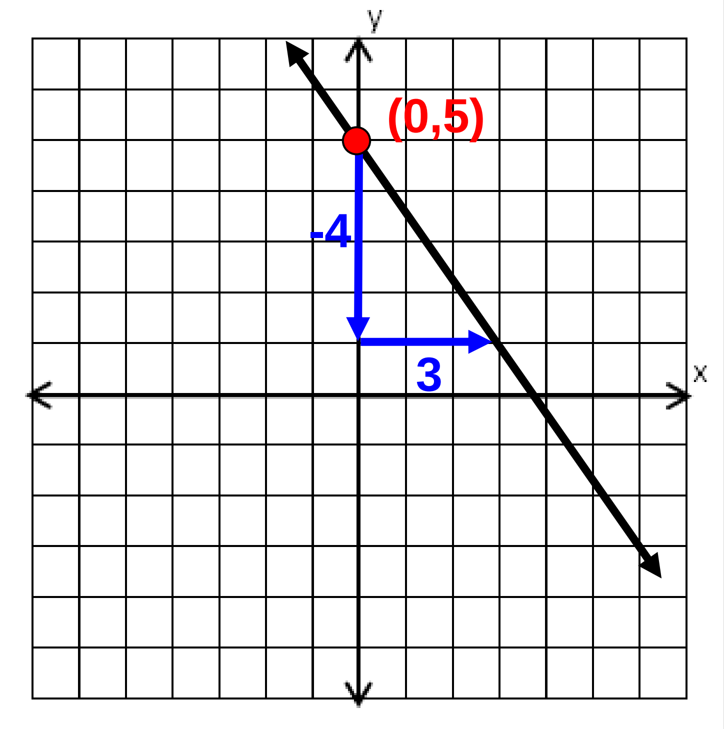
$$y = mx + b$$

(slope is m and y-intercept is b)

Example: $y = \frac{-4}{3}x + 5$

$m = \frac{-4}{3}$

$b = 5$



Linear Equation: Point-Slope Form

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1, y_1) is the point

Example:
Write an equation for the line that passes through the point (⁻4,1) and has a slope of 2.

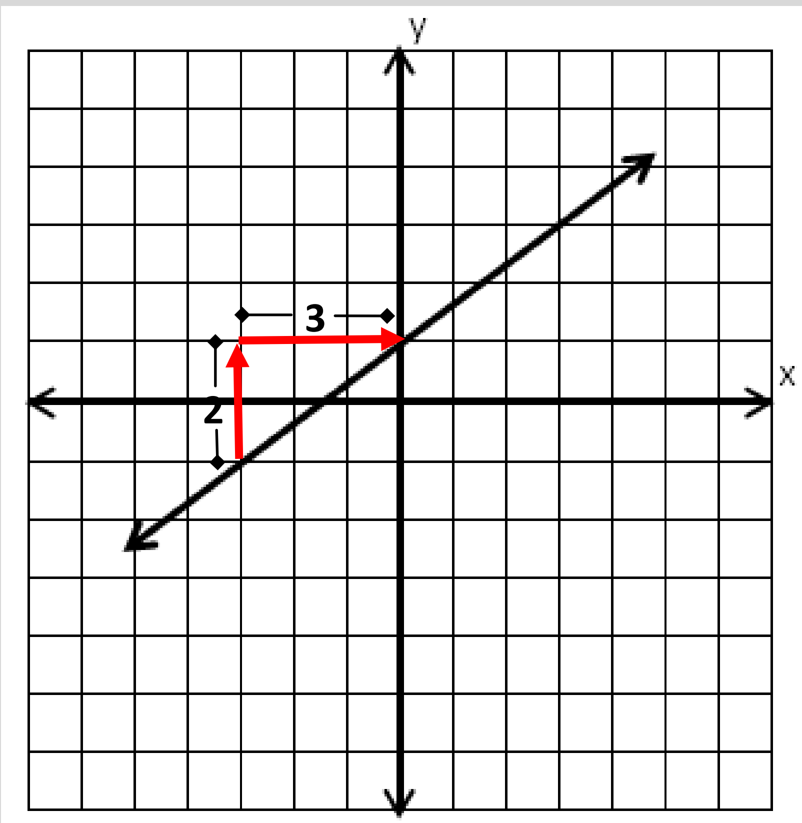
$$y - 1 = 2(x - {}^{-}4)$$

$$y - 1 = 2(x + 4)$$

$$y = 2x + 9$$

Slope

A number that represents the rate of change in y for a unit change in x

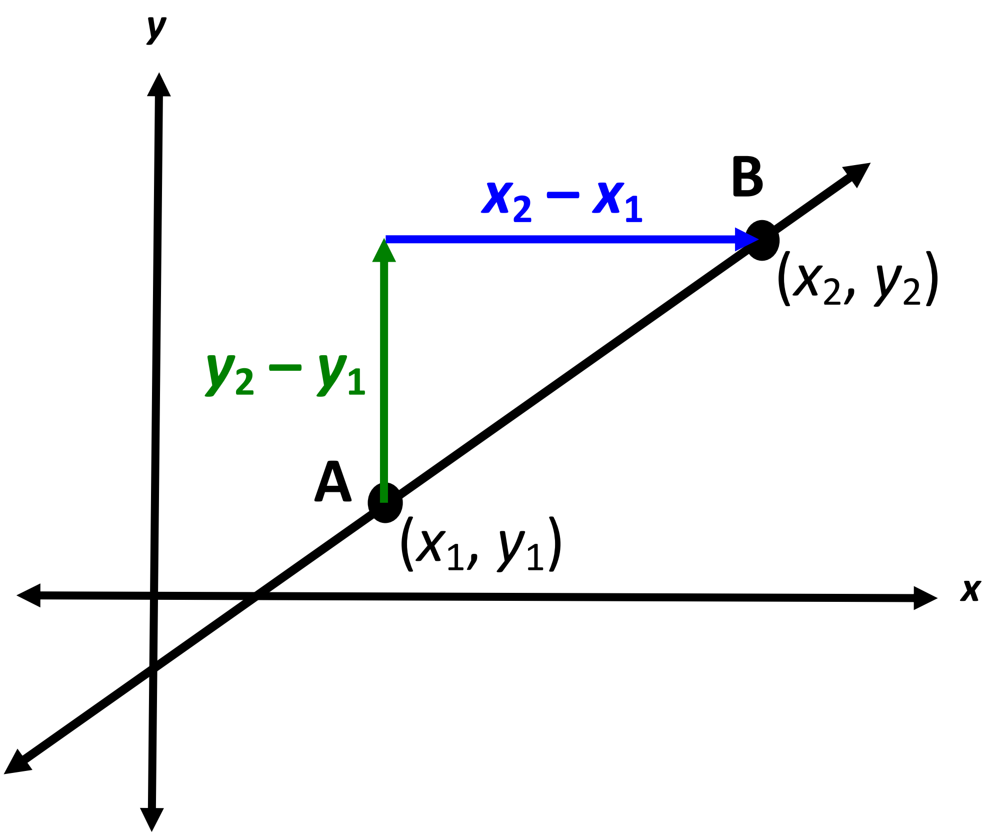


Slope = $\frac{2}{3}$

The slope indicates the steepness of a line.

Slope Formula

The ratio of vertical change to horizontal change

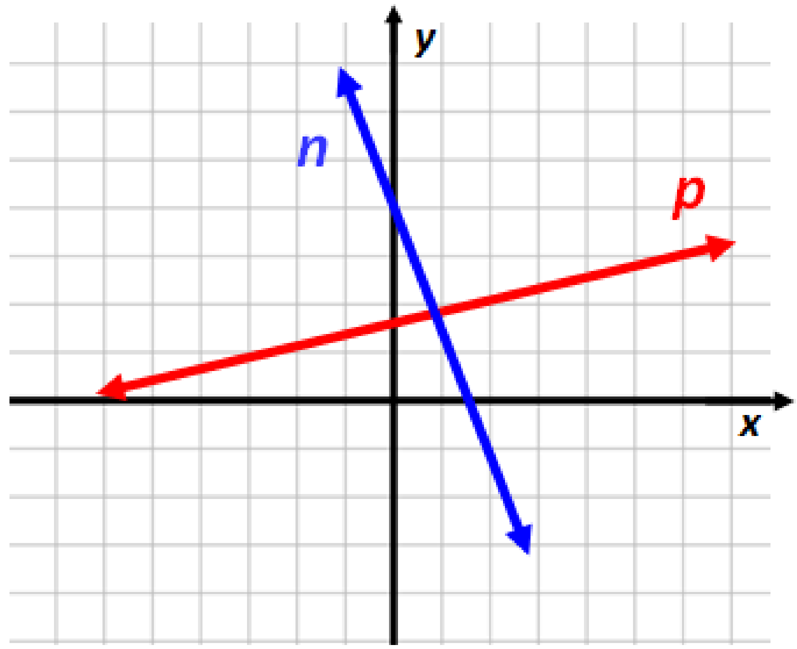


$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slopes of Lines

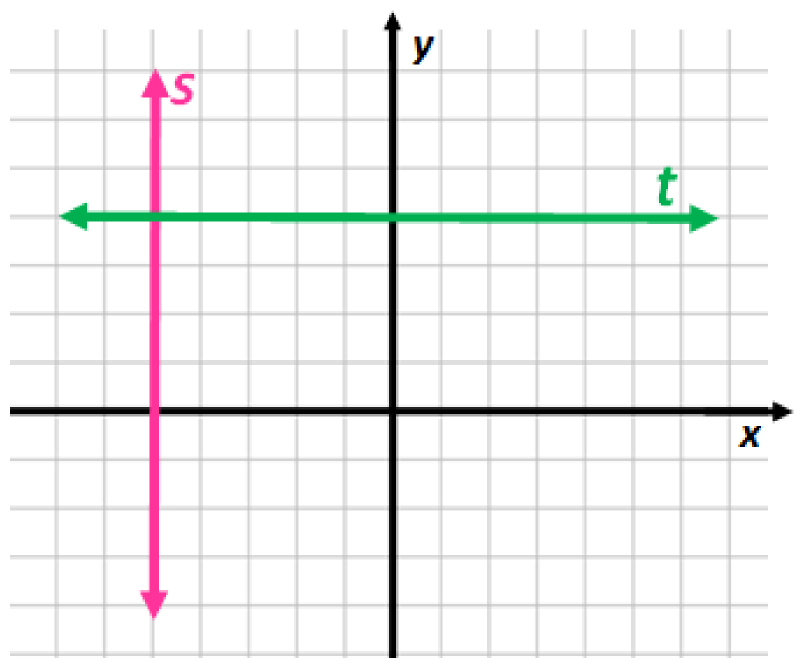
Line ***p***
has a positive
slope.

Line ***n***
has a negative
slope.



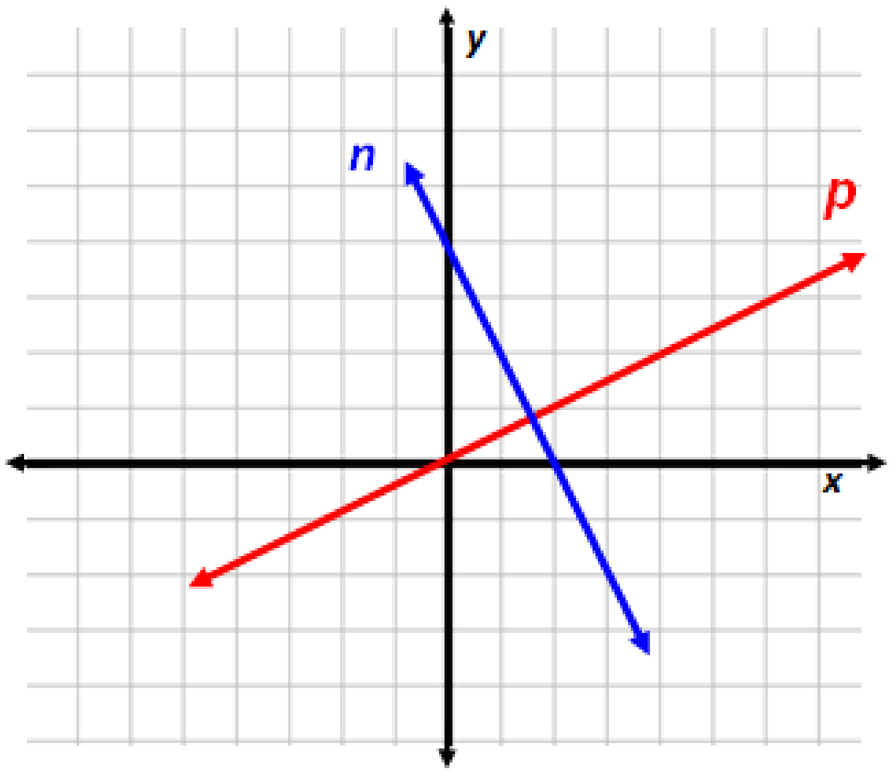
Vertical line ***s*** has
an undefined
slope.

Horizontal line ***t***
has a zero slope.



Perpendicular Lines

Lines that intersect to form a right angle



Perpendicular lines (not parallel to either of the axes) have slopes whose product is -1.

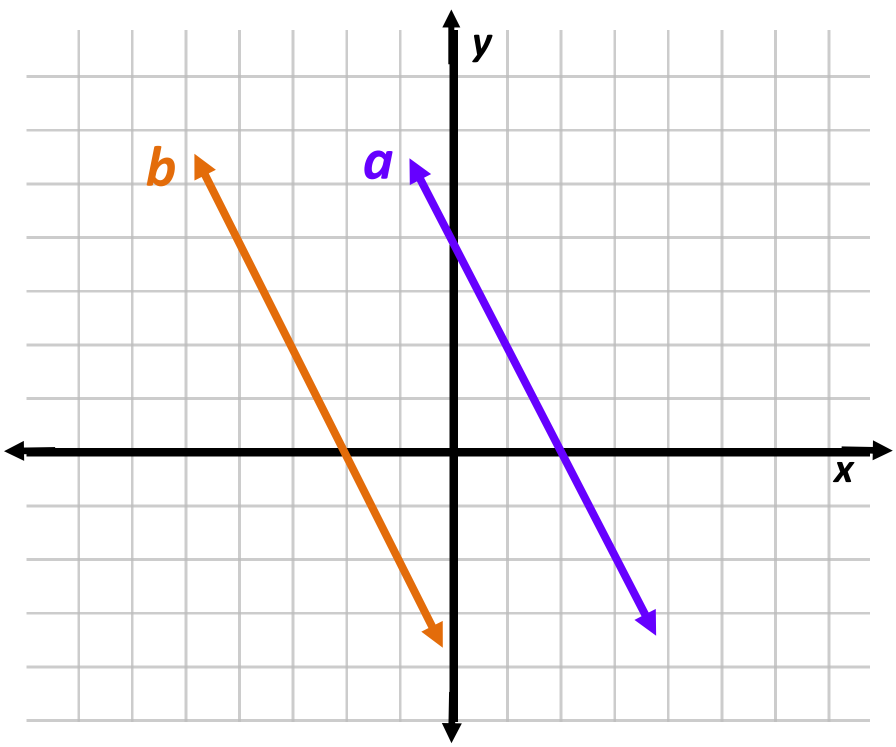
Example:

The slope of line $n = -2$. The slope of line $p = \frac{1}{2}$.

$-2 \cdot \frac{1}{2} = -1$, therefore, n is perpendicular to p.

Parallel Lines

Lines in the same plane that do not intersect are parallel.
Parallel lines have the same slopes.



Example:

The slope of line *a* = -2.

The slope of line *b* = -2.

-2 = -2, therefore, *a* is parallel to *b*.

