

2023 AP Calculus BC Summer Assignment

Dear AP Calculus BC Students,

Welcome to AP Calculus BC! I look forward to meeting you in August and teaching you this upcoming school year. At least 60% of the topics we will cover are built on the skills you learned and used in AP Calculus AB. Therefore, it is imperative for you to know the basics. In this packet, you will see that the work revolves around some of the topics you learned in AP Calculus AB or dual enrollment Calculus 1.

You may use your TI-84 Graphing Calculator for the problems that state “technology permitted”. With that being said, a TI-84 Graphing Calculator is a requirement for this class.

My recommendation is that you look over this packet when you receive it and that you strategically pace yourself in completing it over the summer. Part 1 of this packet is a review of the skills you should have covered in AP Calculus AB or dual enrollment Calculus 1. If you need to review any of these topics, use Part 2. Then, return to Part 1 and complete any topics you were struggling with. Once you have completed Part 2 of the packet, please email me **one attachment** with all the pages of Part 2. I would like to have Part 2 of this packet by Thursday, August 3, 2023, which is about a week prior to starting school. This date will allow me to assess your AP Calculus AB skill levels and plan my lessons for the first few weeks of school.

The work for Chapters 1-4 and pages 7-8, 11-17 from this packet will be collected on the first day of school, Thursday, August 10, 2023. Both parts will be recorded in SIS as (large) homework assignments.

If you scored a 3 or higher on your AP Calculus AB Exam, Congratulations! If you didn't, you can still earn AP Calculus AB credit using your AP Calculus BC exam with a subscore. I will talk more about this on the first day of school.

If you have any questions or concerns, please contact me as soon as possible.

Sincerely,

Mrs. Littles

Carly.Littles@palmbeachschools.org

PART 1: **DUE: Thursday, August 10, 2023 (First day of school)**

AP CALCULUS AB – Review Problems & Unit Circle

Please complete each chapter's problems on a separate sheet of paper. Be sure to write neatly, number each problem and show your work. Only do problems listed below for each chapter, unless you need additional practice. If you need to review any of these topics while working on Part 1, use Part 2 in the packet.

| Chapter | Book Title | Problems |
|----------------|---------------------------------|-----------------|
| 1 | Limits and Their Properties | #1-10 |
| 2 | Differentiation | #1-12 |
| 3 | Applications of Differentiation | #1-7 |
| 4 | Integration | #1-8 |

What You Need to Know . . .

- The AP[®] Exam, especially the free-response section, stresses the major applications of calculus, rather than the foundational limit concept.
- You are not required to use the formal epsilon-delta definition of a limit on the AP[®] Exam.
- Algebraic limit evaluation methods (Section 1.3) are not explicitly tested on the free-response section of the AP[®] Exam. They are, however, helpful on some multiple-choice questions.
- You should be able to apply the Intermediate Value Theorem, whether the function is presented as an equation or by a table.
- You should be able to find both vertical and horizontal asymptotes given a function or a graph.
- The AP[®] Exam frequently uses limits at infinity as a way of describing horizontal asymptotes.

Practice Questions

Section 1, Part A, Multiple Choice, No Technology

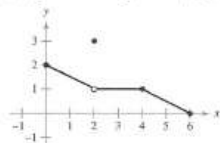
1. What is the limit of $h(x) = 2$ as x approaches π ?
(A) 0 (B) 2
(C) π (D) nonexistent
2. What is the limit of $g(x) = \frac{|x+4|}{x+4}$ as x approaches -4 ?
(A) -4 (B) 0
(C) 1 (D) nonexistent
3. $\lim_{x \rightarrow \pi} \frac{\sin x}{x}$ is
(A) 0. (B) 1.
(C) π . (D) nonexistent.
4. $\lim_{x \rightarrow 2} \frac{3x^2 + 5x + 7}{x - 4}$ is
(A) $-\frac{5}{2}$ (B) $-\frac{3}{2}$
(C) -1 . (D) 3.
5. If $\lim_{x \rightarrow 3} f(x) = 10$ and $\lim_{x \rightarrow 3} g(x) = 1$, what is the limit of $\lim_{x \rightarrow 3} [5f(x) - g(x)]$?
(A) 9 (B) 15
(C) 45 (D) 49
6. $\lim_{x \rightarrow \infty} \frac{4x^3 + 3x^2 - x + 5}{6x^4 + x^3 - x^2 + 4x - 1}$ is
(A) -5 . (B) 0.
(C) $\frac{2}{3}$ (D) nonexistent.

7. What are the horizontal asymptotes of the graph of

$$y = \frac{3 + 4^x}{1 - 4^x}$$

- (A) $y = -1$ only (B) $y = 3$ only
(C) $y = -1$ and $y = 3$ (D) $y = 0$ and $y = 3$

8.



The graph of function g is shown above. Which of the following is true?

- I. $\lim_{x \rightarrow 2} g(x) = 1$
 II. $\lim_{x \rightarrow 2} g(x) = g(2)$
 III. g is continuous at $x = 3$.
 (A) I only (B) I and III only
 (C) III only (D) I, II, and III

Section 1, Part B, Multiple Choice, Technology Permitted

9. $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x}-1}$ is
(A) 0. (B) 1.
(C) 2. (D) nonexistent.
10. $\lim_{x \rightarrow \infty} \frac{x^3}{\sqrt{x^2+4}}$ is
(A) $-\infty$. (B) 0.
(C) 4. (D) ∞ .

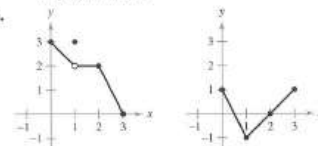
Section 2, Part A, Free Response, Technology Permitted

11. The function f is defined as $f(x) = \frac{10}{1 + \frac{1}{4}e^{-x}}$.
(a) Find $\lim_{x \rightarrow \infty} f(x)$.
(b) Find $\lim_{x \rightarrow 0} (f(x) + 4)$.
(c) State the equation(s) for the horizontal asymptote(s) of the graph of $y = f(x)$. Show the work that leads to your answer.