Mathematical Notation

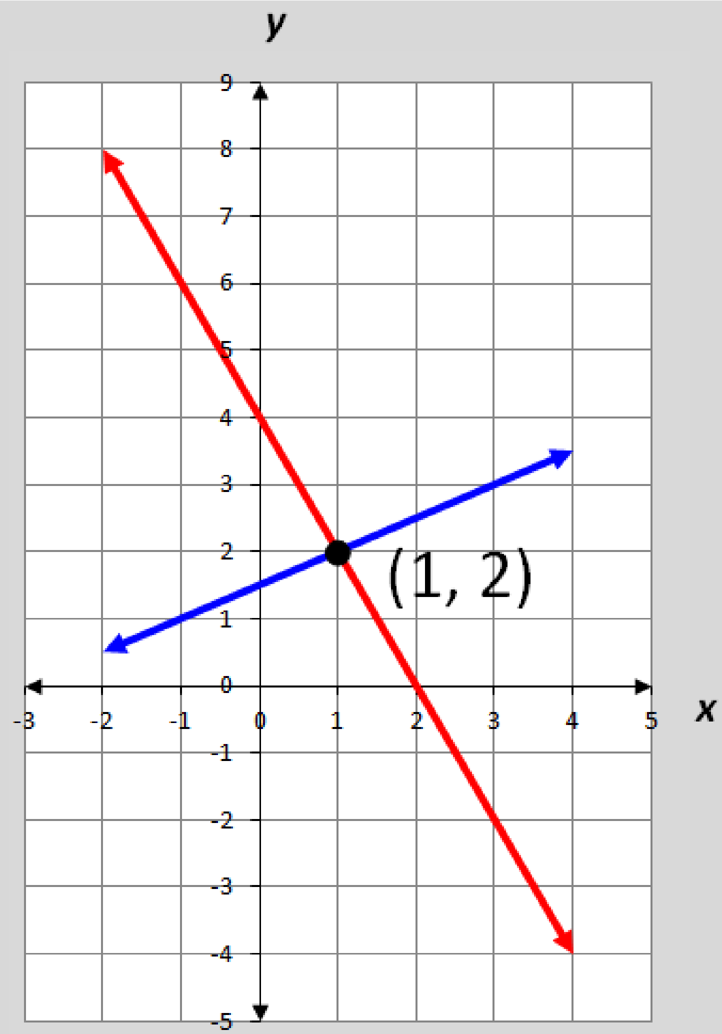
Set Builder Notation	Read	Other Notation
$\{x \mid 0 < x \leq 3\}$	The set of all *x* such that *x* is greater than or equal to 0 and *x* is less than 3.	$0 < x \leq 3$ (0, 3]
$\{y: y \geq -5\}$	The set of all *y* such that *y* is greater than or equal to -5.	$y \geq -5$ [-5, ∞)

System of Linear Equations

Solve by graphing:

$$\begin{cases} -x + 2y = 3 \\ 2x + y = 4 \end{cases}$$

The solution, (1, 2), is the only ordered pair that satisfies both equations (the point of intersection).



System of Linear Equations

Solve by substitution:

$$\begin{cases} x + 4y = 17 \\ y = x - 2 \end{cases}$$

Substitute x – 2 for y in the first equation.

$$x + 4(x - 2) = 17$$

$$x = 5$$

Now substitute 5 for x in the second equation.

$$y = 5 - 2$$

$$y = 3$$

The solution to the linear system is (5, 3),

the ordered pair that satisfies both equations.

System of Linear Equations

Solve by elimination:

$$\begin{cases} -5x - 6y = 8 \\ 5x + 2y = 4 \end{cases}$$

Add or subtract the equations to eliminate one variable.

$$\begin{array}{r} -5x - 6y = 8 \\ +\ 5x + 2y = 4 \\ \hline -4y = 12 \\ y = -3 \end{array}$$

Now substitute -3 for y in either original equation to find the value of x, the eliminated variable.

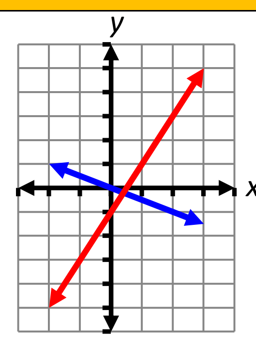
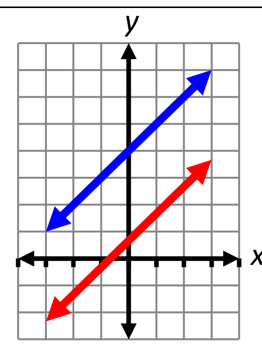
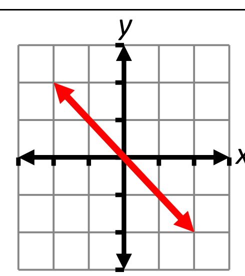
$$-5x - 6(-3) = 8$$

$$x = 2$$

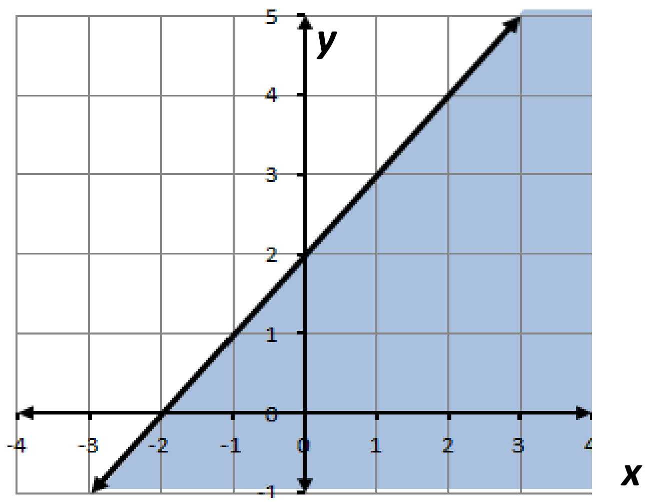
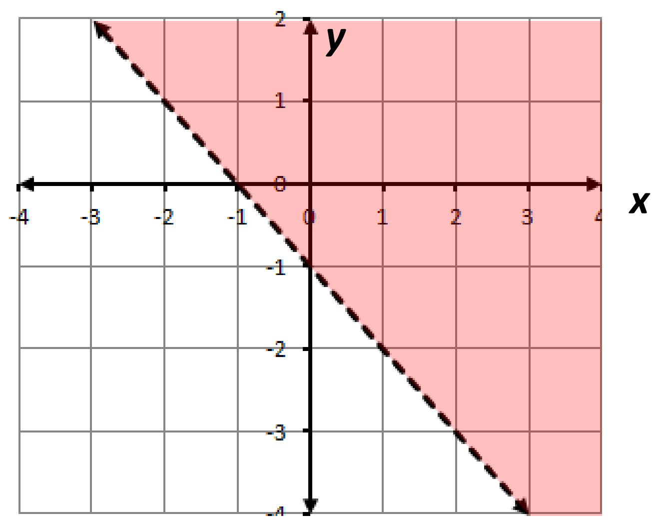
The solution to the linear system is (2,-3), the ordered pair that satisfies both equations.

System of Linear Equations

Identifying the Number of Solutions

Number of Solutions	Slopes and y-intercepts	Graph
One solution	Different slopes	
No solution	Same slope and different y-intercepts	
Infinitely many solutions	Same slope and same y-intercepts	

Graphing Linear Inequalities

Example	Graph
$y \leq x + 2$	
$y > -x - 1$	

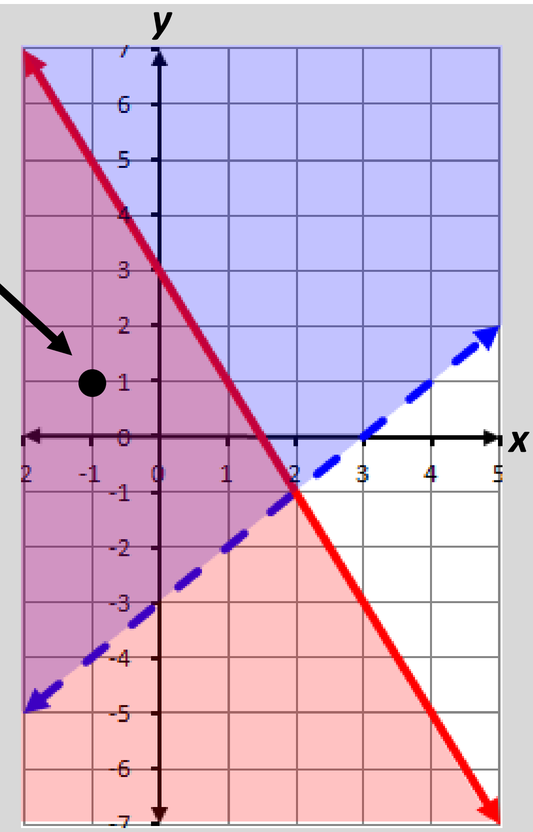
System of Linear Inequalities

Solve by graphing:

$$\begin{cases} y > x - 3 \\ y \leq -2x + 3 \end{cases}$$

The solution region contains all ordered pairs that are solutions to both inequalities in the system.

(-1,1) is one solution to the system located in the solution region.



Linear Programming

An optimization process consisting of a system of constraints and an objective quantity that can be maximized or minimized

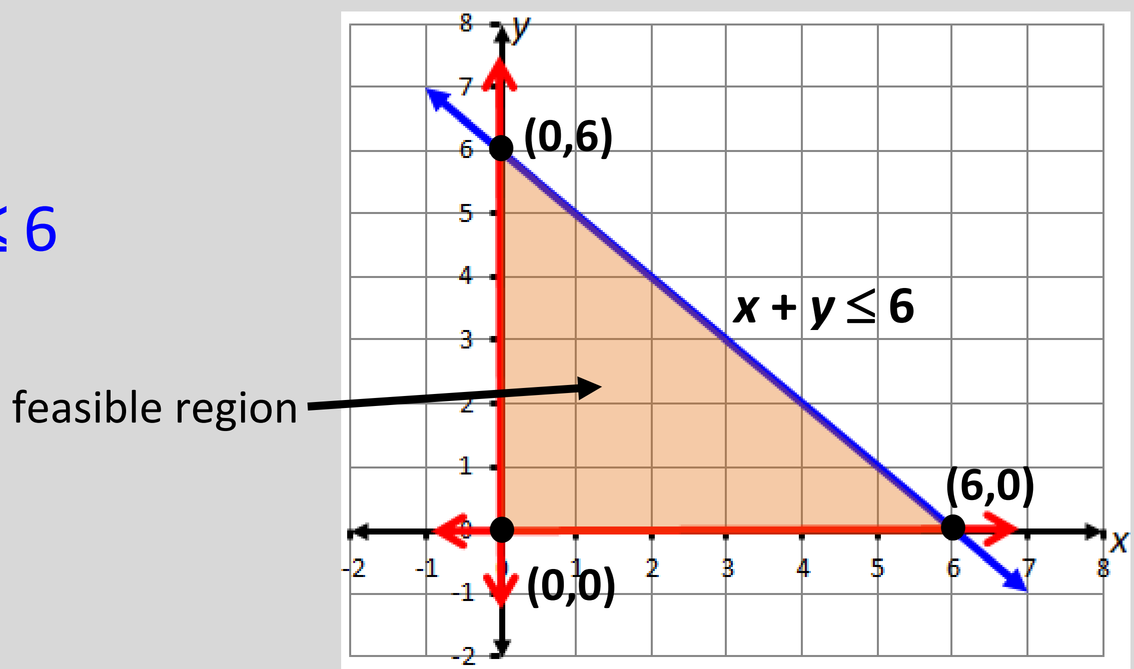
Example:

Find the minimum and maximum value of the objective function C = 4x + 5y, subject to the following constraints.

$x \geq 0$

$y \geq 0$

$x + y \leq 6$



The maximum or minimum value for C = 4x + 5y will occur at a corner point of the feasible region.

Dependent and Independent Variable

x, independent variable
(input values or domain set)

Example:

$$y = 2x + 7$$

y, dependent variable
(output values or range set)

Dependent and Independent Variable

Determine the distance a car will travel going 55 mph.

$$d = 55h$$

independent	h	d	dependent
	0	0	
	1	55	
	2	110	
	3	165	

Graph of a Quadratic Equation

$$y = ax^2 + bx + c$$

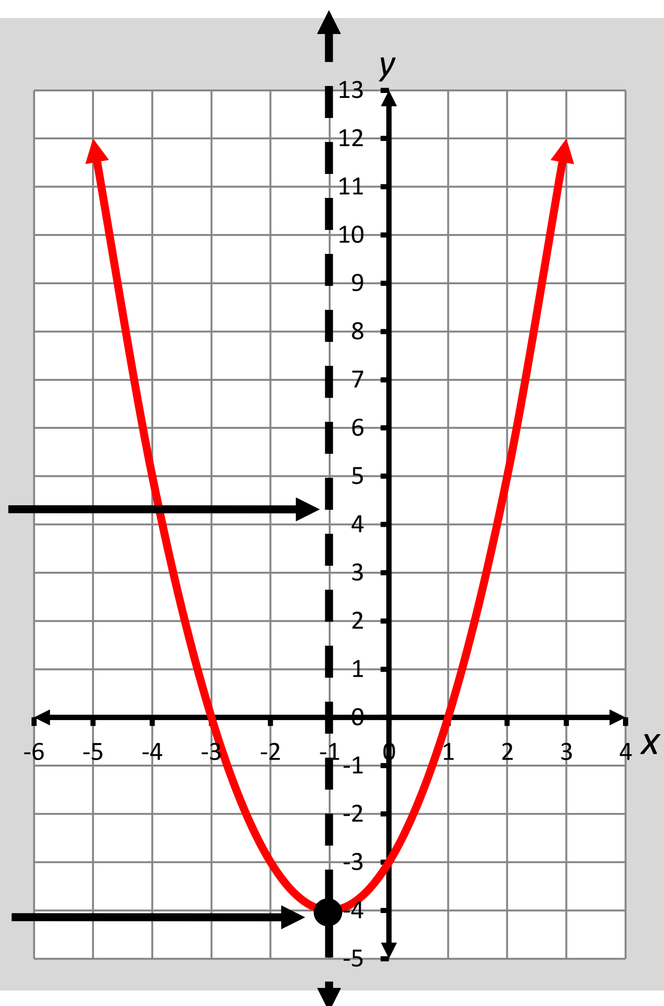
$$a \neq 0$$

Example:

$y = x^2 + 2x - 3$

line of symmetry

vertex



The graph of the quadratic equation is a curve (parabola) with one line of symmetry and one vertex.

Quadratic Formula

Used to find the solutions to any quadratic equation of the form, $y = ax^2 + bx + c$

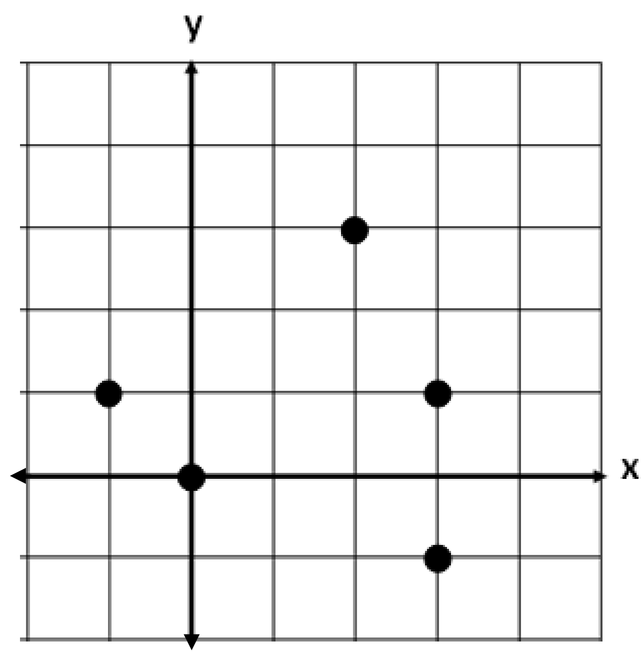
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relations

Representations of relationships

x	*y*
-3	4
0	0
1	-6
2	2
5	-1

Example 1



Example 2

$\{(0,4), (0,3), (0,2), (0,1)\}$

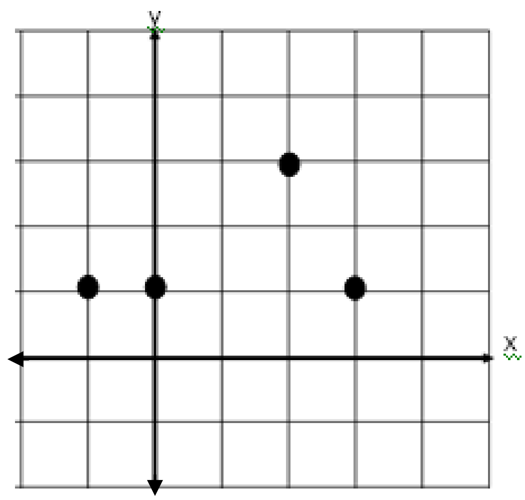
Example 3

Functions

Representations of functions

x	*y*
3	2
2	4
0	2
-1	2

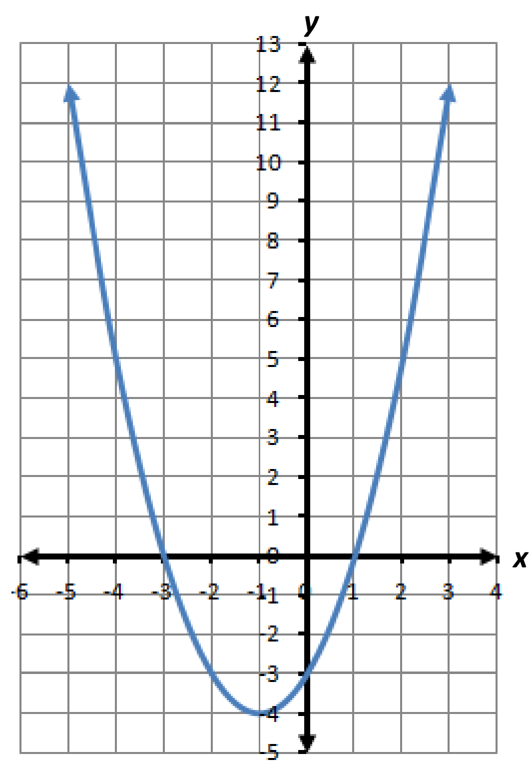
Example 1



Example 2

{(-3,4), (0,3), (1,2), (4,6)}

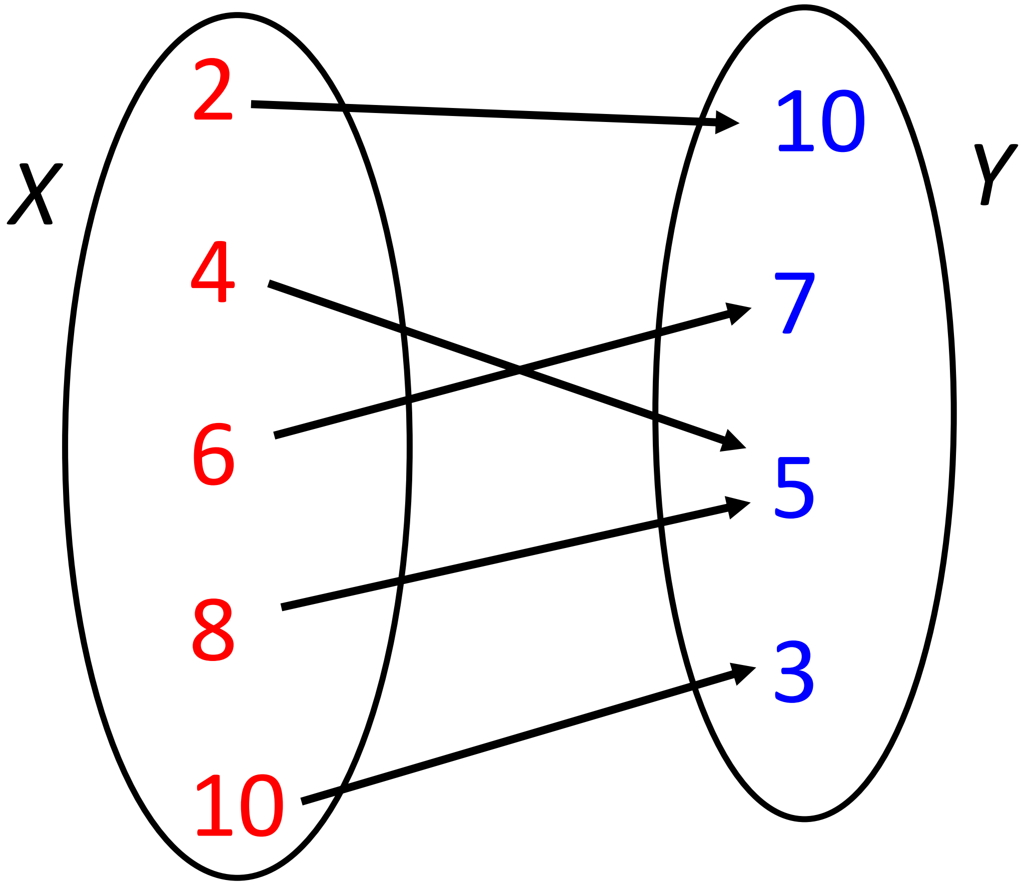
Example 3



Example 4

Function

A relationship between two quantities in which every input corresponds to exactly one output



A relation is a function if and only if each element in the domain is paired with a unique element of the range.

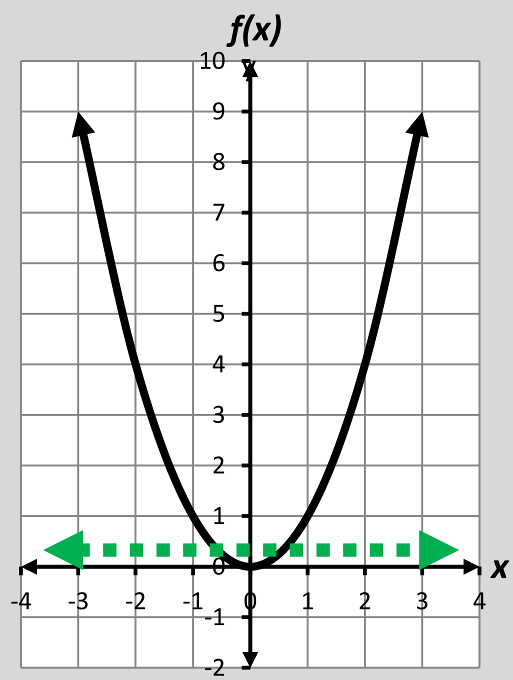
Domain

A set of input values of a relation

Examples:

input	output
x	***g*(*x*)**
-2	**0**
-1	**1**
0	**2**
1	**3**

The **domain** of *g*(*x*) is {**-2, -1, 0, 1**}.



The **domain** of *f*(*x*) is **all real numbers**.

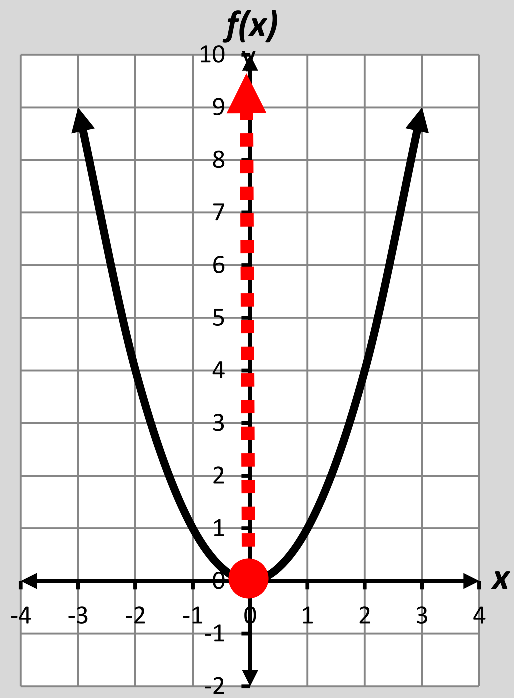
Range

A set of output values of a relation

Examples:

input	output
x	***g*(*x*)**
-2	**0**
-1	**1**
0	**2**
1	**3**

The **range** of g(x) is {**0, 1, 2, 3**}.



The **range** of *f*(*x*) is **all real numbers greater than or equal to zero**.

Function Notation

$f(x)$

$f(x)$ is read
"the value of f at x" or "f of x"

Example:

$f(x) = -3x + 5$, find $f(2)$.