Section 2, Part B, Free Response, No Technology

12. The function f is defined as $f(x) = \frac{x^2 + 5x + 6}{2x^2 + 7x + 3}$.
(a) State the value(s) of x for which f is not continuous.
(b) Evaluate $\lim_{x \rightarrow -1} f(x)$. Justify your answer.
(c) State the equation(s) for the vertical asymptote(s) of the graph of $y = f(x)$.
(d) State the equation(s) for the horizontal asymptote(s) of the graph of $y = f(x)$. Show the work that leads to your answer.

13.



Graph of f

Graph of g

The graphs of functions f and g are shown above. Evaluate each limit using the graphs provided. Show the computations that lead to your answer.

- $\lim_{x \rightarrow 1} [f(x) + 4]$
 - $\lim_{x \rightarrow 3} \frac{5}{g(x)}$
 - $\lim_{x \rightarrow 2} [f(x) \cdot g(x)]$
 - $\lim_{x \rightarrow 2} \frac{f(x)}{g(x) - 1}$ (Assume that f and g are linear on the interval $[2, 3]$)
14. Let f be a function defined by $f(x) = \begin{cases} e^{2x}, & x \leq 0 \\ 4 - 3 \cos x, & x > 0 \end{cases}$
- Find $\lim_{x \rightarrow -1} f(x)$.
 - Show that f is continuous at $x = 0$.
 - Find $\lim_{x \rightarrow -\infty} f(x)$.

15. A hot cup of tea is placed on a counter and left to cool. The temperature (in degrees Fahrenheit) of the tea x minutes after the cup is placed on the counter is modeled by a continuous function $T(x)$ for $0 \leq x < 10$. Values of $T(x)$ at various times x are shown in the table.

| x | 0 | 3 | 4 | 6 | 8 | 9 |
|--------|------|------|------|------|------|------|
| $T(x)$ | 180° | 174° | 172° | 168° | 164° | 162° |

- Find $\lim_{x \rightarrow 4} T(x)$. Justify your answer.
 - Use the data to find the average rate of change in the temperature of the tea for $3 \leq x \leq 8$. Include units in your final answer.
 - Use the data to identify the shortest interval during which there must exist a time x for which the temperature of the tea is 166.5°. Justify your answer.
 - Use the data to find the best estimate of the slope of the line tangent to the graph of T at $x = 8$.
16. The position function $s(t) = -4.9t^2 + 396.9$ gives the height (in meters) of an object that has fallen from a height of 396.9 meters after t seconds.
- Explain why there must exist a time t , $1 < t < 2$, at which the height of the object must be 382 meters above the ground.
 - After how many seconds does the object hit the ground?
 - Find the average rate of change in s over the interval $[8, 9]$. Include units of measure. Explain why this is a good estimate of the velocity at which the object hits the ground. How can this estimate be improved?
 - Find

$$\lim_{t \rightarrow 3} \frac{s(t) - s(3)}{t - 3}$$

Show the work that leads to your answer. Include units.

17. Let a and b represent real numbers. Define

$$f(x) = \begin{cases} ax^2 + x - b, & \text{if } x \leq 2 \\ ax + b, & \text{if } 2 < x < 5. \\ 2ax - 7, & \text{if } x \geq 5 \end{cases}$$

- Find the values of a and b such that f is continuous on the entire real number line.
- Evaluate $\lim_{x \rightarrow 5} f(x)$.
- Let $g(x) = \frac{f(x)}{x-1}$. Evaluate $\lim_{x \rightarrow 1} g(x)$.

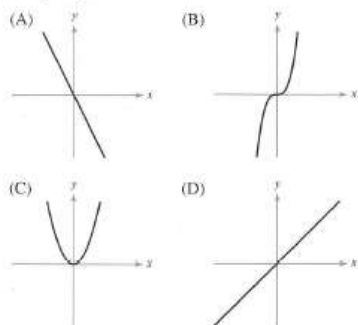
What You Need to Know...

- The definition of the derivative is primarily tested on the multiple-choice section of the AP[®] Exam.
- The AP[®] Exam requires that you have proficiency with using a function's equation, table of values, or graph when finding the average velocity or average rate of change.
- Extremely complex examples of the Product Rule, Quotient Rule, and Chain Rule do not typically appear on the AP[®] Exam. The more difficult problems in this chapter will help you master and remember the concepts.
- Although you should know the derivatives of the six trigonometric functions, the derivatives of the sine, cosine, and tangent functions are the most commonly tested.
- Related rate problems make frequent appearances on the AP[®] Exam because they represent a powerful application of implicit derivatives.

Practice Questions

Section 1, Part A, Multiple Choice, No Technology

- What is an equation of the tangent line to the graph of $f(x) = 4e^x - x + 6$ at $(0, 10)$?
 (A) $y = 4x + 10$
 (B) $y = 4x - 10$
 (C) $y = 10x - 4$
 (D) $y = 3x + 10$



- If $y = \frac{6x^4 - 3x^2 + 5x^3}{x^3}$, then $\frac{d^2y}{dx^2} =$
 (A) $6 - 6x$
 (B) 6
 (C) $6x$
 (D) -6

- If $h(x) = [2x - 5]$, which of the following is true?
 (A) h is continuous but is not differentiable at $x = \frac{5}{2}$.
 (B) h is not continuous but is differentiable at $x = \frac{5}{2}$.
 (C) h is continuous and differentiable at $x = \frac{5}{2}$.
 (D) h is neither continuous nor differentiable at $x = \frac{5}{2}$.