$f(2) = -3(2) + 5$

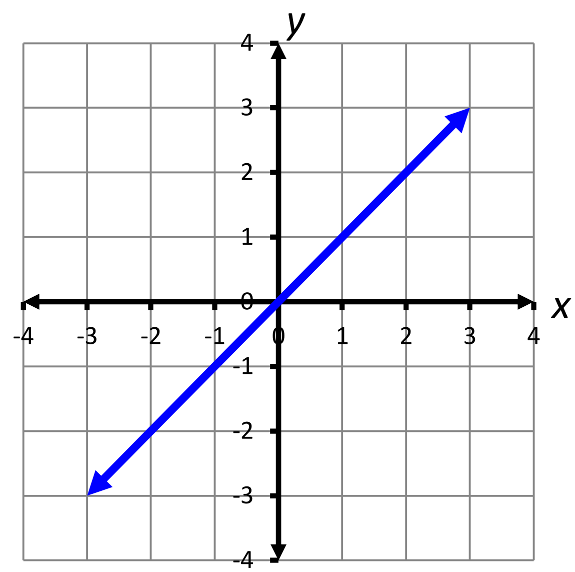
$f(2) = -6$

Letters other than f can be used to name functions, e.g., $g(x)$ and $h(x)$

Parent Functions

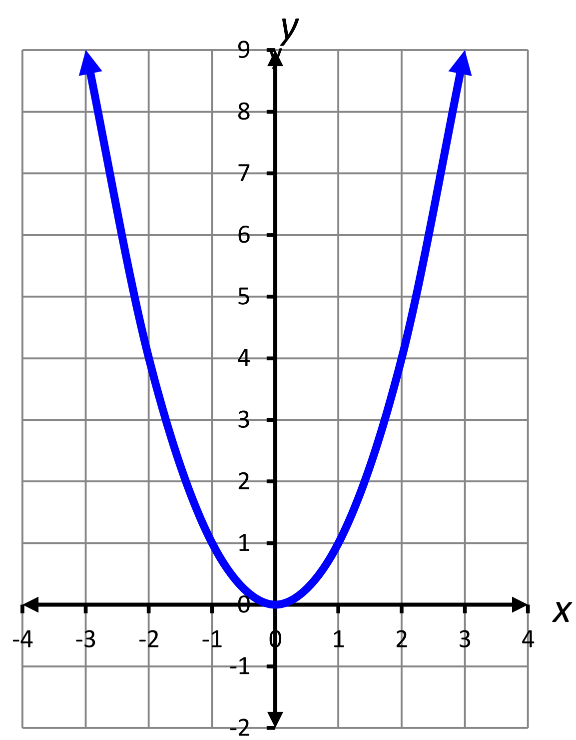
Linear

$f(x) = x$



Quadratic

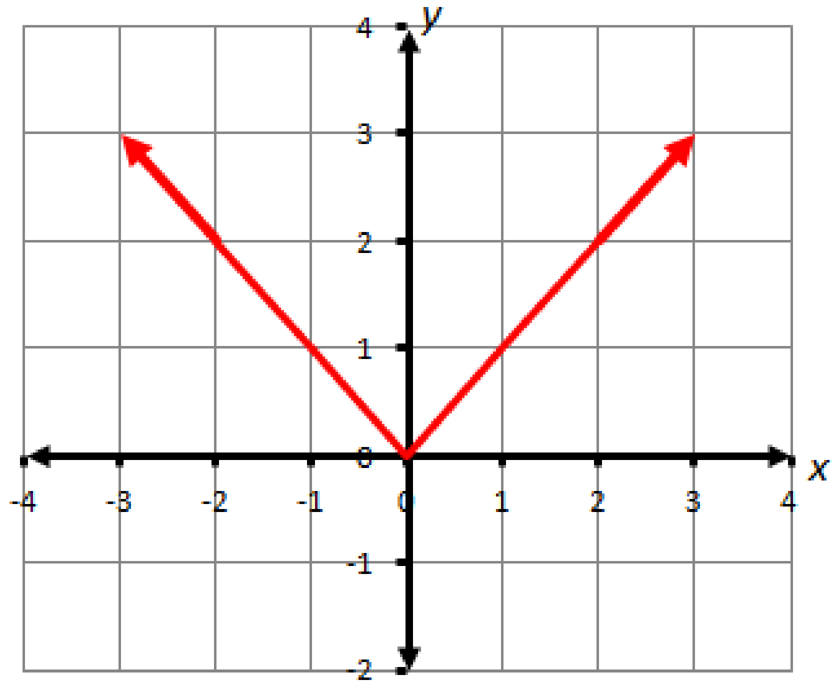
$f(x) = x^2$



Parent Functions

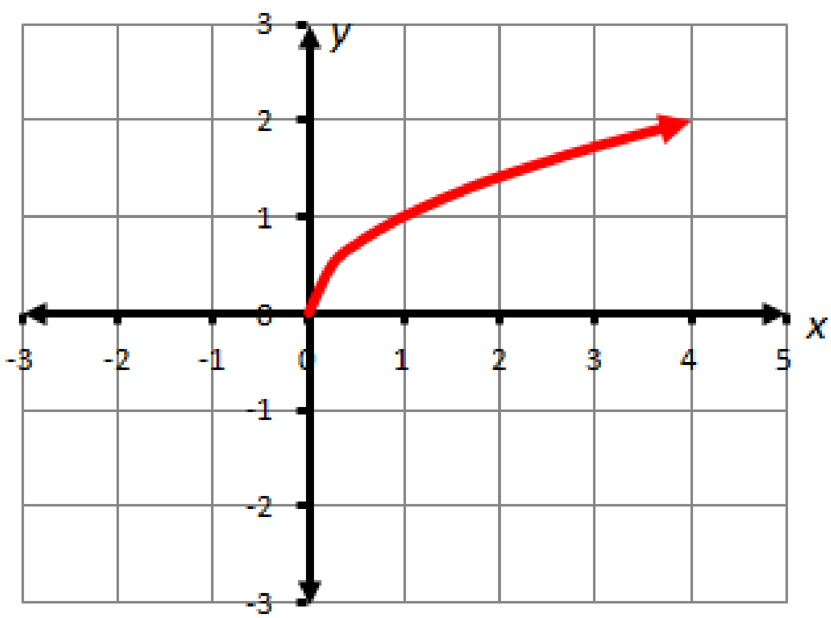
Absolute Value

$f(x) = |x|$



Square Root

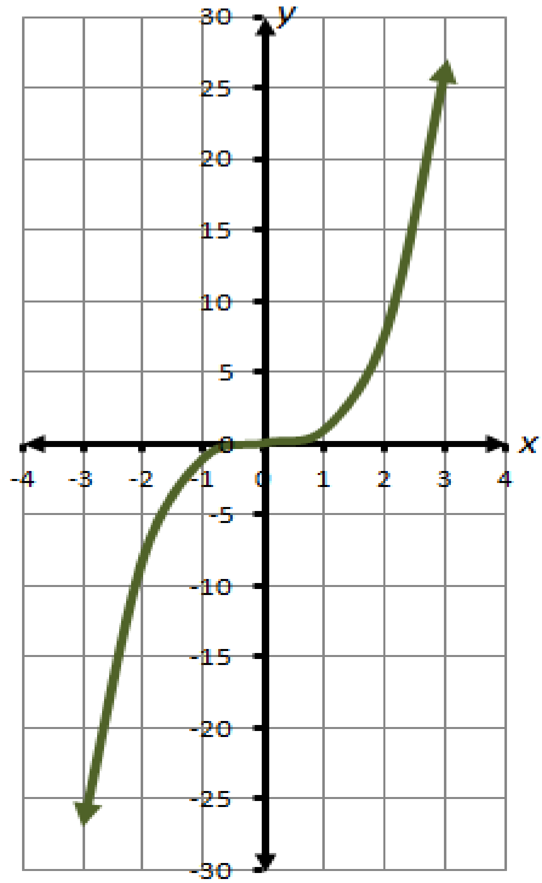
$f(x) = \sqrt{x}$



Parent Functions

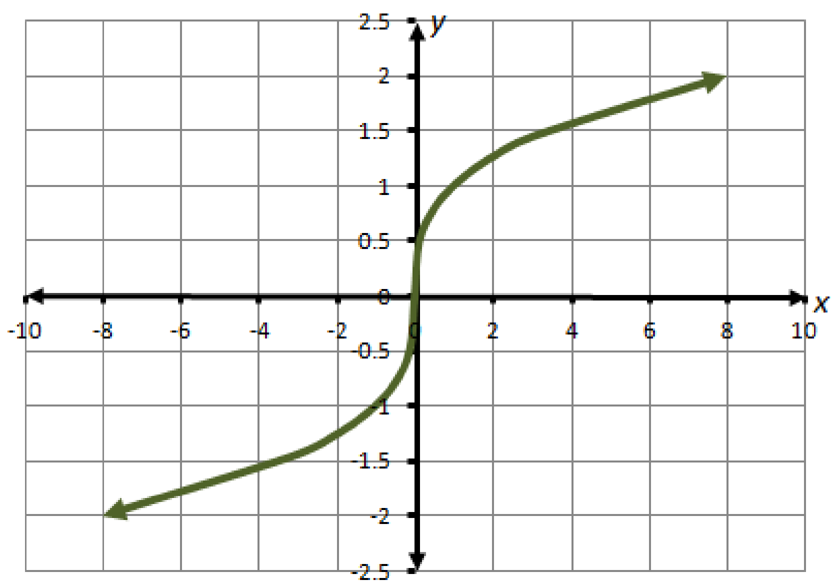
Cubic

$$f(x) = x^3$$



Cube Root

$$f(x) = \sqrt[3]{x}$$

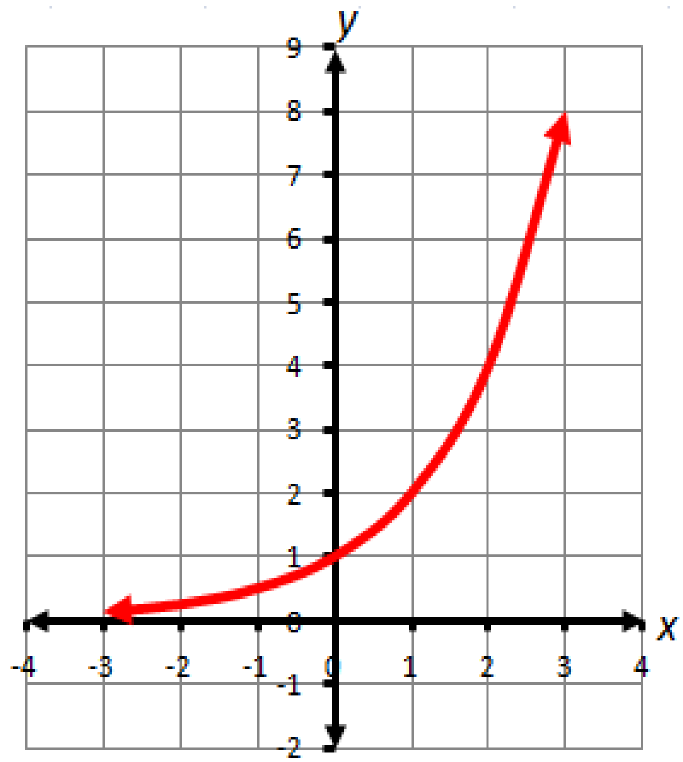


Parent Functions

Exponential

$f(x) = b^x$

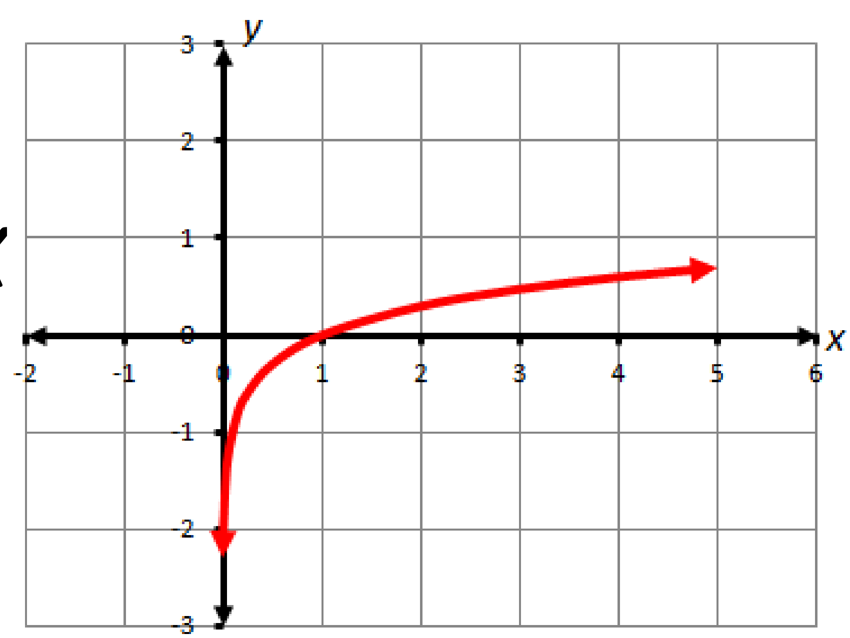
b > 1



Logarithmic

$f(x) = \log_b x$

b > 1



Transformations of Parent Functions

Parent functions can be transformed to create other members in a family of graphs.

Translations	$g(x) = f(x) + k$ is the graph of $f(x)$ **translated vertically** –	k units **up** when $k > 0$.
		k units **down** when $k < 0$.
	$g(x) = f(x - h)$ is the graph of $f(x)$ **translated horizontally** –	h units **right** when $h > 0$.
		h units **left** when $h < 0$.

Transformations of Parent Functions

Parent functions can be transformed to create other members in a family of graphs.

Reflections		
	$\boldsymbol{g(x) = -f(x)}$ is the graph of $f(x)$ –	**reflected** over the ***x*-axis**.
	$\boldsymbol{g(x) = f(-x)}$ is the graph of $f(x)$ –	**reflected** over the ***y*-axis**.

Transformations of Parent Functions

Parent functions can be transformed to create other members in a family of graphs.

Dilations	$g(x) = a \cdot f(x)$ is the graph of $f(x)$ –	**vertical dilation** (stretch) if $a > 1$.
		vertical dilation (compression) if $0 < a < 1$.
	$g(x) = f(ax)$ is the graph of $f(x)$ –	**horizontal dilation** (compression) if $a > 1$.
		horizontal dilation (stretch) if $0 < a < 1$.

Transformational Graphing

Linear functions

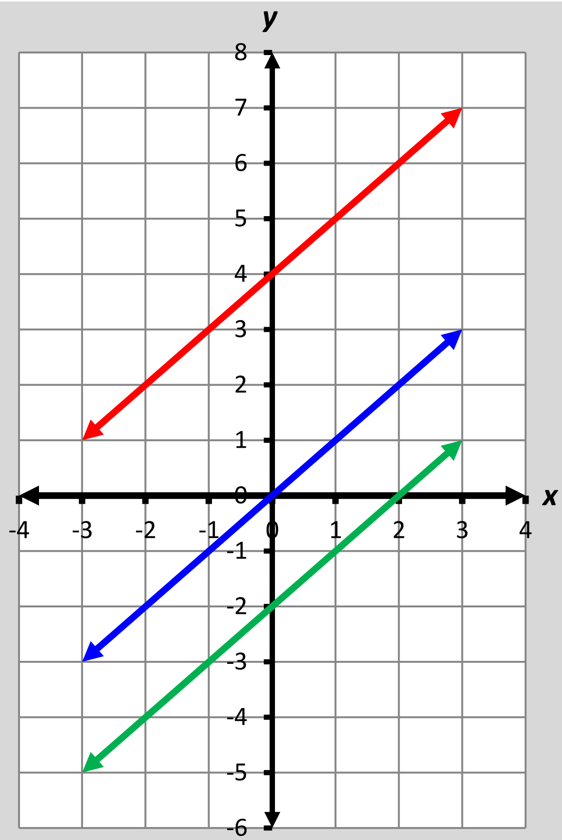
$g(x) = x + b$

Examples:

$f(x) = x$

$t(x) = x + 4$

$h(x) = x - 2$



Vertical translation of the parent function, $f(x) = x$

Transformational Graphing

Linear functions

$g(x) = mx$

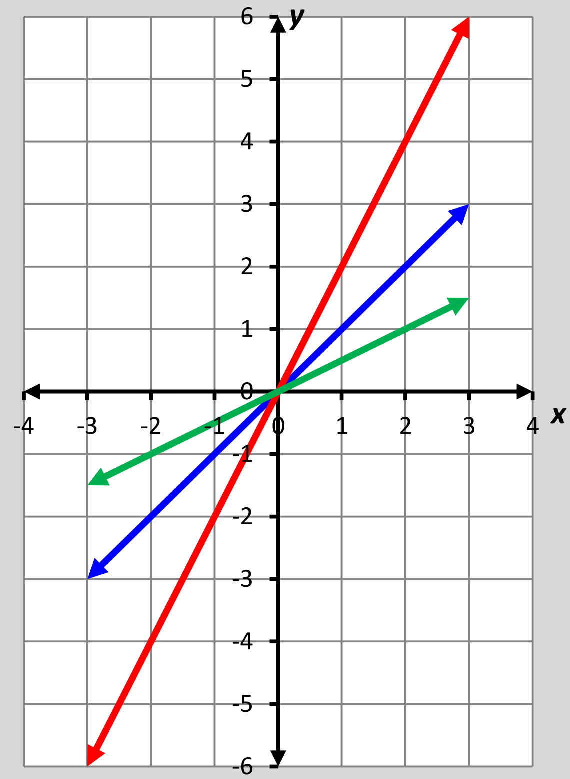
$m > 0$

Examples:

$f(x) = x$

$t(x) = 2x$

$h(x) = \frac{1}{2}x$



Vertical dilation (stretch or compression) of the parent function, $f(x) = x$

Transformational Graphing

Linear functions

$g(x) = mx$

$m < 0$

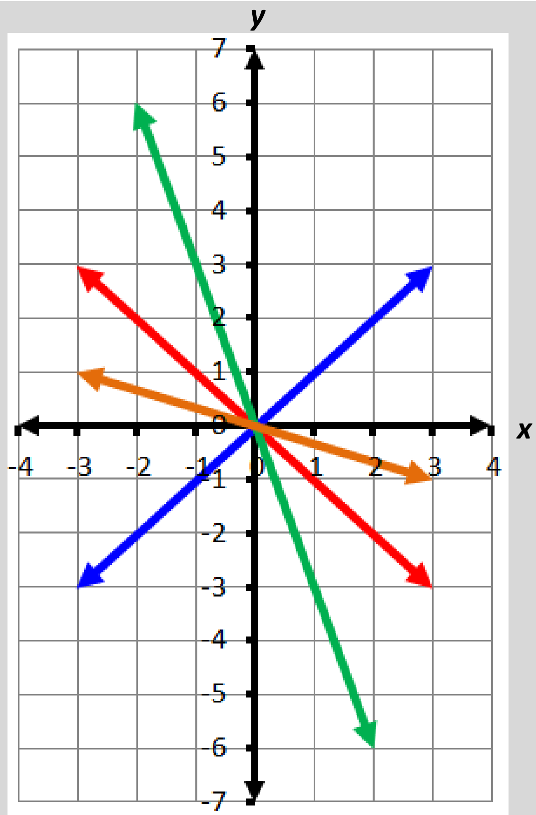
Examples:

$f(x) = x$

$t(x) = -x$

$h(x) = -3x$

$d(x) = -\frac{1}{3}x$



Vertical dilation (stretch or compression) with a reflection of $f(x) = x$

Transformational Graphing

Quadratic functions

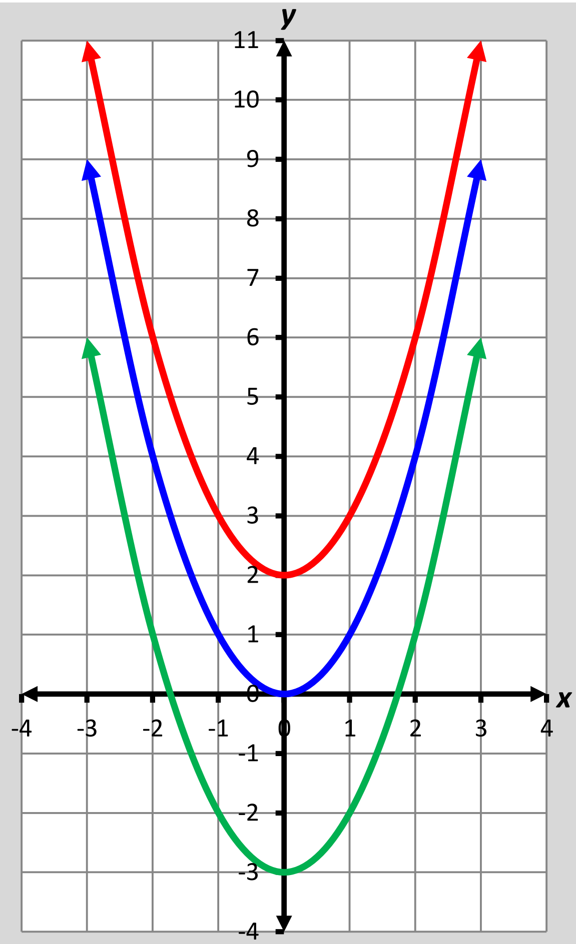
$h(x) = x^2 + c$

Examples:

$f(x) = x^2$

$g(x) = x^2 + 2$

$t(x) = x^2 - 3$



Vertical translation of $f(x) = x^2$

Transformational Graphing

Quadratic functions

$h(x) = ax^2$

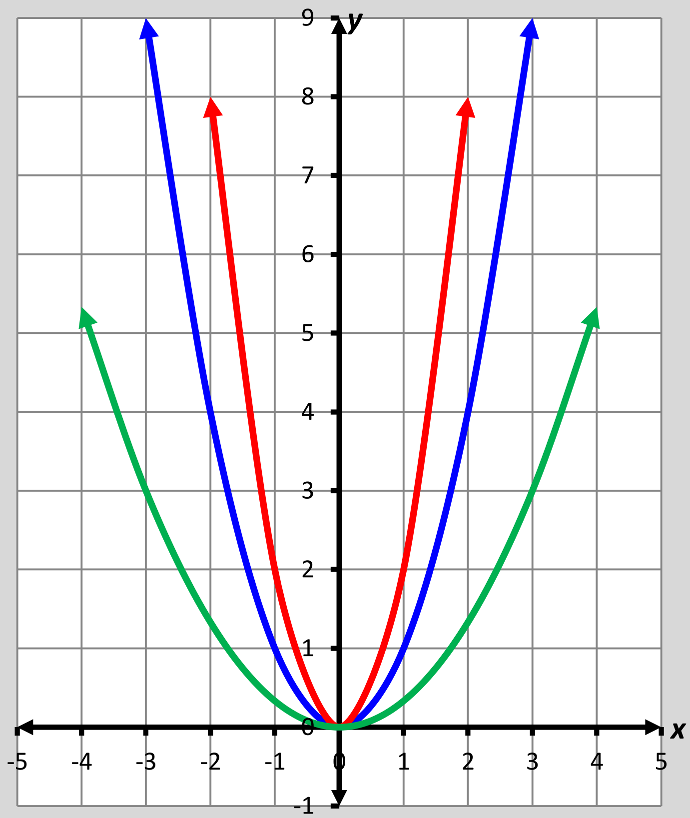
$a > 0$

Examples:

$f(x) = x^2$

$g(x) = 2x^2$

$t(x) = \frac{1}{3}x^2$



Vertical dilation (stretch or compression) of $f(x) = x^2$

Transformational Graphing

Quadratic functions

$h(x) = ax^2$

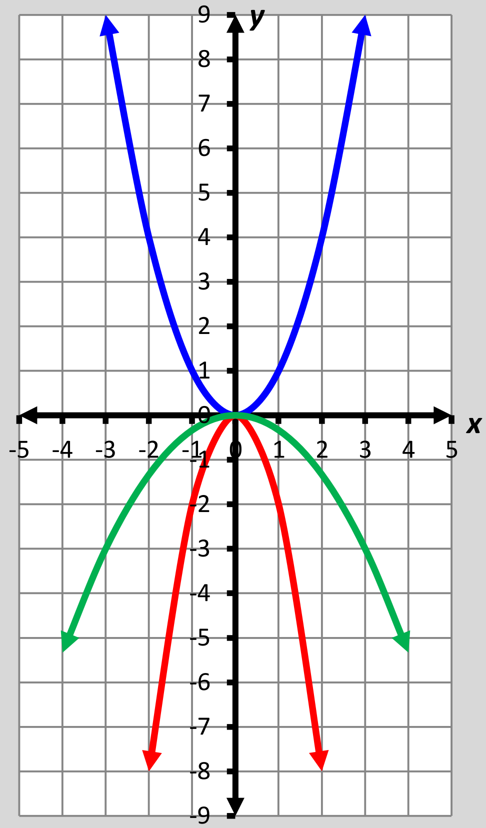
$a < 0$

Examples:

$f(x) = x^2$

$g(x) = -2x^2$

$t(x) = -\frac{1}{3}x^2$



Vertical dilation (stretch or compression) with a reflection of $f(x) = x^2$

Transformational Graphing

Quadratic functions

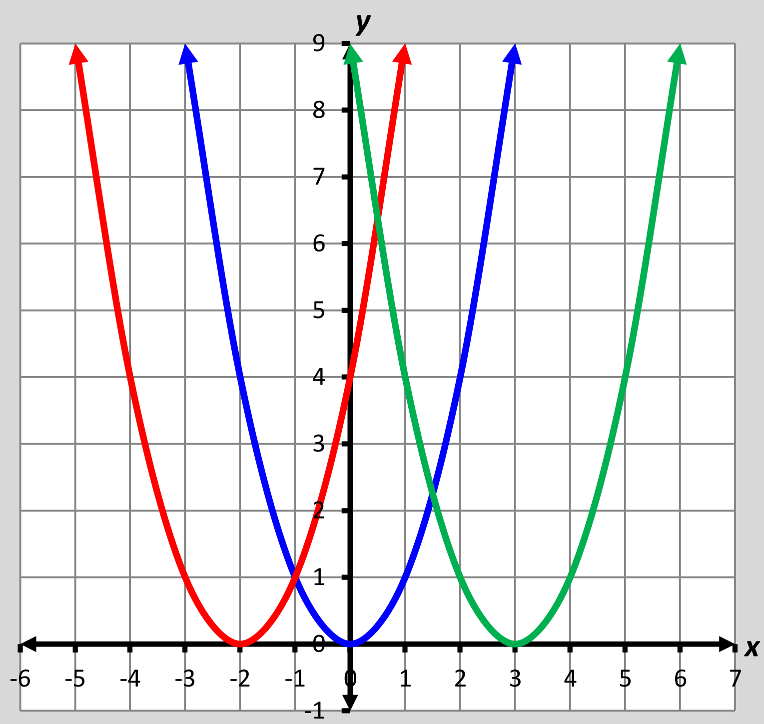
$h(x) = (x + c)^2$

Examples:

$f(x) = x^2$

$g(x) = (x + 2)^2$

$t(x) = (x - 3)^2$



Horizontal translation of $f(x) = x^2$

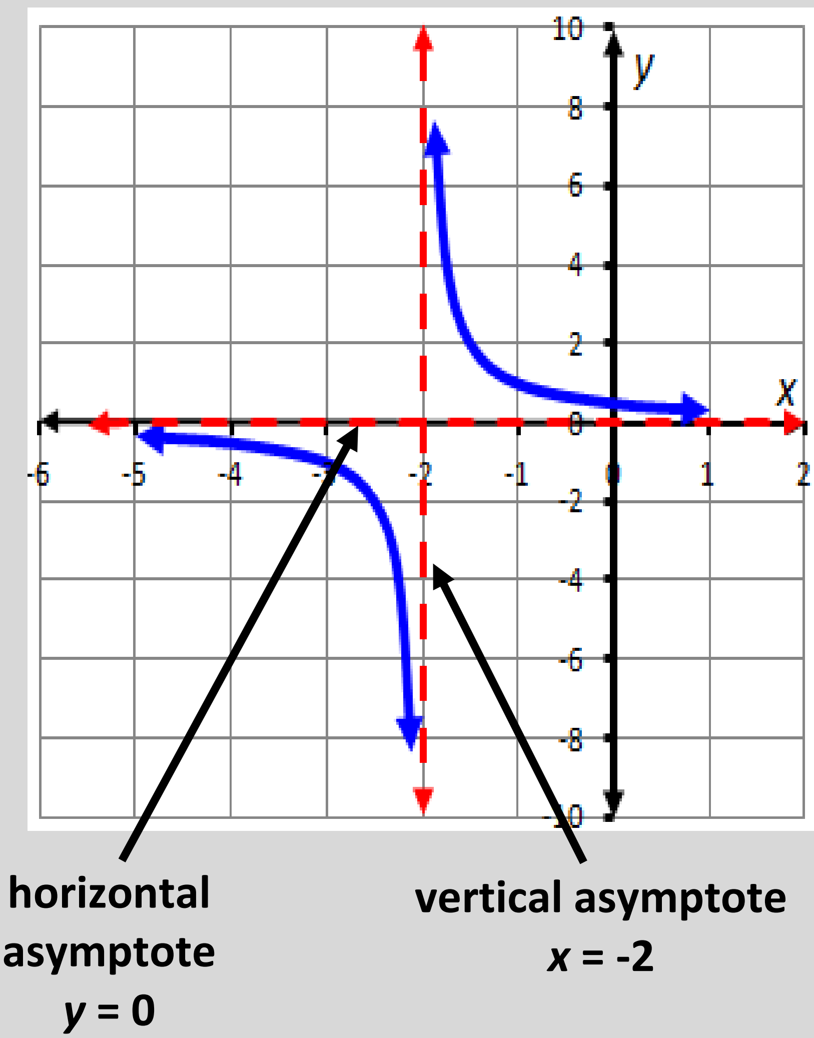
Discontinuity

Vertical and Horizontal Asymptotes

Example:

$f(x) = \frac{1}{x+2}$

f(-2) is not defined, so *f(x)* is discontinuous.



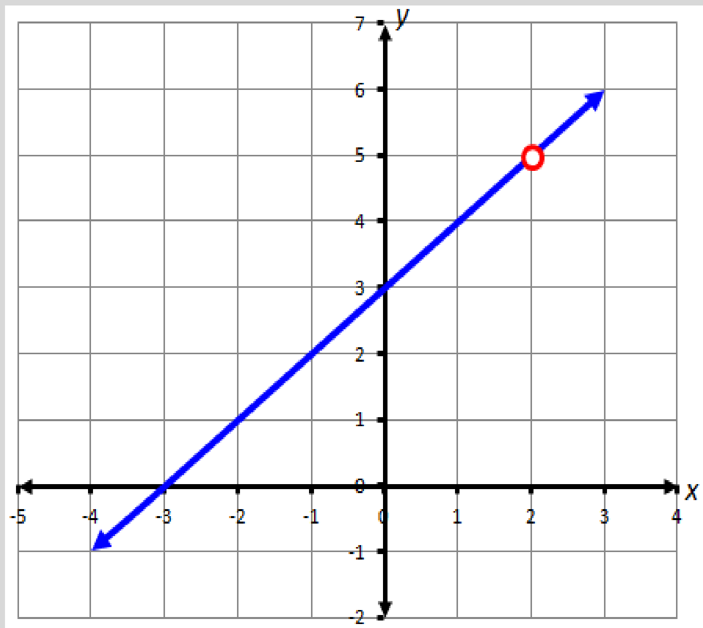
Discontinuity

Removable Discontinuity
Point Discontinuity

Example:

$$f(x) = \frac{x^2 + x - 6}{x - 2}$$

$f(2)$ is not defined.



x	$f(x)$
-3	0
-2	1
-1	2
0	3
1	4
2	error
3	6

$$f(x) = \frac{x^2 + x - 6}{x - 2}$$

$$= \frac{(x + 3)(x - 2)}{x - 2}$$

$$= x + 3,\ x \neq 2$$

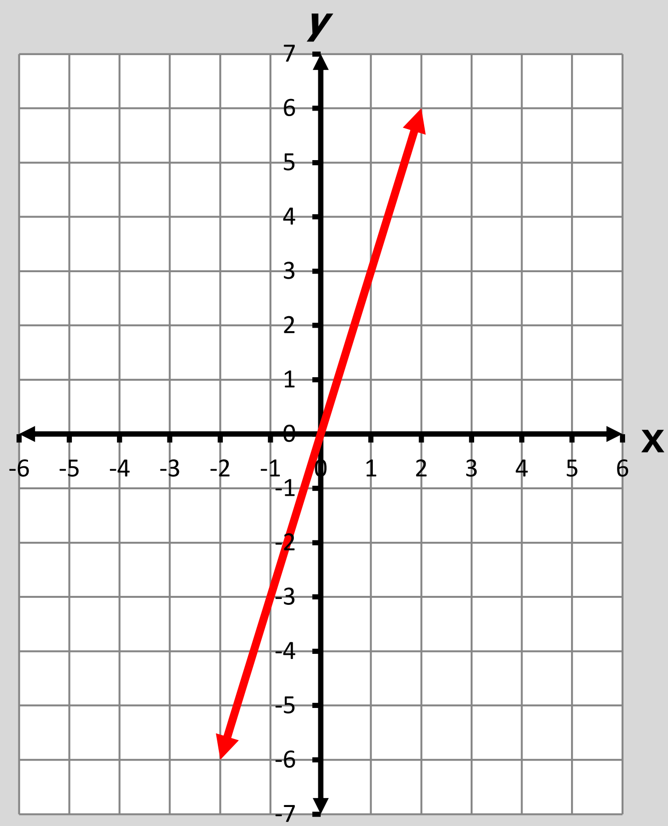
Direct Variation

$$y = kx \quad \text{or} \quad k = \frac{y}{x}$$

constant of variation, $k \neq 0$

Example:

$y = 3x$ or $3 = \frac{y}{x}$



The graph of all points describing a direct variation is a line passing through the origin.

Arithmetic Sequence

A sequence of numbers that has a common difference between every two consecutive terms

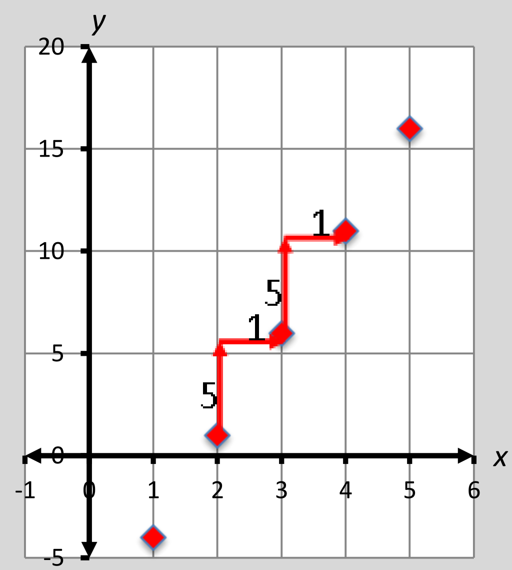
Example: -4, 1, 6, 11, 16 …

+5 +5 +5 +5

common difference

Position x	Term y
1	-4
2	1
3	6
4	11
5	16

+5
+5
+5
+5



The common difference is the slope of the line of best fit.

Geometric Sequence

A sequence of numbers in which each term after the first term is obtained by multiplying the previous term by a constant ratio

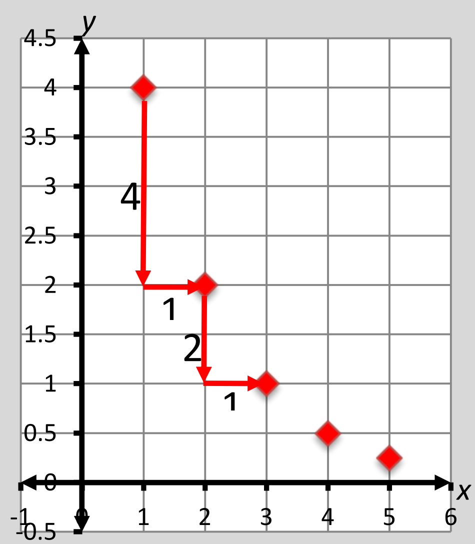
Example: 4, 2, 1, 0.5, 0.25 ...

$\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$

common ratio

Position x	Term y
1	4
2	2
3	1
4	0.5
5	0.25

$\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$ $\times\frac{1}{2}$

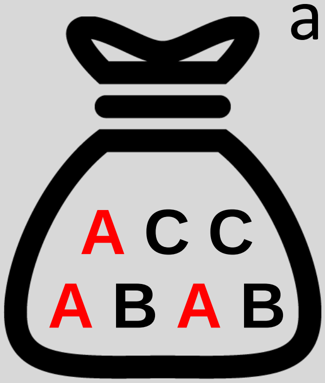


Probability

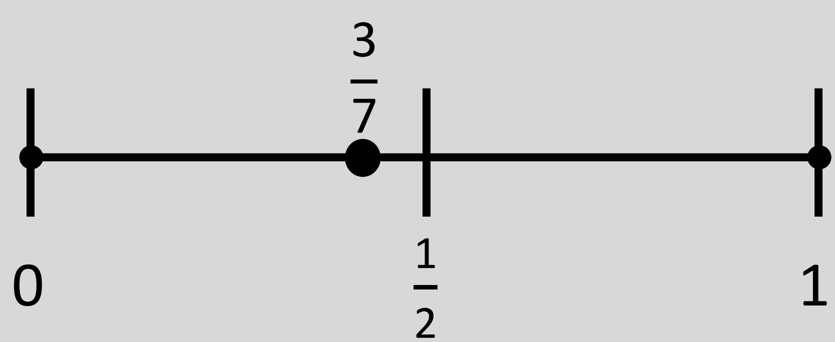
The likelihood of an event occurring

$$\text{probability of an event} = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$

Example: What is the probability of drawing an A from the bag of letters shown?