- If $f(x) = \frac{\sin x}{x^2}$, then $f'(x) =$
 (A) $\frac{\cos x}{2x}$
 (B) $\frac{x \cos x - 2 \sin x}{x^2}$
 (C) $\frac{x \cos x - 2 \sin x}{x^3}$
 (D) $\frac{\cos x - 2 \sin x}{x^2}$

- If $y = \sqrt[4]{8x + 3}$, then $y' =$
 (A) $\frac{2}{(8x + 3)^{3/4}}$
 (B) $\frac{1}{4(8x + 3)^{3/4}}$
 (C) $\frac{1}{4}(8x + 3)^{3/4}$
 (D) $\frac{8}{(8x + 3)^{3/4}}$

- If $y = 6 \cos 2x$, then $y^{(6)} =$
 (A) $384 \cos 2x$ (B) $-384 \cos 2x$
 (C) $384 \sin 2x$ (D) $-384 \sin 2x$

- The table shows the position $s(t)$ of a particle that moves along a straight line at several times t , where t is measured in seconds and s is measured in meters.

| | | | | |
|--------|-----|-----|------|------|
| t | 2.0 | 2.7 | 3.2 | 3.8 |
| $s(t)$ | 5.2 | 7.8 | 10.6 | 12.2 |

Which of the following best estimates the velocity of the particle at $t = 3$?

- (A) 3.7 m/sec (B) 3.9 m/sec
 (C) 5.6 m/sec (D) 7.8 m/sec

- If $2y^3 - 3xy + x^2 = 4$, then $\frac{dy}{dx} =$

- (A) $-\frac{2x}{6y^2 - 3}$ (B) $\frac{2x - 3y}{3x - 6y^2}$
 (C) $\frac{2x - 3}{6y^2}$ (D) $-\frac{2x}{6y^2 - 3x}$

- The volume of a cylinder with radius r and height h is given by $V = \pi r^2 h$. The radius of the cylinder is increasing at a rate of $1/3$ centimeter per second and the height of the cylinder is increasing at a rate of $1/2$ centimeter per second. At what rate, in cubic centimeters per second, is the volume of the cylinder increasing when its height is 9 centimeters and the radius is 4 centimeters?

- (A) $\frac{4\pi}{3}$ (B) $\frac{8\pi}{3}$ (C) 6π (D) 32π

Section 1, Part B, Multiple Choice, Technology Permitted

- Two roads intersect at right angles. You are standing 25 meters north of the intersection on one of the roads. You are watching a car traveling west at 30 meters per second. At how many meters per second is the car traveling away from you 3 seconds after it passes through the intersection?

- (A) 23.047 (B) 28.906
 (C) 29.032 (D) 30

- The position $s(t)$ of a particle moving along the x -axis at time t is given by $s(t) = -t^3 + 2t^2 + \frac{1}{3}$, where s is measured in meters and t is measured in seconds. At what time is the particle's instantaneous velocity equal to its average velocity on the interval $[0, 4]$?

- (A) 1.097 seconds (B) 2 seconds
 (C) 2.333 seconds (D) 2.431 seconds

Section 2, Part A, Free Response, Technology Permitted

- The function f is defined as $f(x) = 3e^{2x^3}$.
 (a) Find $f'(x)$.
 (b) For what value of x is the slope of the tangent line to the graph of f equal to 2?
 (c) For what value(s) of x does the tangent line to the graph of f intersect the x -axis at the point $(\frac{1}{2}, 0)$?

Section 2, Part B, Free Response, No Technology

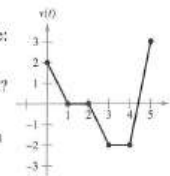
- Evaluate each limit analytically.

- (a) $\lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$ (b) $\lim_{h \rightarrow 0} \frac{\sqrt[3]{x+h} - \sqrt[3]{x}}{h}$
 (c) $\lim_{h \rightarrow 0} \frac{\sqrt{16+h} - 4}{h}$ (d) $\lim_{h \rightarrow 0} \frac{1}{5+h} - \frac{1}{5}$

- Given:

| x | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 2 | -3 | 1 | 5 | -2 |
| 5 | 4 | 7 | -1 | 2 |

- (a) If $h(x) = \frac{f(x)}{g(x)}$, find $h'(2)$.
 (b) If $f(x) = f(g(x))$, find $f'(2)$.
 (c) If $h(x) = \sqrt{f(x)}$, find $h'(5)$.
- The figure below shows the graph of the velocity, in feet per second, for a particle moving along the line $x = 4$.



- (a) During which time interval(s) is the particle:
 (i) moving upward?
 (ii) moving downward?
 (iii) at rest?
 (b) What is the acceleration of the particle at
 (i) $t = 0.75$ and
 (ii) $t = 4.2$? Be sure to include units.
- Given: $g(x) = f(x) \cdot \tan x + kx$, where k is a real number. f is differentiable for all x ; $f(\pi/4) = 4$; $f'(\pi/4) = -2$.
 (a) For what values of x , if any, in the interval $0 < x < 2\pi$ will the derivative of g fail to exist? Justify your answer.
 (b) If $g'(\frac{\pi}{4}) = 6$, find the value of k .

What You Need to Know . . .

- On some free-response questions, there may be more than one way of applying derivatives and theorems to justify your answer.
- Be prepared to apply the Mean Value Theorem. It may be referred to directly, or it may be necessary to use the theorem to justify your answer.
- Questions that involve position, velocity, and acceleration functions are very common on the AP[®] Exam.
- Be prepared to apply the Second Derivative Test to justify whether a point is a local maximum, a local minimum, or a point of inflection. For a point of inflection, make sure to also check for a sign change.
- Tangent line approximations, and whether such an approximation overestimates or underestimates a function value, is commonly tested on the free-response section.

Practice Questions

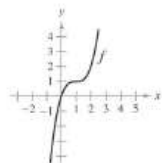
Section 1, Part A, Multiple Choice, No Technology

1. What are the critical numbers of

$$f(x) = 4x^3 + 6x^2 - 72x - 9?$$

- (A) $x = -2$ and $x = 3$
 (B) $x = -3$ and $x = 2$
 (C) $x = -2$
 (D) $x = -3$

2. The graph of the function f is shown. Which of the following is true?



- I. $f'(x) > 0$ on the entire real number line.
 II. $f''(x) < 0$ on the interval $(-\infty, 1)$.
 III. $f''(x) > 0$ on the interval $(1, \infty)$.

- (A) I only
 (B) II and III only
 (C) I and III only
 (D) I, II and III

3. The position of an object along a vertical line is given by $s(t) = -t^3 + 3t^2 + 9t + 5$, where s is measured in feet and t is measured in seconds. The maximum velocity of the object in the time interval $[0, 4]$ is

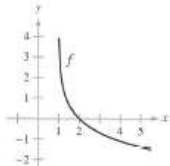
- (A) 9 feet per second.
 (B) 12 feet per second.
 (C) 16 feet per second.
 (D) 32 feet per second.

4. The function g is continuous and differentiable on the interval $[2, 6]$. The table shows selected values of g on $[2, 6]$. Which of the following statements must be true?

| | | | | | |
|--------|---|---|---|---|---|
| x | 2 | 3 | 4 | 5 | 6 |
| $g(x)$ | 7 | 4 | 1 | 4 | 7 |

- (A) The minimum value of g on $[2, 6]$ is 1.
 (B) The maximum value of g on $[2, 6]$ is 7.
 (C) There exists a number c , with $2 < c < 6$, for which $g'(c) = 0$.
 (D) $g'(x) < 0$ for $2 < x < 4$

5. Consider the graph of $y = f(x)$ shown below. If f is a function such that f' and f'' are defined in a region around $x = 2$, then which of the following must be true?



- (A) $f''(2) < f(2)$
 (B) $f''(2) < f'(2)$
 (C) $f(2) = f'(2)$
 (D) $f''(2) > f(2)$

6. If $y = \arctan 4x$, then $dy =$

- (A) $\frac{4}{1 + 16x^2} dx$.
 (B) $\frac{4x}{1 + 16x^2} dx$.
 (C) $-\frac{4x}{1 + 16x^2} dx$.
 (D) $-\frac{4}{1 + 16x^2} dx$.

Section 1, Part B, Multiple Choice, Technology Permitted

7. If the Mean Value Theorem is applied to the function $f(x) = \ln(x - 3)$ on the interval $[4, 8]$, then the number c that must exist in $(4, 8)$ is
 (A) 5.485.
 (B) 5.885.
 (C) 6.
 (D) 6.368.

Section 2, Part A, Free Response, Technology Permitted

8. The table below shows the behavior of a function f that is continuous on the entire real number line. For the function, $f(2) = 4$, and $\lim_{x \rightarrow \infty} f(x) = 0$.

| | | | |
|----------|----------|----------------|----------|
| | $x < 4$ | $x = 4$ | $x > 4$ |
| $f'(x)$ | positive | does not exist | negative |
| $f''(x)$ | negative | does not exist | positive |

- (a) For what values of x is f increasing?
 (b) Does f have a relative maximum at $x = 4$? Explain.
 (c) If possible, name the x -coordinate of the point of inflection on the graph of f . Justify your answer.
 (d) Does the Mean Value Theorem apply over the interval $[3, 5]$? Justify your answer.
 (e) Sketch a possible graph of f .

9. Consider the function $f(x) = \frac{x^3}{2} - \sin x + 1$.

- (a) Approximate the relative extrema of f .
 (b) Find the tangent line approximation of f at $\frac{\pi}{2}$.
 (c) Use your tangent line approximation to approximate the value of $f(1.5)$. Is your approximation an underestimate or an overestimate of the actual value of $f(1.5)$? Justify your answer.

Section 2, Part B, Free Response, No Technology

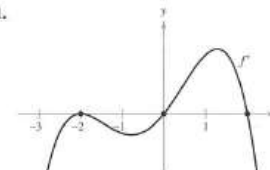
10. Consider the function

$$f(x) = 2x + \cos 2x$$

on the interval $[0, \pi]$.

- (a) Find the maximum value of f . Justify your answer.
 (b) Explain how the conditions of the Mean Value Theorem are satisfied by f for $0 \leq x \leq \pi$. Find the value of x whose existence is guaranteed by the Mean Value Theorem.

- 11.



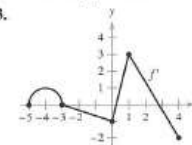
The figure above shows the graph of f' , the derivative of f . The function f is a twice differentiable function on $x \in (-\infty, \infty)$, $f''(-0.8) = 0$, and $f''(1.3) = 0$.

- (a) For what values of x is f increasing?
 (b) For what values of x is the graph of f concave downward? Justify your answer.
 (c) Is $\frac{f(-0.5) - f(0)}{-0.5 - 0}$ positive or negative? Justify your answer.

12. Consider the function $f(x) = \frac{1 - 4x^2}{x}$.

- (a) For what values of x is f decreasing?
 (b) For what values of x is the graph of f concave downward? Justify your answer.
 (c) Does the graph of f have any points of inflection? Justify your answer.

- 13.



The figure above shows the graph of f' , the derivative of f , on the interval $[-5, 4]$. The function f is differentiable on the interval and $f''(-4) = 0$.

- (a) Find $f'(-1)$ and $f''(-1)$.
 (b) At which x -values does f have a relative extrema on the interval $(-5, 0)$? Justify your answer.
 (c) Find all intervals on which the graph of f is concave downward. Explain your reasoning.
 (d) Find the x -coordinate of each of the points of inflection of the graph of f . Justify your answer.
 (e) If $g(x) = f(x) + \sin^2 x$, is g increasing or decreasing at $x = -\pi/4$? Justify your answer.

What You Need to Know . . .

- Approximating the area under a curve using rectangles and basic geometry is often tested on the AP[®] Exam.
- Be prepared to approximate definite integrals using left, right, or midpoint Riemann sums, or trapezoidal sums.
- An alternative form of the Fundamental Theorem of Calculus, $f(b) = f(a) + \int_a^b f'(x) dx$, is also emphasized on the AP[®] Exam.
- Some questions where technology is permitted not only encourage but also require the use of a graphing utility in evaluating definite integrals. This is the case for functions with no elementary antiderivative.