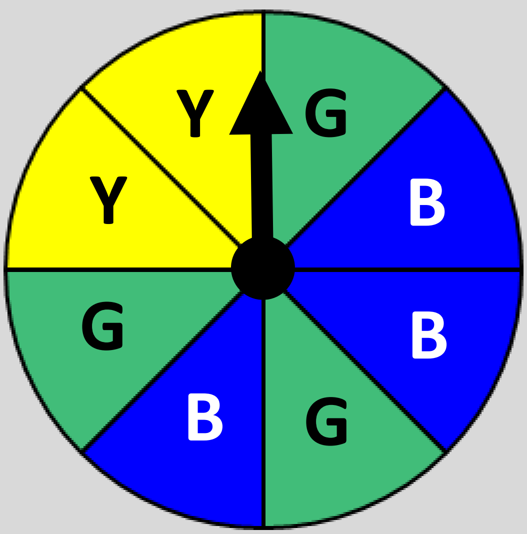


$$P(A) = \frac{3}{7}$$



Probability of Independent Events

Example:



What is the probability of landing on green on the first spin and then landing on yellow on the second spin?

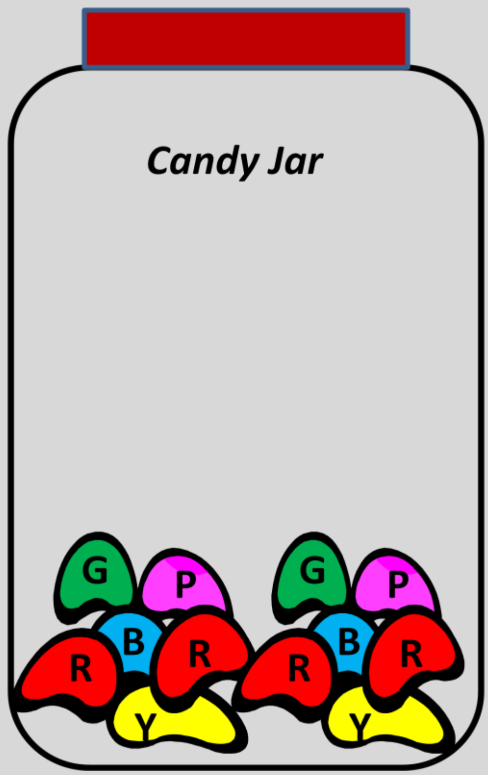
P(green and yellow) =

$$P(\text{green}) \cdot P(\text{yellow}) = \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{32}$$

Probability of Dependent Events

Example:

What is the probability of selecting a red jelly bean on the first pick and without replacing it, selecting a blue jelly bean on the second pick?



P(red and blue) =

$$P(\text{red}) \cdot P(\text{blue}|\text{red}) = \frac{4}{12} \cdot \frac{2}{11} = \frac{8}{132} = \frac{2}{33}$$

"blue after red"

Probability

Mutually Exclusive Events

Events are mutually exclusive if they cannot occur at the same time.

Example:
In a single card draw from a deck of cards, what is the probability of selecting

- a king and an ace? P(king and ace) = 0
- a king or an ace? P(king or ace) = P(king) + P(ace)

$P(\text{king}) = \frac{4}{52}$

$P(\text{ace}) = \frac{4}{52}$

$P(\text{king}) + P(\text{ace}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52}$

If two events A and B are mutually exclusive, then

- P(A and B) = 0; and
- P(A or B) = P(A) + P(B).

Fundamental Counting Principle

If there are m ways for one event to occur and n ways for a second event to occur, then there are $m \cdot n$ ways for both events to occur.

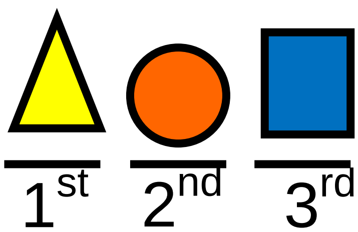
Example:

How many outfits can Joey make using 3 pairs of pants and 4 shirts?

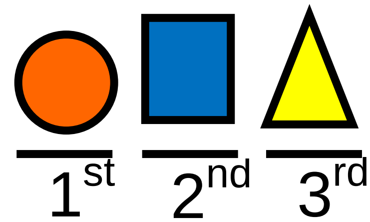
$3 \cdot 4 = 12$ outfits

Permutation

An ordered arrangement of a group of objects



is different from



Both arrangements are included in possible outcomes.

Example:

5 people to fill 3 chairs (order matters). How many ways can the chairs be filled?

1^{st} chair – 5 people to choose from

2^{nd} chair – 4 people to choose from

3^{rd} chair – 3 people to choose from

possible arrangements are $5 \cdot 4 \cdot 3 = 60$

Permutation

To calculate the number of permutations

$$_nP_r = \frac{n!}{(n-r)!}$$

n and *r* are positive integers, n ≥ r, and *n* is the total number of elements in the set and *r* is the number to be ordered.

Example: There are 30 cars in a car race. The first-, second-, and third-place finishers win a prize. How many different arrangements of the first three positions are possible?

$$_{30}P_3 = \frac{30!}{(30-3)!} = \frac{30!}{27!} = 24360$$

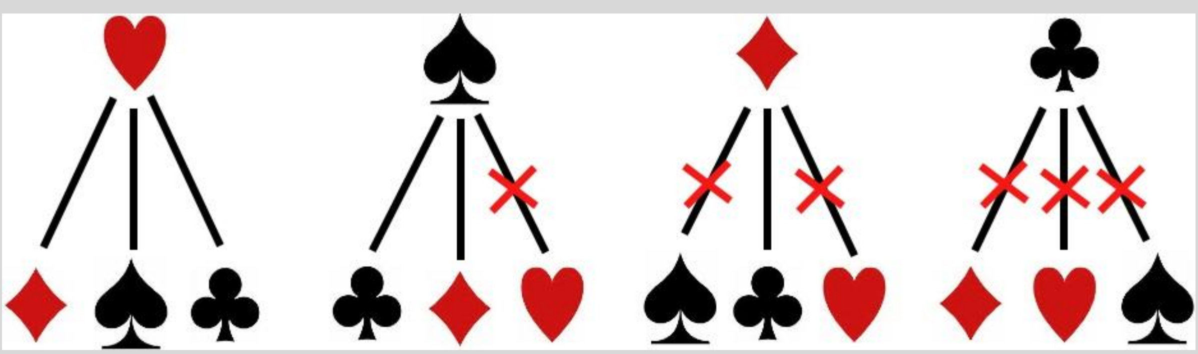
Combination

The number of possible ways to select or arrange objects when there is no repetition and order does not matter

Example: If Sam chooses 2 selections from heart, club, spade and diamond. How many different combinations are possible?

Order (position) does not matter so

♦ ♣ is the same as ♣ ♦



There are 6 possible combinations.

Combination

To calculate the number of possible combinations using a formula

$$_nC_r = \frac{n!}{r!(n-r)!}$$

n and *r* are positive integers, n ≥ r, and *n* is the total number of elements in the set and *r* is the number to be ordered.

Example: In a class of 24 students, how many ways can a group of 4 students be arranged?

$$_{24}C_4 = \frac{24!}{4!(24-4)!} = 10{,}626$$

Statistics Notation

x_i	i^{th} element in a data set
μ	mean of the data set
σ^2	variance of the data set
σ	standard deviation of the data set
n	number of elements in the data set

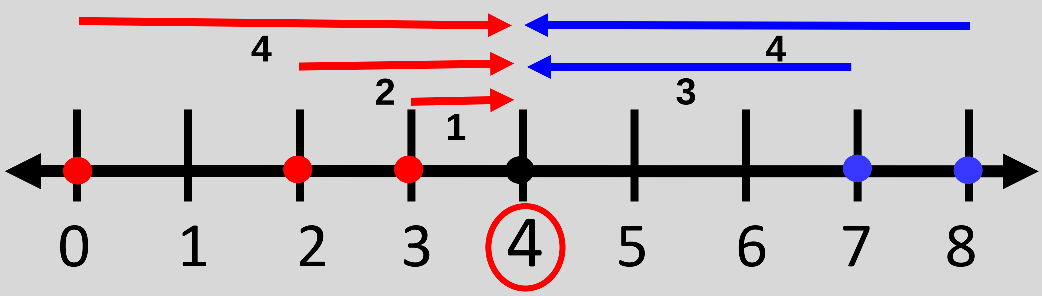
Mean

A measure of central tendency

Example: Find the mean of the given data set.

Data set: 0, 2, 3, 7, 8

Balance Point



Numerical Average

$$\mu = \frac{0+2+3+7+8}{5} = \frac{20}{5} = 4$$

Median

A measure of central tendency

Examples:

Find the median of the given data sets.

Data set: 6, 7, 8, 9, 9

The median is 8.

Data set: 5, 6, 8, 9, 11, 12

The median is 8.5.

Mode

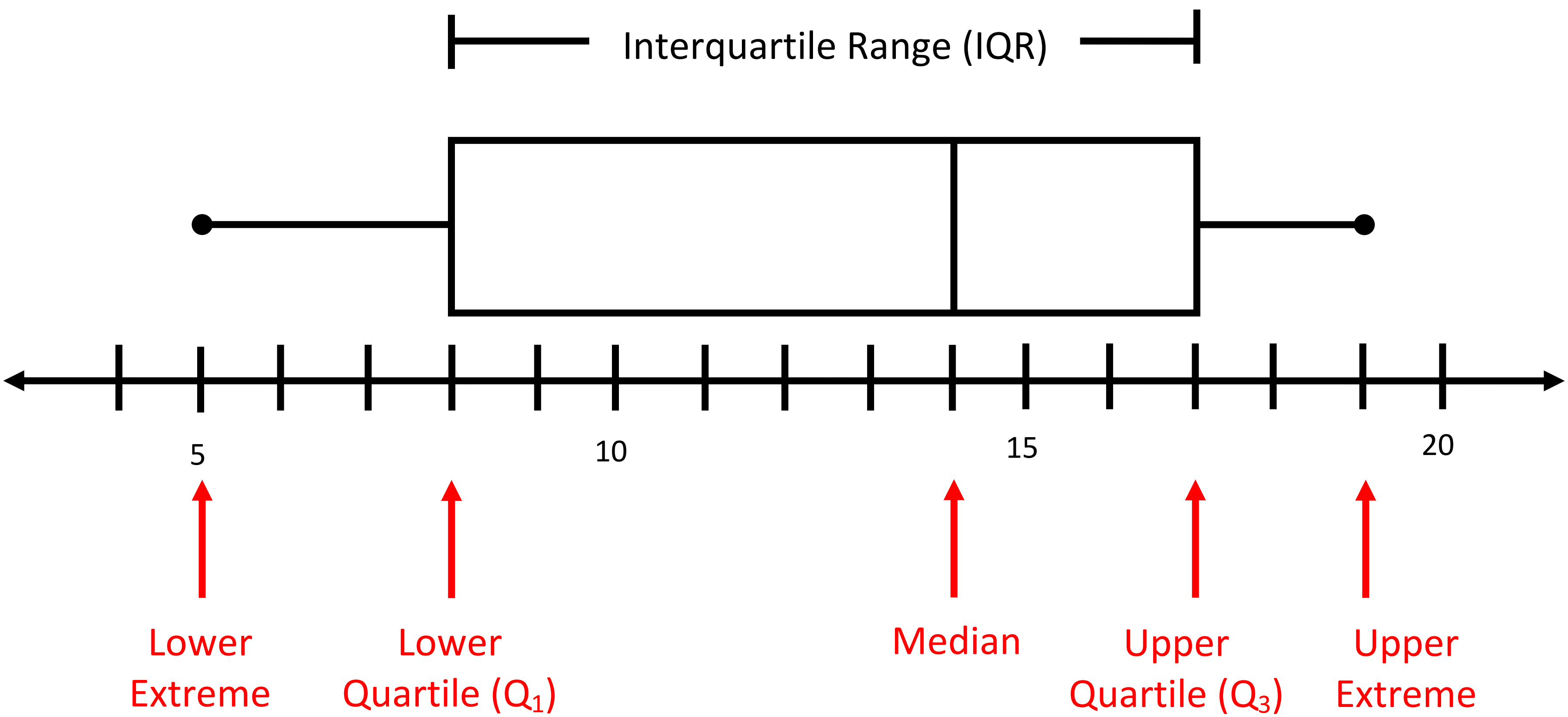
A measure of central tendency

Examples:

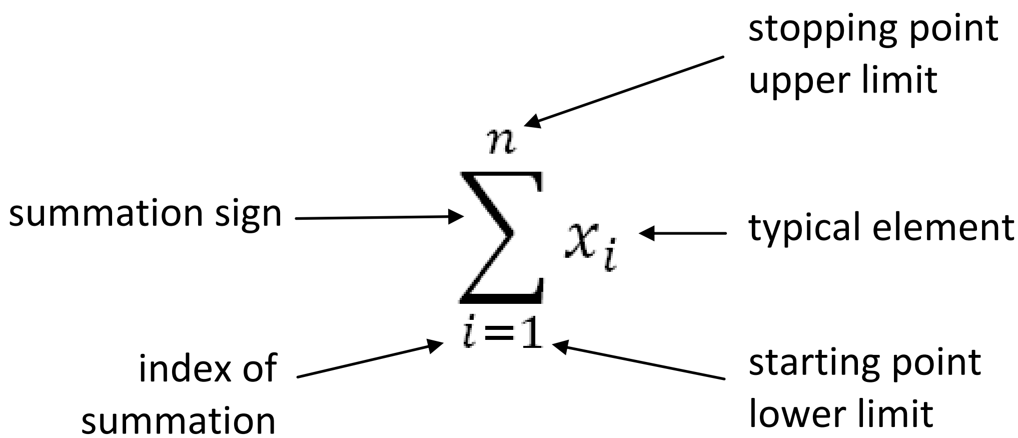
Data Sets	Mode
3, 4, 6, 6, 6, 6, 10, 11, 14	6
0, 3, 4, 5, 6, 7, 9, 10	none
5.2, 5.2, 5.2, 5.6, 5.8, 5.9, 6.0	5.2
1, 1, 2, 5, 6, 7, 7, 9, 11, 12	1, 7 bimodal

Box-and-Whisker Plot

A graphical representation of the five-number summary



Summation



This expression means sum the values of *x*, starting at x_1 and ending at x_n.