Practice Questions

Section 1, Part A, Multiple Choice, No Technology

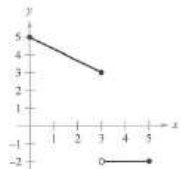
1. The table shows selected values for a continuous function g that is increasing over the interval $[0, 4]$. Which of the following could be the value of $\int_0^4 g(x) dx$?

| | | | | | | | | | |
|--------|---|---------------|---|---------------|----|---------------|----|---------------|----|
| x | 0 | $\frac{1}{2}$ | 1 | $\frac{3}{2}$ | 2 | $\frac{5}{2}$ | 3 | $\frac{7}{2}$ | 4 |
| $g(x)$ | 0 | 3 | 7 | 12 | 18 | 25 | 33 | 42 | 52 |

- (A) 70 (B) 80
(C) 96 (D) 100

2. The graph of f is shown for $0 \leq x \leq 5$. What is the value of $\int_0^3 f(x) dx$?

- (A) -1
(B) 7
(C) 8
(D) 16



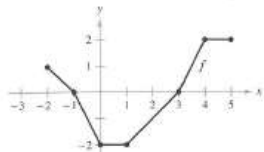
3. $\int \frac{4}{(x-5)^2 + 9} dx =$

- (A) $\frac{4}{3} \tan^{-1} \frac{x-5}{3} + C$ (B) $4 \tan^{-1} \frac{x-5}{3} + C$
(C) $\tan^{-1} \frac{x-5}{3} + C$ (D) $\frac{1}{3} \tan^{-1} \frac{x-5}{3} + C$

4. The velocity of a particle is given by $v(t) = 4t^3 - 4t$ for the times $0 \leq t \leq 2$ in seconds. What is the average velocity of the particle over that interval?

- (A) 4 (B) 5
(C) 10 (D) 24

5.



The graph of a piecewise linear function f is shown above. If g is the function defined by

$$g(x) = \int_a^x f(t) dt$$

find $g(-1)$.

- (A) -6 (B) -4
(C) 4 (D) 6

Section 1, Part B, Multiple Choice, Technology Permitted

6. If $0 \leq b \leq \pi$, and the area under the curve $y = \sin x$ from $x = b$ to $x = \pi$ is 0.4, what is the value of b ?

- (A) 0.927 (B) 1.159
(C) 1.982 (D) 2.214

7. Let $f(x)$ be a continuous function such that $f(1) = 2$ and $f'(x) = \sqrt{x^3 + 6}$. What is the value of $f(5)$?

- (A) 11.446 (B) 13.446
(C) 24.672 (D) 26.672

8. What is the average value of the function $y = x + \sin x$ on the interval

$$\left[0, \frac{3\pi}{2}\right]?$$

- (A) 2.144 (B) 2.356 (C) 2.568 (D) 2.781

Section 2, Part A, Free Response, Technology Permitted

9. As a pot of coffee cools down, the temperature of the coffee is modeled by a differentiable function C , for $0 \leq t \leq 12$, where time t is measured in minutes and the temperature $C(t)$ is measured in degrees Celsius. Selected values of t are shown in the table.

| | | | | | | |
|--------------------------|----|----|----|----|----|----|
| t (minutes) | 0 | 3 | 5 | 7 | 8 | 12 |
| $C(t)$ (degrees Celsius) | 65 | 57 | 50 | 46 | 44 | 40 |

- (a) Evaluate $\int_0^{12} C'(t) dt$. Explain the meaning of your answer in the context of the problem. Indicate units of measure.
(b) Explain the meaning of $\frac{1}{12} \int_0^{12} C(t) dt$ in the context of the problem. Use a trapezoidal sum with 5 subintervals indicated by the table to approximate $\frac{1}{12} \int_0^{12} C(t) dt$. Indicate units of measure.
(c) Use the data in the table to approximate the rate at which the temperature is changing at time $t = 4$. Show the work that leads to your answer.
(d) For $12 \leq t \leq 15$, the rate of cooling is modeled by $C'(t) = -2 \cos(0.5t)$.

Based on the model, what is the temperature of the coffee when $t = 15$? Assume $C(t)$ is continuous at $t = 12$.

10. On a typical day, the snow on a mountain melts at a rate modeled by the function

$$M(t) = \frac{\pi}{6} \sin \frac{\pi t}{12}$$

A snow maker adds snow at a rate modeled by the function

$$S(t) = 0.006t^2 - 0.12t + 0.87.$$

Both M and S have units in inches per hour and t is measured in hours for $0 \leq t \leq 6$. At $t = 0$, the mountain has 40 inches of snow.

- (a) How much snow will melt during the 6 hour period? Indicate units of measure.
(b) Write an expression for $R(t)$, the total number of inches of snow at any time t .
(c) Find the rate of change of the total amount of snow when $t = 3$.
(d) For $0 \leq t \leq 6$, at what time t is the amount of snow a maximum? What is the maximum value? Justify your answers.

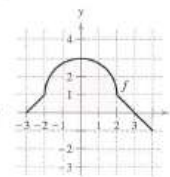
Section 2, Part B, Free Response, No Technology

11. For $0 \leq t \leq 9$, a particle moves along the x -axis. The velocity of the particle is given by $v(t) = \sin(\pi t/4)$. The particle is at position $x = -4$ when $t = 0$.

- (a) For $0 \leq t \leq 9$, when is the particle moving to the right? Justify your answer.
(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to $t = 9$.
(c) Find the acceleration of the particle at time $t = 3$. Is the particle speeding up, slowing down, or neither at $t = 3$? Justify your answer.
(d) Find the position of the particle at time $t = 3$.

12. Let

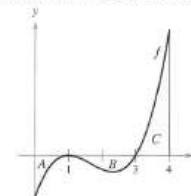
$$F(x) = \int_3^x f(t) dt.$$



The graph of f on the interval $[-3, 4]$ consists of two line segments and a semicircle, as shown in the graph.