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + x_3 + \dots + x_n$$

Example: Given the data set {3, 4, 5, 5, 10, 17}

$$\sum_{i=1}^{6} x_i = 3 + 4 + 5 + 5 + 10 + 17 = 44$$

Mean Absolute Deviation

A measure of the spread of a data set

$$\text{Mean Absolute Deviation} = \frac{\sum_{i=1}^{n} |x_i - \mu|}{n}$$

The mean of the sum of the absolute value of the differences between each element and the mean of the data set

Variance

A measure of the spread of a data set

$$\text{variance}(\sigma^2) = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}$$

The mean of the squares of the differences between each element and the mean of the data set

Standard Deviation

A measure of the spread of a data set

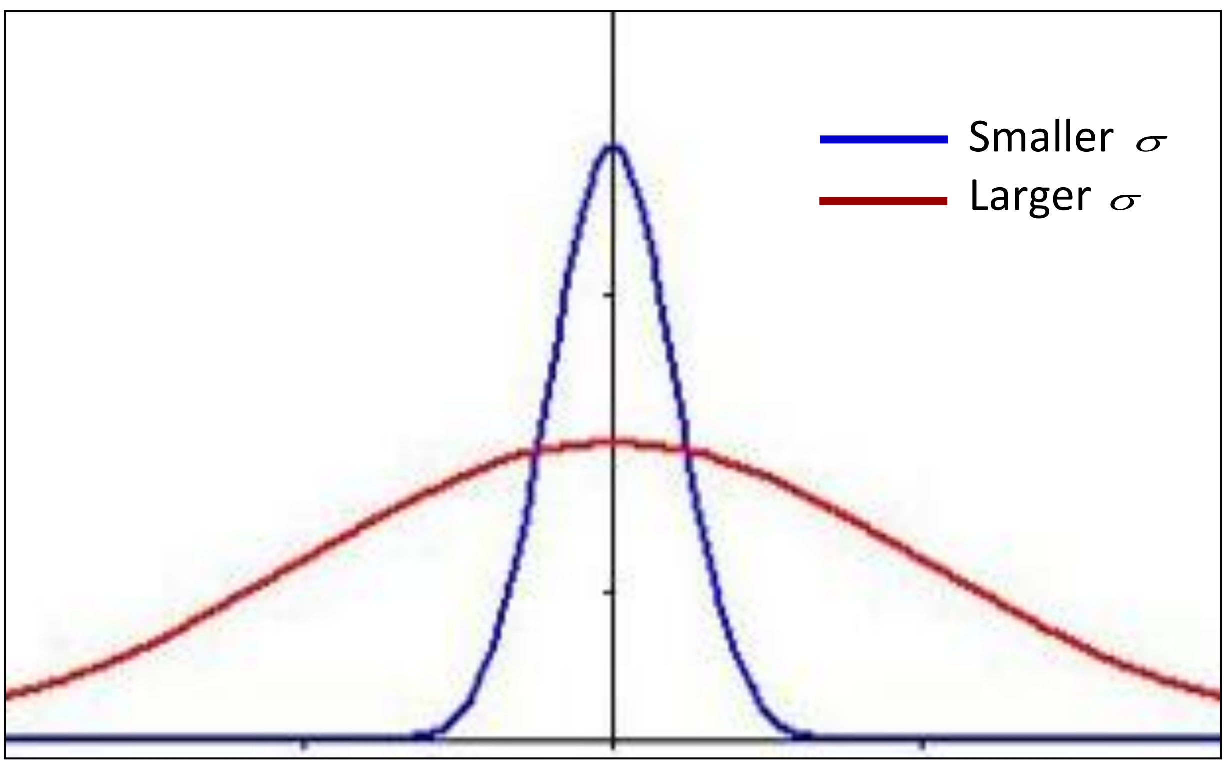
$$\text{standard deviation}\,(\sigma) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$$

The square root of the mean of the squares of the differences between each element and the mean of the data set or the square root of the variance

Standard Deviation

A measure of the spread of a data set

$$\text{standard deviation}(\sigma) = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$$



Comparison of two distributions with same mean and different standard deviation values

z-Score

The number of standard deviations an element is away from the mean

$$\text{z-score (z)} = \frac{x - \mu}{\sigma}$$

where *x* is an element of the data set, *μ* is the mean of the data set, and *σ* is the standard deviation of the data set.

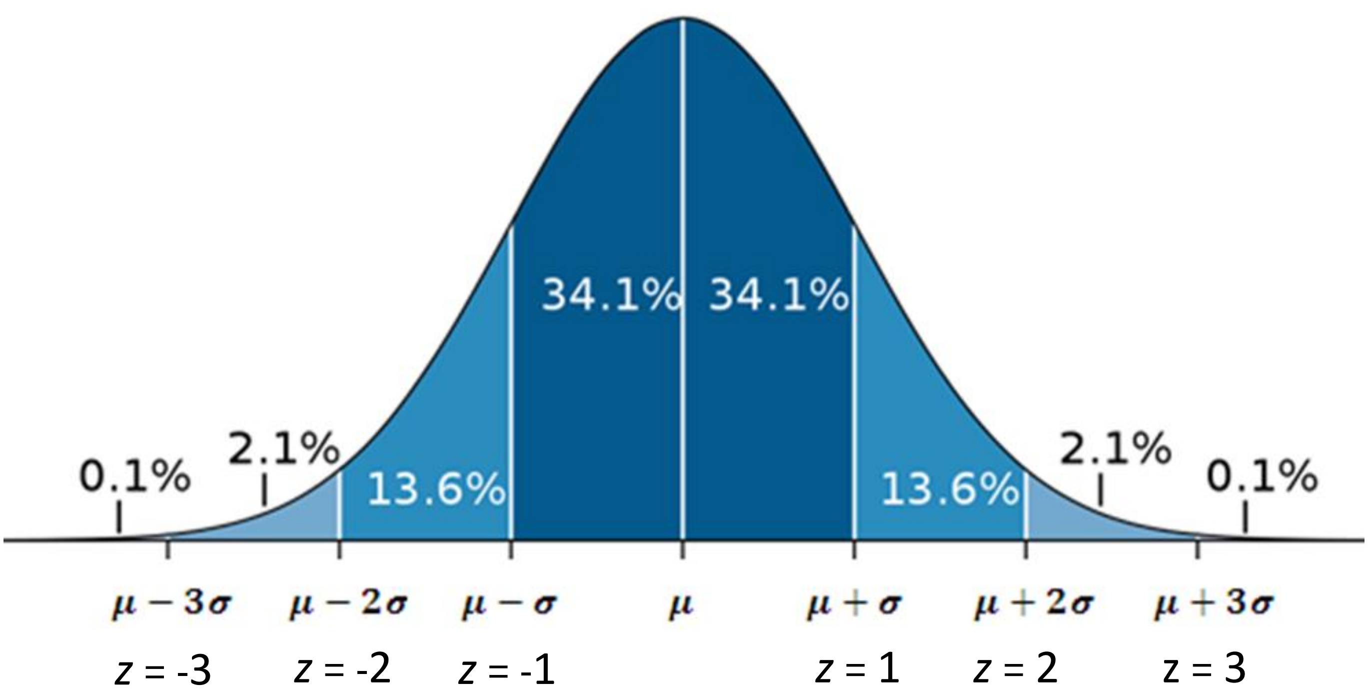
Example: Data set A has a mean of 83 and a standard deviation of 9.74. What is the *z*-score for the element 91 in data set A?

$$z = \frac{91\text{-}83}{9.74} = 0.821$$

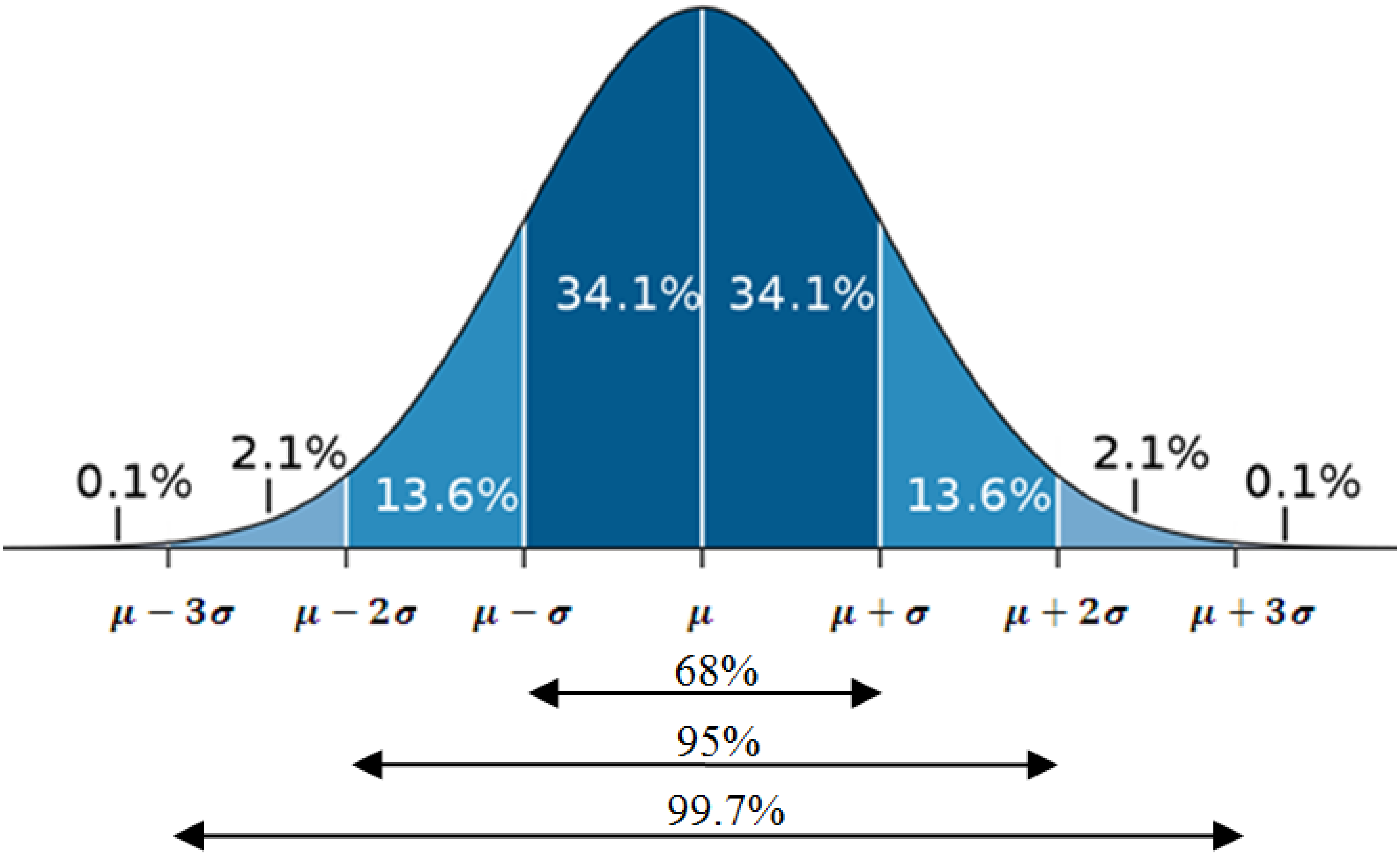
z-Score

The number of standard deviations an element is from the mean

$$\text{z-score (z)} = \frac{x - \mu}{\sigma}$$

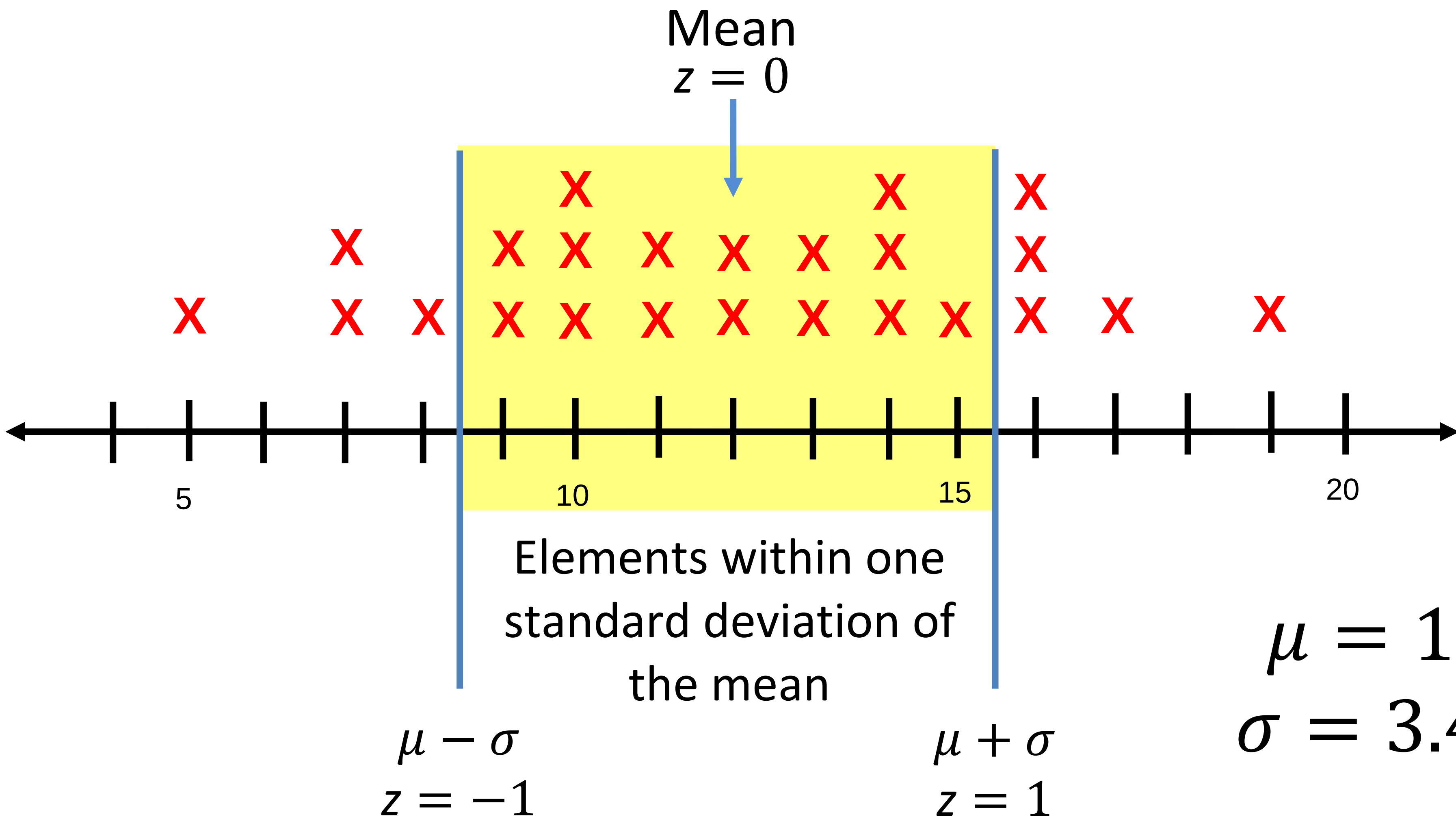


Normal Distribution



approximate percentage of element distribution

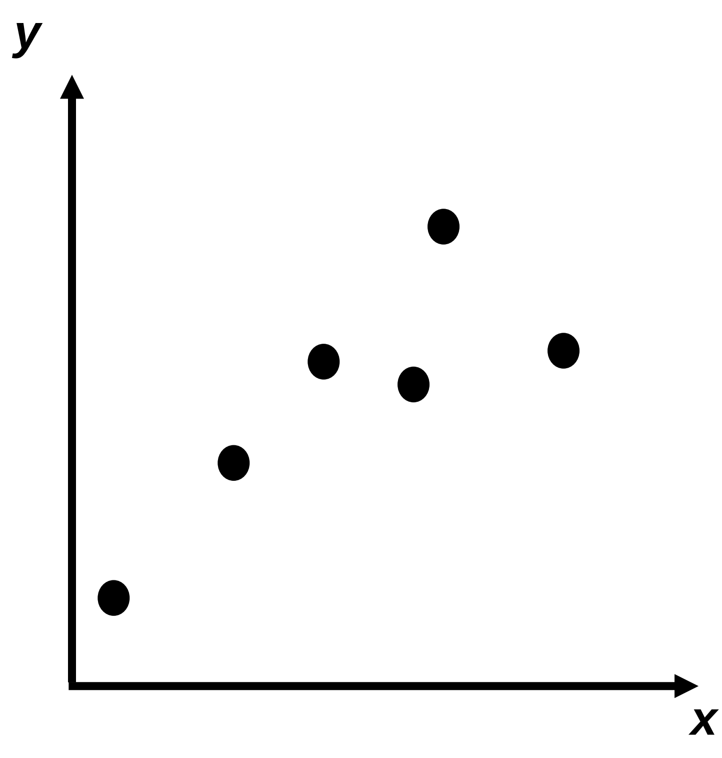
Elements within One Standard Deviation (σ) of the Mean (μ)