- (a) Find $F(0)$, $F'(0)$, and $F(4)$.
(b) Find all relative minimum values of $F(x)$ on the interval $[-3, 4]$. Justify your answer.
(c) Find the x -coordinates of the inflection points of $F(x)$ on the interval $[-3, 4]$. Justify your answer.
(d) Write the equation of the line tangent to the point where $x = 2$.

13. The graph of a continuous function f is shown. The three regions between the graph of f and the x -axis are marked A, B, and C, and have unsigned areas 5.5, 6, and 15.5, respectively. Let $F(x)$ be an antiderivative of f that is differentiable on $(0, 4)$ and with $F(1) = 9$.

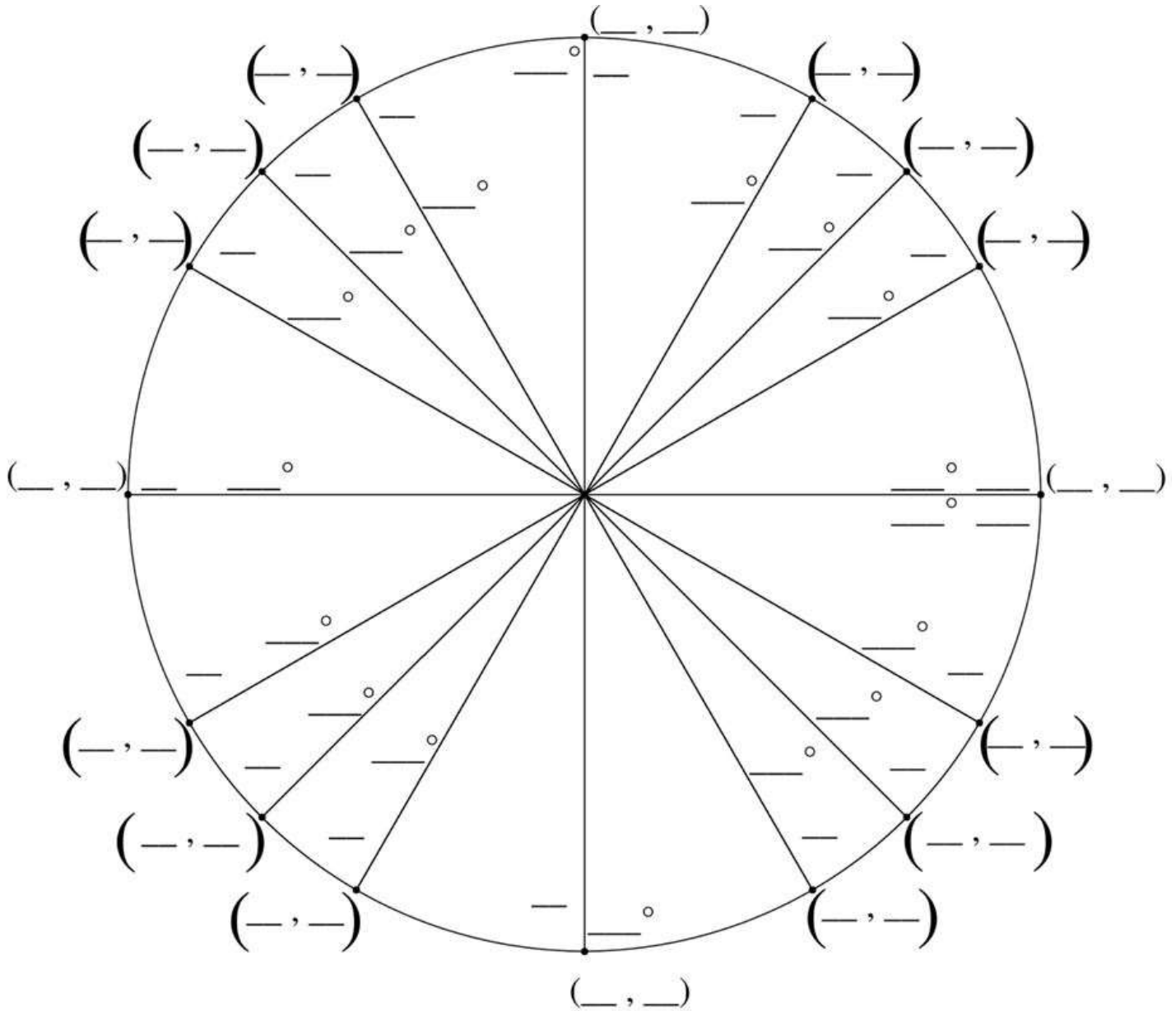


- (a) Find $F(0)$ and $F(4)$.
(b) What is the minimum number of times F equals 5 on the interval $[0, 4]$? Show the work that leads to your answer.
(c) Find all intervals where F is increasing. Justify your answer.

This portion of the summer assignment is all about your basic mathematical skills from your time in AICE Mathematics and/or Precalculus. The skills from these courses are the backbone of the problems we will be working on throughout the school year.

UNIT CIRCLE

Complete the unit circle. No calculators!



BASIC TRIGONOMETRY

Complete the table. Simplify your answers. No calculators!

| θ | $\sin \theta$ | $\cos \theta$ | $\tan \theta$ | $\csc \theta$ | $\sec \theta$ | $\cot \theta$ |
|------------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $-\frac{\pi}{2}$ | | | | | | |
| $-\frac{\pi}{3}$ | | | | | | |
| $-\frac{\pi}{4}$ | | | | | | |
| $-\frac{\pi}{6}$ | | | | | | |
| 0 | | | | | | |
| $\frac{\pi}{6}$ | | | | | | |
| $\frac{\pi}{4}$ | | | | | | |
| $\frac{\pi}{3}$ | | | | | | |
| π | | | | | | |
| $\frac{2\pi}{3}$ | | | | | | |
| $\frac{3\pi}{4}$ | | | | | | |
| $\frac{5\pi}{6}$ | | | | | | |
| π | | | | | | |

PART 2: DUE: As soon as possible, via email. Thursday, August 3, 2023 at the latest.

This portion of the summer assignment is all about your time in AP Calculus AB. Since the skills you used in that course are a large portion of AP Calculus BC, it is imperative that you review certain topics. Instead of me assigning particular topics to you for review, I want you to take a look at the Eight Units listed below (that were covered in AP Calculus AB) and the topics within them to determine which are **not** your strongest. This means that I want you to spend your time for this part to review the topics that you struggled with or need a refresher lesson. You will need to choose at least 5 topics, each from a different unit. Once you have chosen your topics, you can either use AP Daily on AP Central or YouTube to view the “AP Daily 2023” video for that particular topic. You should watch the entire video, taking notes, to fill in any learning gaps on that topic that you might have. To help me sort your lesson notes, please make sure you start each topic on a new sheet of paper, which is provided on the following pages. I have added an additional page (labeled optional) if you do more than 5 topics.

| UNIT 1 | Limits and Continuity | UNIT 2 | Differentiation: Definition and Basic Derivative Rules | UNIT 3 | Differentiation: Composite, Implicit, and Inverse Functions | UNIT 4 | Contextual Applications of Differentiation |
|-------------------|--|-------------------|---|-------------------|---|-------------------|--|
| AP EXAM WEIGHTING | 10–12% AB 4–7% BC | AP EXAM WEIGHTING | 10–12% AB 4–7% BC | AP EXAM WEIGHTING | 9–13% AB 4–7% BC | AP EXAM WEIGHTING | 10–15% AB 6–9% BC |
| CLASS PERIODS | ~22–23 AB ~13–14 BC | CLASS PERIODS | ~13–14 AB ~9–10 BC | CLASS PERIODS | ~10–11 AB ~8–9 BC | CLASS PERIODS | ~10–11 AB ~6–7 BC |
| CHA 2 | 1.1 Introducing Calculus: Can Change Occur at an Instant? | CHA 2 | 2.1 Defining Average and Instantaneous Rates of Change at a Point | FUN 1 | 3.1 The Chain Rule | CHA 1 | 4.1 Interpreting the Meaning of the Derivative in Context |
| LIM 2 | 1.2 Defining Limits and Using Limit Notation | CHA 1 | 2.2 Defining the Derivative of a Function and Using Derivative Notation | FUN 1 | 3.2 Implicit Differentiation | CHA 1 | 4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration |
| LIM 2 | 1.3 Estimating Limit Values from Graphs | CHA 4 | 2.3 Estimating Derivatives of a Function at a Point | FUN 3 | 3.3 Differentiating Inverse Functions | CHA 2 | 4.3 Rates of Change in Applied Contexts Other Than Motion |
| LIM 2 | 1.4 Estimating Limit Values from Tables | CHA 1 | 2.4 Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist | FUN 1 | 3.4 Differentiating Inverse Trigonometric Functions | CHA 1 | 4.4 Introduction to Related Rates |
| LIM 1 | 1.5 Determining Limits Using Algebraic Properties of Limits | FUN 3 | 2.5 Applying the Power Rule | FUN 1 | 3.5 Selecting Procedures for Calculating Derivatives | CHA 3 | 4.5 Solving Related Rates Problems |
| LIM 1 | 1.6 Determining Limits Using Algebraic Manipulation | FUN 1 | 2.6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple | FUN 1 | 3.6 Calculating Higher-Order Derivatives | CHA 1 | 4.6 Approximating Values of a Function Using Local Linearity and Linearization |
| LIM 1 | 1.7 Selecting Procedures for Determining Limits | FUN 1 | 2.7 Derivatives of $\cos x$, $\sin x$, e^x , and $\ln x$ | LIM 3 | | LIM 3 | 4.7 Using L'Hospital's Rule for Determining Limits of Indeterminate Forms |
| LIM 3 | 1.8 Determining Limits Using the Squeeze Theorem | FUN 1 | 2.8 The Product Rule | | | | |
| LIM 2 | 1.9 Connecting Multiple Representations of Limits | FUN 1 | 2.9 The Quotient Rule | | | | |
| LIM 3 | 1.10 Exploring Types of Discontinuities | FUN 1 | 2.10 Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions | | | | |
| LIM 3 | 1.11 Defining Continuity at a Point | | | | | | |
| LIM 1 | 1.12 Confirming Continuity over an Interval | | | | | | |
| LIM 1 | 1.13 Removing Discontinuities | | | | | | |
| LIM 3 | 1.14 Connecting Infinite Limits and Vertical Asymptotes | | | | | | |
| LIM 2 | 1.15 Connecting Limits at Infinity and Horizontal Asymptotes | | | | | | |
| FUN 3 | 1.16 Working with the Intermediate Value Theorem (IVT) | | | | | | |

MATHEMATICAL PRACTICES
Mathematical practices spiral throughout the course.

| | |
|---------------------------------------|------------------------------|
| 1 Implementing Mathematical Processes | 3 Justification |
| 2 Connecting Representations | 4 Communication and Notation |

BIG IDEAS
Big ideas spiral across topics and units.

| | |
|------------|---------------------------|
| CHA Change | FUN Analysis of Functions |
| LIM Limits | |

BC ONLY
The purple shading represents BC only content.

UNIT 5

Analytical Applications of Differentiation

AP EXAM WEIGHTING **15–18% AB 8–11% BC**

CLASS PERIODS **~15–16 AB ~10–11 BC**

| | |
|-----------------|--|
| FUN 3 | 5.1 Using the Mean Value Theorem |
| FUN 3 | 5.2 Extreme Value Theorem, Global Versus Local Extrema, and Critical Points |
| FUN 2 | 5.3 Determining Intervals on Which a Function Is Increasing or Decreasing |
| FUN 3 | 5.4 Using the First Derivative Test to Determine Relative (Local) Extrema |
| FUN 1 | 5.5 Using the Candidates Test to Determine Absolute (Global) Extrema |
| FUN 2 | 5.6 Determining Concavity of Functions over Their Domains |
| FUN 3 | 5.7 Using the Second Derivative Test to Determine Extrema |
| FUN 2 | 5.8 Sketching Graphs of Functions and Their Derivatives |
| FUN 2 | 5.9 Connecting a Function, Its First Derivative, and Its Second Derivative |
| FUN 2 | 5.10 Introduction to Optimization Problems |
| FUN 3 | 5.11 Solving Optimization Problems |
| FUN 1 | 5.12 Exploring Behaviors of Implicit Relations |