$$\mu = 12$$
$$\sigma = 3.49$$

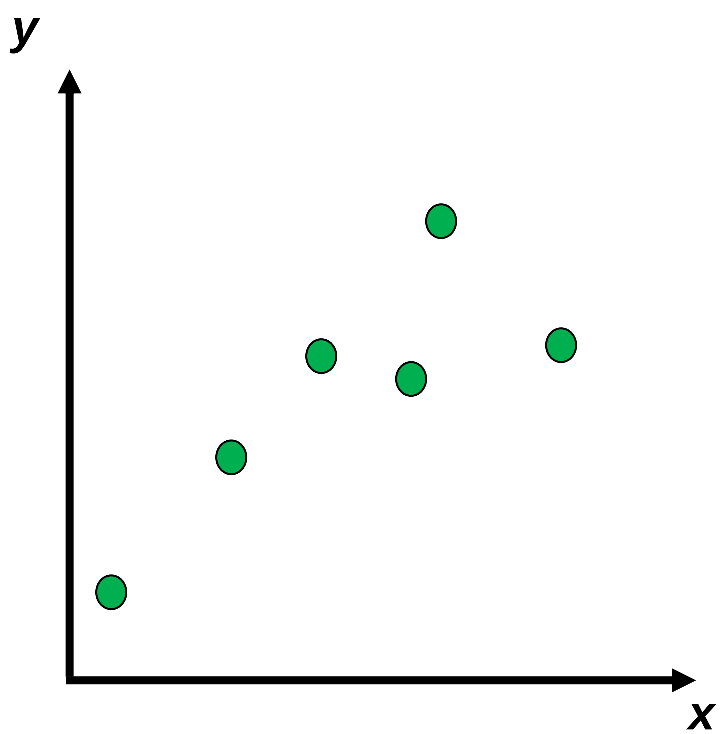
Scatterplot

Graphical representation of the relationship between two numerical sets of data



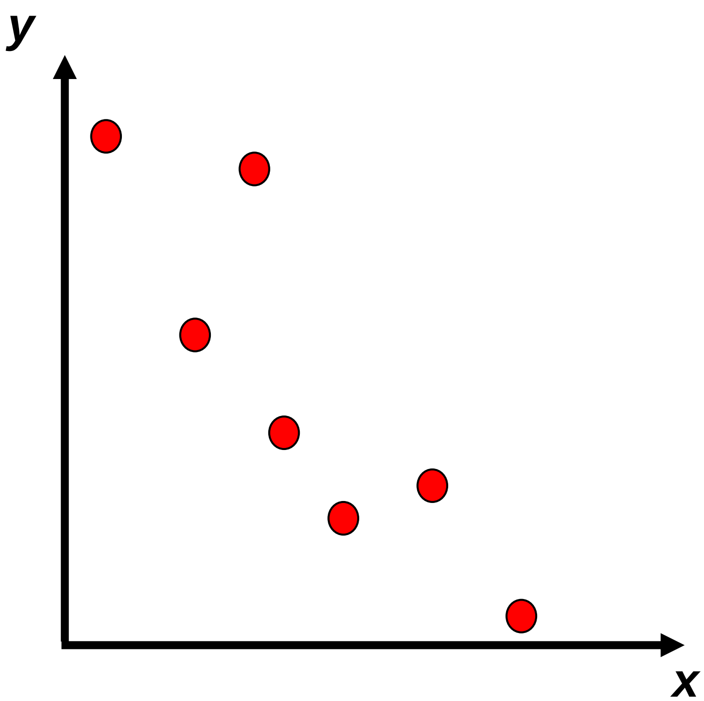
Positive Correlation

In general, a relationship where the dependent (*y*) values increase as independent values (*x*) increase



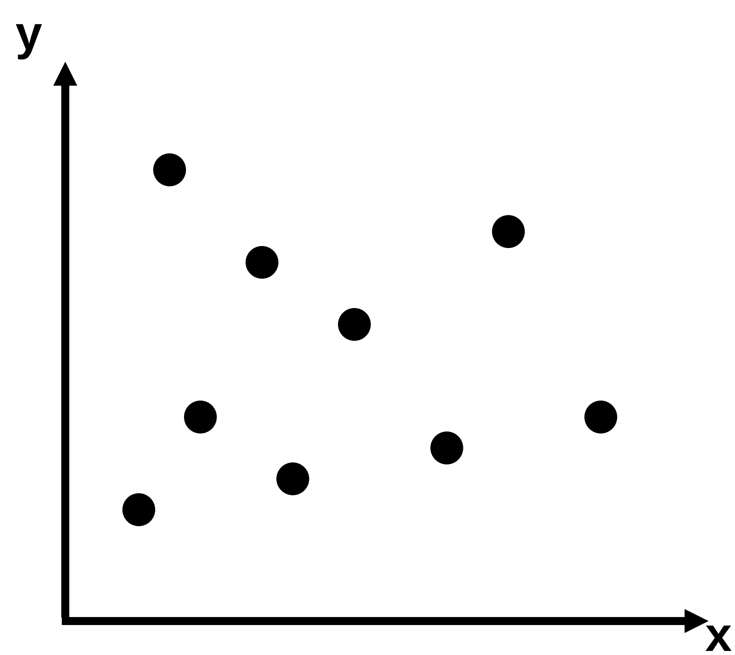
Negative Correlation

In general, a relationship where the dependent (*y*) values decrease as independent (*x*) values increase.

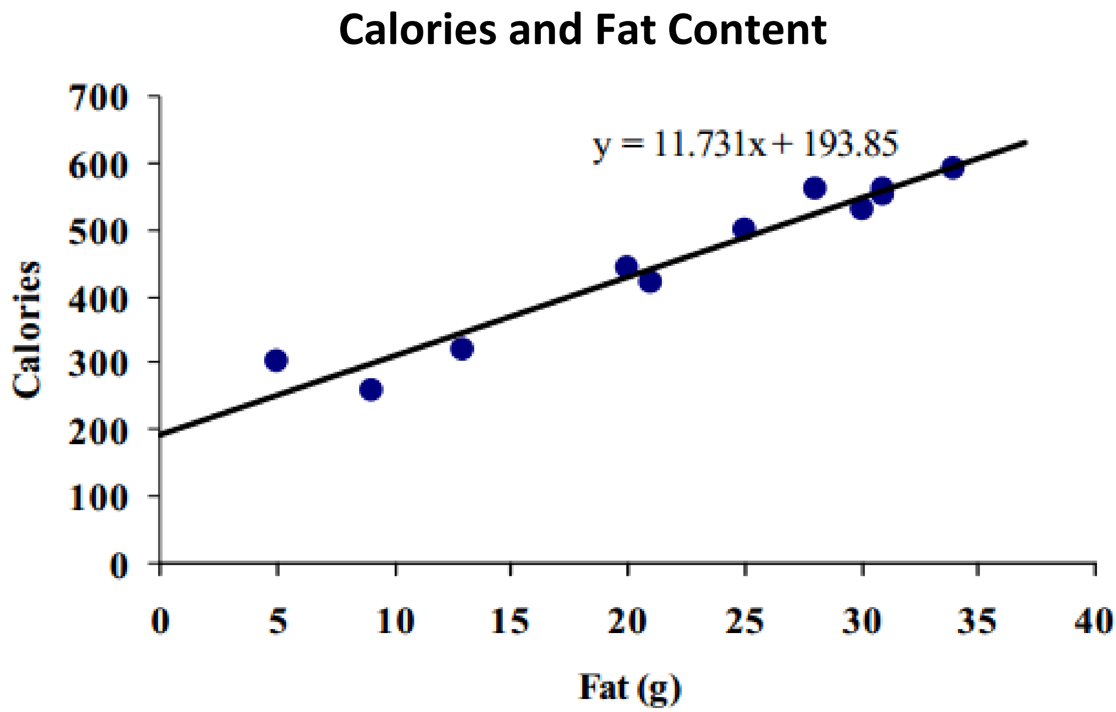


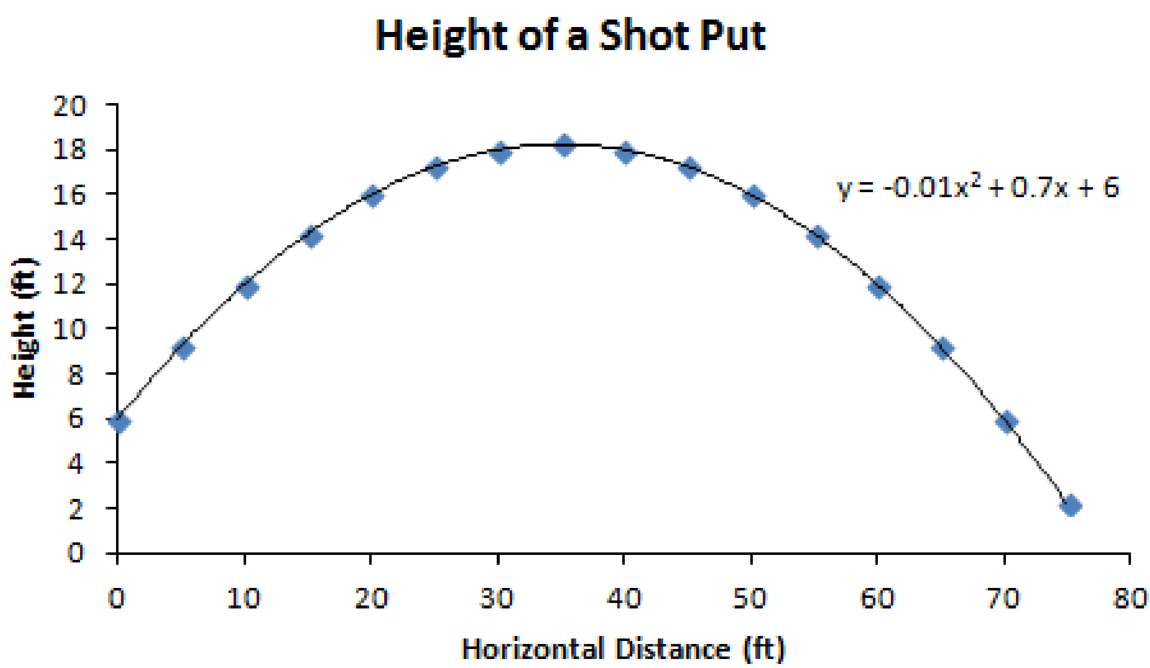
No Correlation

No relationship between the dependent (*y*) values and independent (*x*) values.

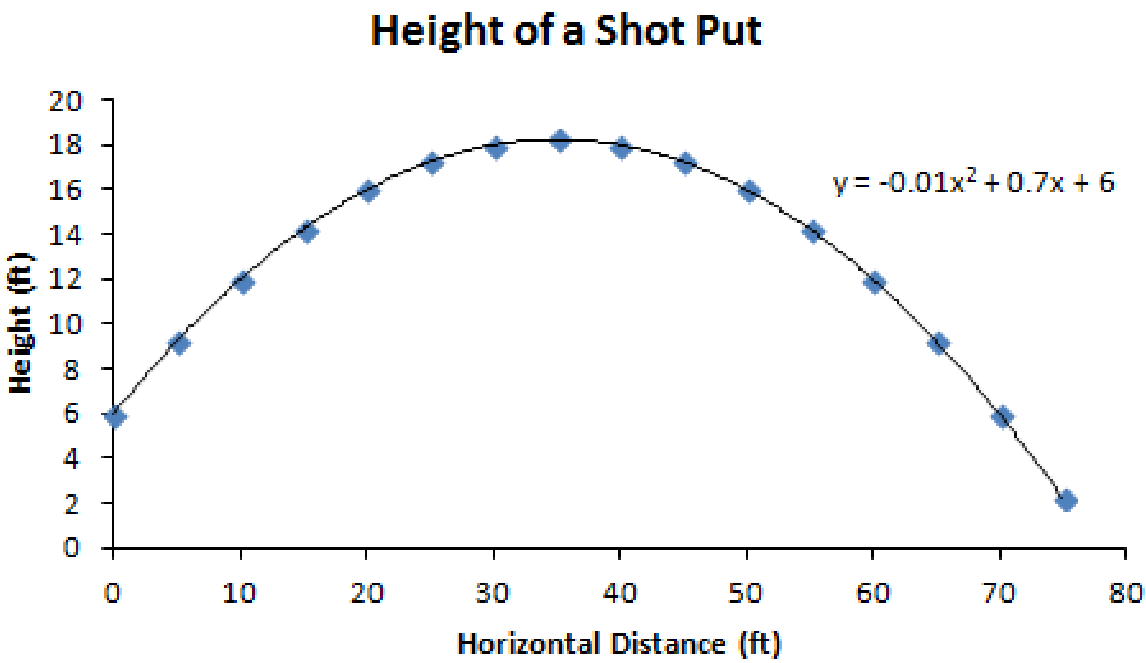


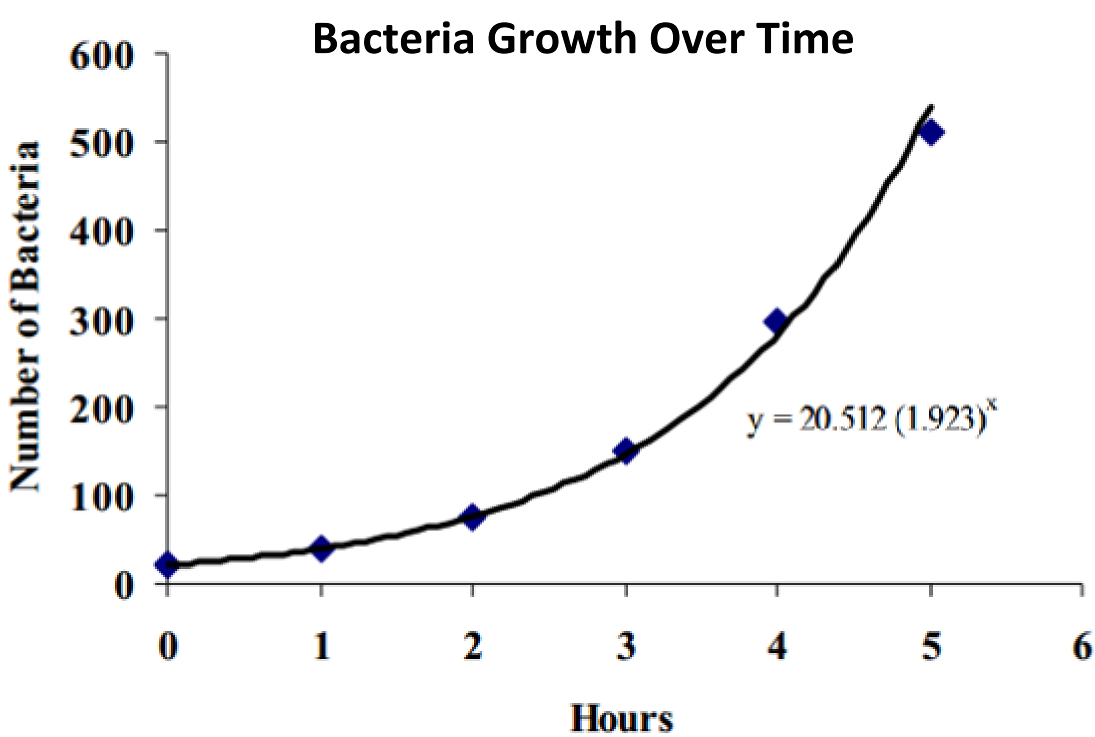
Curve of Best Fit





Curve of Best Fit





Outlier Data