UNIT 6

Integration and Accumulation of Change

AP EXAM WEIGHTING **17–20% AB 17–20% BC**

CLASS PERIODS **~18–20 AB ~15–16 BC**

| | |
|-----------------|---|
| CHA 4 | 6.1 Exploring Accumulations of Change |
| LIM 1 | 6.2 Approximating Areas with Riemann Sums |
| LIM 2 | 6.3 Riemann Sums, Summation Notation, and Definite Integral Notation |
| FUN 1 | 6.4 The Fundamental Theorem of Calculus and Accumulation Functions |
| FUN 2 | 6.5 Interpreting the Behavior of Accumulation Functions Involving Area |
| FUN 3 | 6.6 Applying Properties of Definite Integrals |
| FUN 3 | 6.7 The Fundamental Theorem of Calculus and Definite Integrals |
| FUN 4 | 6.8 Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation |
| FUN 1 | 6.9 Integrating Using Substitution |
| FUN 1 | 6.10 Integrating Functions Using Long Division and Completing the Square |
| FUN 1 | 6.11 Integrating Using Integration by Parts BC ONLY |
| FUN 1 | 6.12 Using Linear Partial Fractions BC ONLY |
| LIM 1 | 6.13 Evaluating Improper Integrals BC ONLY |
| FUN 1 | 6.14 Selecting Techniques for Antidifferentiation |

UNIT 7

Differential Equations

AP EXAM WEIGHTING **6–12% AB 6–9% BC**

CLASS PERIODS **~8–9 AB ~9–10 BC**

| | |
|-----------------|--|
| FUN 2 | 7.1 Modeling Situations with Differential Equations |
| FUN 3 | 7.2 Verifying Solutions for Differential Equations |
| FUN 2 | 7.3 Sketching Slope Fields |
| FUN 4 | 7.4 Reasoning Using Slope Fields |
| FUN 1 | 7.5 Approximating Solutions Using Euler's Method BC ONLY |
| FUN 1 | 7.6 Finding General Solutions Using Separation of Variables |
| FUN 1 | 7.7 Finding Particular Solutions Using Initial Conditions and Separation of Variables |
| FUN 3 | 7.8 Exponential Models with Differential Equations |
| FUN 3 | 7.9 Logistic Models with Differential Equations BC ONLY |

UNIT 8

Applications of Integration

AP EXAM WEIGHTING **10–15% AB 6–9% BC**

CLASS PERIODS **~19–20 AB ~13–14 BC**

| | |
|-----------------|---|
| CHA 1 | 8.1 Finding the Average Value of a Function on an Interval |
| CHA 1 | 8.2 Connecting Position, Velocity, and Acceleration of Functions Using Integrals |
| CHA 3 | 8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts |
| CHA 4 | 8.4 Finding the Area Between Curves Expressed as Functions of x |
| CHA 1 | 8.5 Finding the Area Between Curves Expressed as Functions of y |
| CHA 2 | 8.6 Finding the Area Between Curves That Intersect at More Than Two Points |
| CHA 3 | 8.7 Volumes with Cross Sections: Squares and Rectangles |
| CHA 3 | 8.8 Volumes with Cross Sections: Triangles and Semicircles |
| CHA 3 | 8.9 Volume with Disc Method: Revolving Around the x - or y -Axis |
| CHA 2 | 8.10 Volume with Disc Method: Revolving Around Other Axes |
| CHA 4 | 8.11 Volume with Washer Method: Revolving Around the x - or y -Axis |
| CHA 2 | 8.12 Volume with Washer Method: Revolving Around Other Axes |
| CHA 3 | 8.13 The Arc Length of a Smooth, Planar Curve and Distance Traveled BC ONLY |

MATHEMATICAL PRACTICES

Mathematical practices spiral throughout the course.

- 1** Implementing Mathematical Processes
- 2** Connecting Representations
- 3** Justification
- 4** Communication and Notation

BIG IDEAS

Big ideas spiral across topics and units.

- CHA** Change
- LIM** Limits
- FUN** Analysis of Functions

BC ONLY

The purple shading represents BC only content.

Print Student Name _____

Part 2: 1st Video/Notes

UNIT# ____ TOPIC # ____ TOPIC TITLE _____

If you need additional space, use loose leaf paper and attach after this page. Be sure to label the Unit, Topic #, and Topic Title at the top with your name.

Print Student Name _____

Part 2: 2nd Video/Notes

UNIT# ____ TOPIC # ____ TOPIC TITLE _____

If you need additional space, use loose leaf paper and attach after this page. Be sure to label the Unit, Topic #, and Topic Title at the top with your name.

Print Student Name _____

Part 2: 3rd Video/Notes

UNIT# ____ TOPIC # ____ TOPIC TITLE _____

If you need additional space, use loose leaf paper and attach after this page. Be sure to label the Unit, Topic #, and Topic Title at the top with your name.

Print Student Name _____

Part 2: 4th Video/Notes

UNIT# ____ TOPIC # ____ TOPIC TITLE _____

If you need additional space, use loose leaf paper and attach after this page. Be sure to label the Unit, Topic #, and Topic Title at the top with your name.

Print Student Name _____

Part 2: 5th Video/Notes

UNIT# ____ TOPIC # ____ TOPIC TITLE _____

If you need additional space, use loose leaf paper and attach after this page. Be sure to label the Unit, Topic #, and Topic Title at the top with your name.

Print Student Name _____ OPTIONAL **Part 2:** ___Video/Notes

UNIT# ___ TOPIC # __.____ TOPIC TITLE _____

If you need additional space, use loose leaf paper and attach after this page. Be sure to label the Unit, Topic #, and Topic Title at the top with your name.

